A Blended Sliding Mode Control with Linear Quadratic Integral Control based on Reduced Order Model for a VTOL System

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Abstract: In this paper, a Sliding Mode Control with chattering reduction based on reduced order model using Linear Quadratic Integral Control as sliding surface, is implemented to One Degree of Freedom Vertical Take-Off Landing System (VTOL). The controller performance is measured using Integral of the Square Error index by simulation and real tests. Finally, the Sliding Mode Control with a Linear Quadratic Integral Control as sliding surface performance for reference tracking and, robustness against VTOL system physical parameter uncertainties and external disturbances are verified by experimental results.

1 INTRODUCTION

The Sliding Mode Control (SMC) is a robust controller that deals with high-order nonlinearities, which has been extensively studied due to its ability to reject disturbances (Han et al., 2016; Nawawi et al., 2011), and low sensitivity to uncertainties in the parameters (Prusty et al., 2016), thus it eliminates the necessity of an accurate model of the system (Sabanovic et al., 2004). An LQI controller is a Linear-Quadratic Regulator (LQR) with integral action. The LQI advantages are: simple implementation (Carrière et al., 2008), best possible performance according to the minimization of an index, with a compromise between the response of the variables and the control effort that guarantees the stability of the system (Mohammadbagheri et al., 2011).

In order to achieve a robust control system with the best performance, the advantages of the SMC and LQI controllers can be blended in an robust-optimal controller. In (Dong et al., 2011), an optimal sliding mode control for nonlinear systems with uncertainties is designed, where system stability is ensured by minimizing a performance index. In (Teimoori et al., 2012), an optimal sliding surface with respect a quadratic performance index is selected for a system where the parameters uncertainties are considered. In (Chithra and Koshy, 2016), an integral action LQR is combined with a robust SMC for a Twin MIMO Rotor system, which is evaluated and compared with PID and LQI controllers using simulations.

In (Zhang et al., 2014a), the combination of a method to decouple the dynamics of the VTOL aircraft system and a SMC is presented. The performance of the designed controller and the tracking process are shown with simulation results. In (Mondal and Mahanta, 2013), a second order sliding mode is presented and a sliding surface is designed by an adaptive gain tuning mechanism for stabilizing a single degree of freedom VTOL system.

In this article a robust-optimal controller based on Sliding Mode Control with integral action (SMC-LQI) is designed and implemented on VTOL system. The sliding surface with an optimal criterion by minimizing of a performance index is chosen. The design of the controller is based on a reduced order model of the VTOL system, which allows that implementation in the real system to be simple. This paper is organized as follows. In Section I, a simplified dynamic model of One Degree of Freedom (1-DOF) VTOL is described. Section II, a SMC controller with reduction of the chattering effect is designed, where the sliding surface is chosen via LQI controller approach. In Section III, the performance of the controller is verified by the simulation results. In Section IV, the implementation of the proposed controller on the real platform for experimental tests are shown. Finally the conclusions are presented in Section V.

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2 SYSTEM DESCRIPTION AND MODEL

The Laboratory set-up QNET VTOL trainer with 1-DOF is shown in Fig.1. The system is composed of a variable speed fan (a) mounted on an arm and an adjustable counterweight (c) in other arm's end. The arm pivots around the rotary encoder (b) shaft, that way the VTOL pitch angle position can be adquired. The VTOL system is compatible with the LabVIEW software using NI-Elvis board (d).

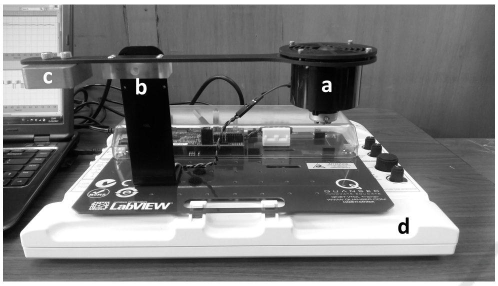

Figure 1: Vertical Take-off landing VTOL system.

In order to reduce the complexity of system, the VTOL can be considered as two subsystems: the motor dynamics of fan and the pitch angle θ dynamics of the VTOL body. Separating into subsystems allows to design two controller loops as it is shown in the Fig.2.

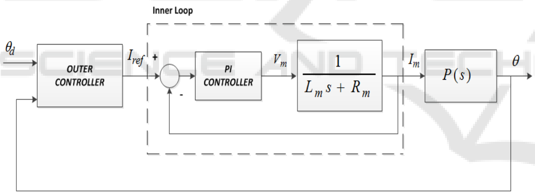

Figure 2: Scheme of two control loops for the VTOL system. (QUANSER, 2011).

The transfer function of motor dynamics is described by:

$$I_m(s) = \frac{V_m(s)}{R_m + L_m s} \tag{1}$$

where $V_m(s)$ is the input voltaje of motor and, $I_m(s)$ is the output current of motor. The PI controller is used as an inner control loop, which regulates the current of motor around a current reference I_{ref}. The PI controller simplifies the outer-loop control (QUANSER, 2011).

The pitch angle motion of the VTOL system with respect to current is given by (QUANSER, 2011)

$$J\ddot{\theta} + B\dot{\theta} + K\theta = k_t I_m \tag{2}$$

Taking the laplace transform, the transfer fuction is obtained:

$$P(s) = \frac{\theta(s)}{I_m(s)} = \frac{k_t}{Js^2 + Bs + K} \tag{3}$$

Where the VTOL system is a second order system and based on this an outer controller is designed. In table 1 the QNET VTOL trainer and PI controler parameteres are shown.

The state-space representation of a linear invariant-time system is given by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{4}$$

By choosing the state variable as $\mathbf{x} = \begin{bmatrix} \theta & \dot{\theta} \end{bmatrix}^T = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, the system inpunt is $u = I_m$ and ouput $y = \theta$. The state-space matrix representation of VTOL body dynamic model can be described as:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\dfrac{K}{J} & -\dfrac{B}{J} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \dfrac{k_t}{J} \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases} \tag{5}$$

Analyzing the system described in (5), the states matrices (A,B) are controllable and, the pair (A,C) are observable. In addition the matriz A is nonsingular therefore, the system eigenvalues are different to zero.

3 CONTROLLER DESIGN

In this section, a SMC with reduction of chattering effect using a LQI control approach as sliding surface is proposed.

3.1 LQI Controller

In order to reduce the steady-state error for a desired reference, integral of the error ε is added to the state feedback control as is illustrated in Fig. 3.

Where $K_r = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$ is the state-feedback control gains vector, k_i is the integral error constant and, the error is given by:

Table 1: VTOL and PI Parameters (QUANSER, 2011).

Description	Parameter	Unit
Torque constant	$k_t = 0.0108$	Nm/A
Moment of equivalent inertia along pitch axis	$J = 0.00347$	kgm^2
Viscous Friction Constant	$B = 0.002$	Nms/rad
Stiffness	$K = 0.0373$	Nm/rad
DC Motor Resistance	$R_m = 3$	Ω
DC Motor Inductance	$L_m = 0.0583$	H
Proporcional Constant PI	$K_P = 0.25$	
Integral Constant PI	$K_I = 100$	

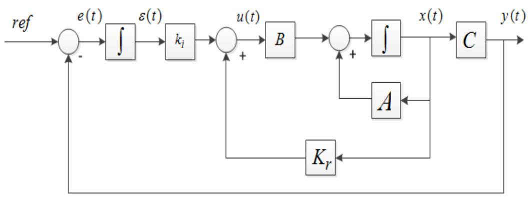

Figure 3: Feedback state control with integrator.

$$e(t) = ref - Cx(t) \tag{6}$$

furthermore, the error can be replaced by

$$e(t) = \dot{\varepsilon}(t) \tag{7}$$

Introducing ε as new state, the system (4) and combinig with (7) an extended system can be written as :

$$\begin{bmatrix} \dot{x} \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ \varepsilon \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ ref \end{bmatrix} \tag{8}$$

The LQR method is used to design an optimal linear state-feeback control. The objetive is to minimize the quadratic cost function:

$$J = \int_0^\infty \left[x^T(t)Qx(t) + u^T(t)Ru(t)dt \right] \tag{9}$$

where $Q \geq 0$ is the weight matrix that penalizes the states and $R > 0$ penalizes the input. The algebraic Riccati equation is given by:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{10}$$

where the vector P is solution of (10). The state-feedback optimal gains are determined by (Zhang et al., 2014b).

$$K_{ri} = R^{-1}B^T P \tag{11}$$

In this way the state-feedback control law is defined by:

$$u(t) = -K_{ri}x(t) \tag{12}$$

Where $K_{ri} = \begin{bmatrix} K_r & k_i \end{bmatrix}$ is the state-feedback gains vector. The cost of function (9) depends of Q and R matrices for this purpose choosing these for VTOL system as:

$$Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}, \quad R = [\rho_1] \tag{13}$$

A relationship between the response time, damping and control effort can be achieved by tuning the individual weights q_i and ρ_1. These can be selected as the inverse of the square of the error value for x_i or u (Murray, 2009). Thus, control law (12) for outer loop control of the VTOL system is rewritten as:

$$u(t) = -k_1\theta(t) - k_2\dot{\theta}(t) + k_i \int_0^t e_\theta(t)dt \tag{14}$$

where, the error is defined as:

$$e_\theta(t) = \theta_d - \theta(t) \tag{15}$$

3.2 SMC Controller

The aim of Sliding Mode Control is to select a suitable surface that allows to slide the system by a desired trajectory. SMC control law is composed of two parts: a continuous $u_{eq}(t)$ and a discontinuous $u_D(t)$.

$$u_{SMC}(t) = u_{eq}(t) + u_D(t) \tag{16}$$

The first step to design SMC control is to define a sliding surface $\mathbb{S}$, which allows to obtain a desired behavior (Camacho and Smith, 2000) for instace: stability, null steady state error, response time, damping. Hence, It is reasonable to choose a LQI control law (14) as a sliding surface.

$$\mathbb{S}(t) = -k_1\theta(t) - k_2\dot{\theta}(t) + k_i \int_0^t e_\theta(t)dt \tag{17}$$

For the purpose of finding the equivalent control law u_{eq} the procedure is followed: The control must ensure that the output reaches a desired set point, thus $\mathbb{S}$ reaches a constant value, the aim is to keep the output equal to a desired reference θ_d for all time which means (Camacho and Smith, 2000):

$$\dot{\mathbb{S}}(t) = 0 \tag{18}$$

This is called sliding condition, by applying in (17) it is obtained:

$$\dot{\mathbb{S}}(t) = -k_1\dot{\theta} - k_2\ddot{\theta} + k_i e_\theta = 0 \tag{19}$$

by Replacing (2), into (19), it is obtained:

$$\dot{\mathbb{S}}(t) = -k_1\dot{\theta} - k_2\left(\frac{-B\dot{\theta} - K\theta + k_t u}{J}\right) + k_i e_\theta = 0 \tag{20}$$

Considering $u = u_{eq}$, therefore from the sliding condition the continuous part u_{eq} of the SMC can be writted as:

$$u_{eq} = \left(\frac{B}{k_t} - \frac{Jk_1}{k_2 k_t}\right)\dot{\theta} + \frac{K}{k_t}\theta + \frac{Jk_i}{k_2 k_t}e_\theta \tag{21}$$

and, by to complete the SMC control law, the discontinuous part is added:

$$u_{SMC} = \left(\frac{B}{k_t} - \frac{Jk_1}{k_2k_t}\right)\dot{\theta} + \frac{K}{k_t}\theta + \frac{Jk_i}{k_2k_t}e_\theta + K_d sign(\mathbb{S}(t)) \tag{22}$$

Where u_D is responsible for reaching sliding surface, which is composed of a non-linear fuction $sign(\mathbb{S}(t))$ that switches about the sliding surface and, K_d is a controller gain parameter.

To design u_D, a function of Lyapunov is defined as:

$$V = \frac{1}{2}\mathbb{S}^2 \tag{23}$$

Obviously, for all $\mathbb{S}(t) \neq 0$ where V is positive-define function, and $\dot{V}$ must be a negative-define function:

$$\dot{V} = \mathbb{S}\dot{\mathbb{S}} < 0 \tag{24}$$

Substituting (20) and, (22) into (24), it can be found:

$$\left(-\frac{k_2k_t}{J}K_d sign(\mathbb{S}(t))\right)\mathbb{S} < 0 \tag{25}$$

where:

$$k_2 > 0, \quad k_t > 0, \quad J > 0 \tag{26}$$

Therefore, by analyzing

$$\begin{aligned} if \quad \mathbb{S}(t) > 0 \quad &\rightarrow \quad sign(\mathbb{S}(t)) > 0 \\ if \quad \mathbb{S}(t) < 0 \quad &\rightarrow \quad sign(\mathbb{S}(t)) < 0 \end{aligned} \tag{27}$$

In order to satisfy (25), K_d should be

$$K_d > 0 \tag{28}$$

for the purpose to reducing the chattering effect, u_D can be rewritten as a sigmoidal function (Camacho and Smith, 2000)

$$u_D(t) = \frac{K_d\mathbb{S}}{|\mathbb{S}| + \delta} \tag{29}$$

Where δ, it is a tunning parameter to reduce the chattering problem. Finally, the complete law of SMC control with reduction of chattering effect is given by:

$$U_{SMC} = \left(\frac{B}{k_t} - \frac{Jk_1}{k_2k_t}\right)\dot{\theta} + \frac{K}{k_t}\theta + \frac{Jk_i}{k_2k_t}e_\theta + \frac{K_d\mathbb{S}}{|\mathbb{S}| + \delta} \tag{30}$$

The proposed control scheme of the system is illustrated in Fig. 4.

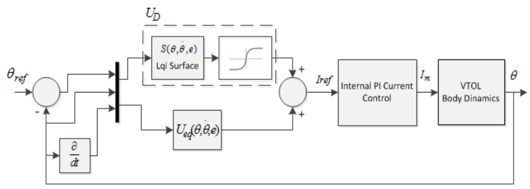

Figure 4: SMC-LQI controller scheme.

4 SIMULATION RESULTS

The simulation using Simulink-MatLab® software has been made to examine the controller behavior. First, the performance of the designed controller is tested by step change reference. Matrices Q and R (13) are selected considering an error of $\theta = \frac{\pi}{30}\,[rad]$ and, $\varepsilon = \frac{\pi}{30}\,[radsec]$, which can be written as

$$Q = \begin{bmatrix} \left(\frac{30}{\pi}\right)^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \left(\frac{30}{\pi}\right)^2 \end{bmatrix}, \quad R = [1] \tag{31}$$

By solving (9) with the LQI MatLab fuction, the feedback gains K_{ri} (11) is obtained:

$$K_{ri} = \begin{bmatrix} 8.73 & 2.19 & -9.55 \end{bmatrix} \tag{32}$$

Finally, the complete SMC control law is computed with tunning parameters by trial and error for lower ISE $\delta = 0.3$ and $K_d = 1$, which is given by:

$$u_{SMC} = 3.4537\theta - 0.9486\dot{\theta} + 5.8255e_\theta + \frac{\mathbb{S}}{|\mathbb{S}| + 0.3} \tag{33}$$

In figure 5 shows the time reponse of pitch angle θ and action control u with LQI and SMC-LQI controllers for a step change reference of $\theta_d = \frac{\pi}{6}[rad]$.

The SMC-LQI has a lower setting time and the control action of the two controllers is below the physical values allowed by the system $(\pm 3[A])$.

The robustness of the controller to uncertainties in the physical parameters is tested using the ISE performance index, which is calculated with:

$$ISE = \int_0^t e^2(t)dt \tag{34}$$

where: $e(t)$: represents the error between θ_d and θ

In figure 6 shows the evolution of the ISE to variations of the moment of inertia along the pitch axis J of $J_1 = 3J$, $J_1 = 9J$ and $J_3 = 15J$.

The ISE index of the SMC-LQI control is maintained for the variations of the J parameter, while the ISE index of the LQI control is incremented for $J_3 = 15J$.

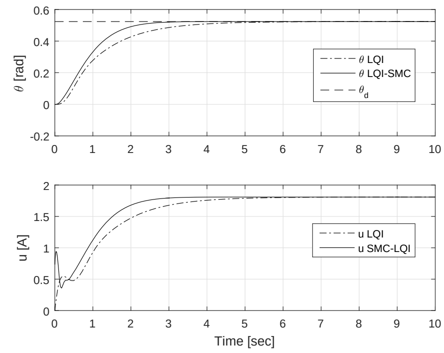

Figure 5: Simulation results of LQI and SMC-LQI Controllers for a step change reference.

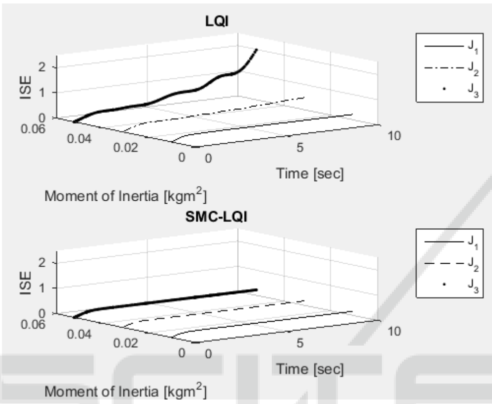

Figure 6: ISE index of LQI and SMC-LQI for Moment of Inertia variation.

5 EXPERIMENTAL RESULTS

In this section, experimental tests are presented. The controllers are implemented in QNET VTOL trainer, this is compatible with LabVIEW software. The following tests were considered: step change in the reference case, robustness to uncertainties in physical parameters and external disturbances.

5.1 Step Change Reference Test

The pitch angle position response for a desired step change reference $\theta_d = \pi/6[rad]$ and, the control action u with LQI and, SMC-LQI controller are shown in Fig. 7 and 8 respectively.

In Table 2 shows the ISE comparison between both controllers for a step change reference, where the SMC-LQI presents lower ISE than LQI for simulation and, experimental results.

The SMC-LQI controller presents lower settling time than LQI controller. In the first three seconds the LQI controller is oscillating near the current limit allowed by the system

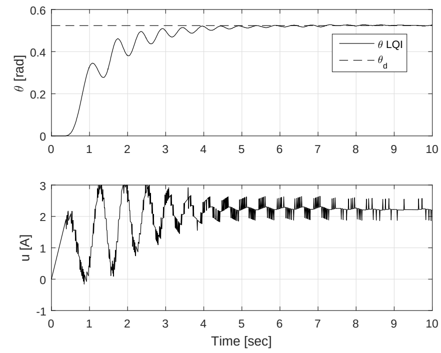

Figure 7: Experimental results of LQI Controller for step a change reference.

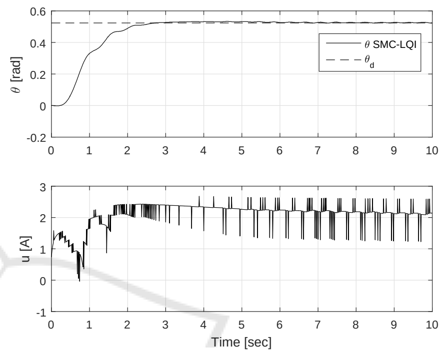

Figure 8: Experimental results of SMC-LQI Controller for a step change reference.

Table 2: ISE Step change Reference.

	LQI	SMC-LQI	Δ %
Simulation Results	0.21	0.16	23.8
Experimental Results	1.91	2.87	50.2

5.2 Uncertain in Physical Parameter Test

For this test, a small iron mass is added the VTOL system physical body as is illustrated in Fig. 9.

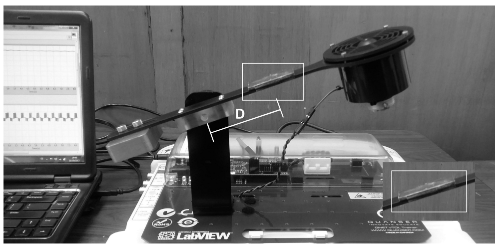

Figure 9: VTOOL system under uncertain parameter test.

where the iron mass of $m = 0.2312[k_g]$ is located a distance of $D = 0.085[m]$ from pivot point of the VTOL system body. It was added a mass to modify the moment of inertia along pitch axis of the system.

In figure 10 shows the time response of pitch angle position θ for a step change reference of $\theta_d = \frac{\pi}{18}[rad]$ considering LQI and SMC-LQI controllers under the uncertain parameter test.

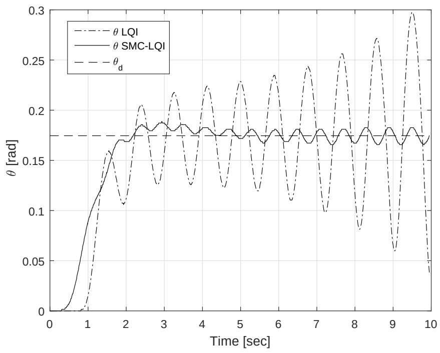

Figure 10: Experimental results of LQI and, SMC-LQI Controllers for uncertain parameter test.

The LQI control for the uncertain parameter test has an unstable behavior, while the SCM-LQI controller is able to reach the desired reference and to maintain on this with a small oscillation.

5.3 External Distubance Test

An external disturbance test is considered. A rubber ball is used as illustrated in the Fig.11, where for an initial pitch angle position $\theta_d = 0[rad]$. A external disturbance produced by a rubber ball of mass $m = 0.1481[k_g]$ falling from an altitude of $L = 0.005[m]$ is applied.

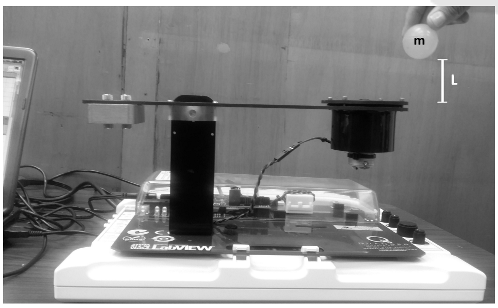

Figure 11: VTOL under external disturbance test.

Figure 12 shows the pitch angle position time response for the LQI and SMC-LQI controllers to the external disturbance test. The LQI control presents an unstable behavior and it is stopped around $6[sec]$ for this there is not physical damage to the system, while the SMC-LQI control presents a stable behaivor.

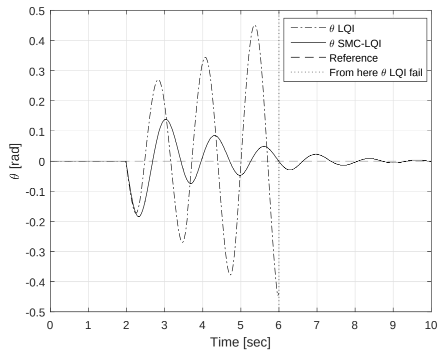

Figure 12: Experimental results of LQI and, SMC-LQI Controller for an external disturbance

6 CONCLUSIONS

In this article, a LQI controller and a robust-optimal controller with integral action SMC-LQI based on a reduced order model are successfully designed and implemented on the 1-DOF VTOL system.
The simulation and experimental results on the 1-DOF VTOL system show that LQI and SMC-LQI controllers are capable to reach a desired reference for the step change reference test, but the SMC-LQI presents a lower settling time and ISE performance index than LQI. For the uncertain parameter and the external disturbance tests on 1-DOF VTOL system, the SMC-LQI controller presents better robustness than LQI controller. It is verified on simulation and experimental results.

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