New Formulations for the Unit Commitment Problem
Optimal Control and Switching-Time Parameterization Approaches

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Abstract: The Unit Commitment Problem (UCP) is a well-known combinatorial optimization problem in power systems. The main goal in the UCP is to schedule a subset of a given group of electrical power generating units and also to determine their production output in order to meet energy demands at minimum cost. In addition, a set of technological and operational constraints must be satisfied. A large variety of optimization methods addressing the UCP is available in the literature. This panoply of methods includes exact methods (such as dynamic programming, branch-and-bound) and heuristic methods (tabu search, simulated annealing, particle swarm, genetic algorithms). This paper proposes two non-traditional formulations. First, the UCP is formulated as a mixed-integer optimal control problem with both binary-valued control variables and real-valued control variables. Then, the problem is formulated as a switching time dynamic optimization problem involving only real-valued controls.

1 INTRODUCTION

The unit commitment problem (UCP) is well-studied and practically relevant in the electrical power industry. This problem involves both the scheduling of power units (i.e., the decisions when each unit is turned on or turned off along a predefined time horizon) and the economic dispatch problem (the problem of deciding how much should produce each unit). The scheduling of the units is typically seen as an integer programming problem and the economic dispatch problem is a nonlinear (real-valued) programming problem. The UCP is then a nonlinear, nonconvex, and mixed-integer optimization problem (Dang and Li, 2007). The objective of the UCP is the minimization of the total operating costs over the scheduling horizon while satisfying demand, the spinning reserve requirements, and other generation and technological constraints such as capacity limits, ramp rate limits, and minimum uptime/downtime. The objective function is expressed as the sum of the fuel, startup, and shutdown costs.

Several methods have been employed to find solutions for the UCP. The available approaches for solving unit commitment problem can usually be classified into mathematical programming and heuris-

tic methods (Salam, 2007). In the past, the proposed approaches were essentially based on exact methods and include dynamic programming (Hobbs et al., 1988; Patra et al., 2009), Lagrangian relaxation (Zhuang et al., 1992), Benders decomposition (Ma and Shahidehpour, 1998), and mixed integer programming (Frangioni et al., 2008; Frangioni et al., 2009). More recently, several metaheuristic approaches have been used such as particle swarm optimization (Zhao et al., 2006), simulated annealing (Simopoulos et al., 2006; Jenkins and Purushothama, 2003), tabu search (Rajan and Mohan, 2004) and genetic algorithms (Dang and Li, 2007; Roque et al., 2014; Roque et al., 2011). A bibliographical survey can be found in (Saravanan et al., 2013). Although the UCP is a highly researched problem with dynamical and multi-period characteristics, it appears that it has not been addressed before by optimal control methods, except in (Fontes et al., 2012; Fontes et al., 2014; Tuffaha and Gravdahl, 2016). In (Fontes et al., 2012), the authors have formulated the UCP as a discrete mixed-integer optimal control problem, which has then been converted into one with only real-valued controls through a variable-time transformation method. A mixed-integer state-space model to solve both the UC and Economic Dispatch prob-

lems is reported in (Tuffaha and Gravdahl, 2016). The output power generation, status of the generating unit, and up and down time counters are considered state variables.

Here, we discuss two formulations of the UCP as an Optimal Control (OC) model. Initially, we formulate the UCP as a discrete-time mixed-integer optimal control problem. Then, we propose a new optimal control switching time approach. The model derived is a continuous one and involves only real-valued decision variables (controls).

The main contributions of the proposed modelling approach are twofold. Firstly, since it allows decisions to be taken at any time moment, and not only at specific points in time (usually, hourly), it may render better solutions. It should be noticed that the proposed approach allows for decisions on unit commitment/decommitment and power production variation at any moment in time. Secondly, it no longer forces utilities to treat demand variations as instantaneous, i.e., time steps. In addition, if one chooses to use the approximated hourly data, as usual in the literature, the solution strategies (both regarding unit commitment/decommitment and power production) of the proposed model will approximate the discrete-time solutions since actions are only required to be taken hourly.

The remaining of this paper is organized as follows. The mixed-integer optimal control formulation is given in section 2. Section 3 introduces a formulation of the UCP as a switching time optimal control problem including only real-valued controls, which is proposed here for the first time. Finally, in section 4 some conclusions are drawn.

2 UCP AS A DISCRETE-TIME MIXED-INTEGER OPTIMAL CONTROL MODEL

In this section a mixed–integer optimal control model (OCM) is proposed to the UCP problem. The mixed–integer optimal control model has two types of decision/control variables: on the one hand, it considers the binary control variables $\beta_j(t)$ and $\gamma_j(t)$, which are the start-up and shut down indicators at time period $t+1$, respectively. $\beta_j(t)(\gamma_j(t))$ are either set to 1, meaning that unit j is turned on-line (off-line) at time period $t+1$, or otherwise set to zero. On the other hand, it also considers real-valued variables $\Delta_j(t)$, which enable to control the production, by increasing or decreasing the power produced by unit j at time t.

The binary variables satisfy

$$\beta_j(t-1) = (1 - u_j(t-1))\, u_j(t)$$

or

$$\beta_j(t-1) = u_j(t) - u_j(t-1) + \gamma_j(t-1)$$

and

$$\gamma_j(t-1) = (1 - u_j(t))\, u_j(t-1)$$

or

$$\gamma_j(t-1) = u_j(t-1) - u_j(t) + \beta_j(t-1).$$

There are three types of state variables:

1. real variables $y_j(t)$, which represent the power generated by unit j at time t; and $y_j^s(t)$, which indicate the contribution for spinning reserve by unit j at time t;

2. integer variables $T_j^{on/off}(t)$, which represent the number of time periods for which unit j has been continuously online/off-line until time t; and

3. binary variables $u_j(t)$, which are either set to 1, meaning that unit j is committed at time t, or otherwise set to zero.

The parameters used in the equations are defined as follows:

T: Number of time periods (hours) of the scheduling time horizon

N: Number of generation units

R(t): System spinning reserve requirements at time t, in $[MW]$

D(t): Load demand at time t, in $[MW]$

Ymin$_j$: Minimum generation limit of unit j, in $[MW]$

Ymax$_j$: Maximum generation limit of unit j, in $[MW]$

T$_{c,j}$: Cold start time of unit j, in $[hours]$

T$_{min,j}^{on/off}$: Minimum uptime/downtime of unit j, in $[hours]$

T$_{0,j}^{on}$: Initial state of unit j at time 0, time since the last status switch off/on, in $[hours]$

T$_{0,j}^{off}$: Initial state of unit j at time 0, time since the last status switch on/off, in $[hours]$

$\Delta_j^{dn/up}$: Maximum allowed output level decrease/increase in consecutive periods for unit j, in $[MW]$.

For convenience, let us also define the index sets: $\mathcal{T} := \{1, \ldots, T\}$ and $\mathcal{J} := \{1, 2, \ldots, N\}$.

The UC problem can now be formulated as a mixed-integer optimal control model.

Objective Function. The objective function has three cost components: generation costs, start-up costs, and shutdown costs. The generation costs, also known as

the fuel costs, are conventionally given by the following quadratic cost function:

$$F_j(y_j(t)) = a_j \cdot (y_j(t))^2 + b_j \cdot y_j(t) + c_j, \quad (1)$$

where a_j, b_j, c_j are the cost coefficients of unit j. The start-up costs, that depend on the number of time periods during which the unit has been off, are represented by $S_j(t)$. The shutdown costs Sd_j for each unit, whenever considered in the literature, are constant. Therefore, the cost incurred with an optimal scheduling is given by the minimization of the total costs for the whole planning period.

$$\text{Minimize} \sum_{t=1}^{T} \sum_{j=1}^{N} (F_j(y_j(t))u_j(t) +$$
$$S_j(t) \cdot \beta_j(t-1) + Sd_j \cdot \gamma_j(t-1)). \quad (2)$$

The State Dynamics. The production of each unit, at time t, depends on the amount produced in the previous time period and is limited by the maximum allowed decrease and increase of the output that can occur during one time period:

$$y_j(t) = y_j(t-1) + \Delta_j(t-1) \cdot (u_j(t) - \beta_j(t-1))$$
$$+ \Delta_j^{SU} \cdot \beta_j(t-1) - \Delta_j^{SD} \cdot \gamma_j(t-1),$$

for all $t \in \mathcal{T}$ and $j \in \mathcal{J}$.
Also

$$u_j(t) - \beta_j(t-1) = u_j(t-1) - \gamma_j(t-1).$$

Thus,

$$y_j(t) = y_j(t-1) + \Delta_j(t-1) \cdot (u_j(t-1) - \gamma_j(t-1))$$
$$+ \Delta_j^{SU} \cdot \beta_j(t-1) - \Delta_j^{SD} \cdot \gamma_j(t-1), \quad (3)$$

for all $t \in \mathcal{T}$ and $j \in \mathcal{J}$.
The term $T_j^{on}(t-1) \cdot \gamma_j(t-1)$ can be used to drive the state variable $T_j^{on}(t)$ to zero whenever the unit j is turned off-line at time period t. Then, the number of time periods for which unit j has been continuously online until time t is given by

$$T_j^{on}(t) = T_j^{on}(t-1) + u_j(t-1) + \beta_j(t-1) - \gamma_j(t-1)$$
$$- T_j^{on}(t-1) \cdot \gamma_j(t-1), \quad (4)$$

for $t \in \mathcal{T}$ and $j \in \mathcal{J}$.

The number of time periods for which unit j has been continuously off-line until time t is given by

$$T_j^{off}(t) = T_j^{off}(t-1) + 1 - u_j(t-1) - \beta_j(t-1)$$
$$+ \gamma_j(t-1) - T_j^{off}(t-1) \cdot \beta_j(t-1), \quad (5)$$

for all $t \in \mathcal{T}$ and $j \in \mathcal{J}$, where the term $T_j^{off}(t-1) \cdot \beta_j(t-1)$ is used to drive the state variable $T_j^{off}(t)$

to zero whenever the unit j is turned on-line at time period t.

The dynamics concerning the unit status (binary) indicator $u_j(t)$ is given by

$$u_j(t) = u_j(t-1) + \beta_j(t-1) - \gamma_j(t-1), \quad (6)$$
for $t \in \mathcal{T}$ and $j \in \mathcal{J}$.

Pathwise Constraints. The pathwise constraints are given by inequalities (7) to (27) and the ramp rate constraints are handled by the equations (3) and control constraints.

$$\Delta_j(t) \in \left[-\Delta_j^{dn}, \Delta_j^{up} \right], \text{ for } t \in \mathcal{T} \text{ and } j \in \mathcal{J}. \quad (7)$$

In addition the control binary variables β_j and γ_j are forced to take the 0 or 1 values using the inequations (8-11).

$$0 \le \beta_j(t-1) \le 1 \text{ for } t \in \mathcal{T}, j \in \mathcal{J}. \quad (8)$$
$$\beta_j(t-1) \cdot (1 - \beta_j(t-1)) \le 0 \text{ for } t \in \mathcal{T}, j \in \mathcal{J}. \quad (9)$$
$$0 \le \gamma_j(t-1) \le 1 \text{ for } t \in \mathcal{T}, j \in \mathcal{J}. \quad (10)$$
$$\gamma_j(t-1) \cdot (1 - \gamma_j(t-1)) \le 0 \text{ for } t \in \mathcal{T}, j \in \mathcal{J}. \quad (11)$$

In a similar way the auxiliary binary variables u_j are forced to take either the value 0 or the value 1.

$$0 \le u_j(t-1) \le 1 \text{ for } t \in \mathcal{T}, j \in \mathcal{J}, \quad (12)$$
$$u_j(t-1) \cdot (1 - u_j(t-1)) \le 0 \text{ for } t \in \mathcal{T}, j \in \mathcal{J}. \quad (13)$$

The unit output range limits are expressed by

$$y_j(t) - Ymin_j \cdot u_j(t) \ge 0, \qquad \forall t \in \mathcal{T}, \forall j \in \mathcal{J} \quad (14)$$

$$y_j(t) - Ymax_j \cdot u_j(t) \le 0, \qquad \forall t \in \mathcal{T}, \forall j \in \mathcal{J} \quad (15)$$

The limits for the number of time periods continuously on-line are given by

$$T_j^{on}(t) \ge 0, \quad (16)$$
$$T_j^{on}(t) - K \cdot T \cdot u_j(t) \le 0, \quad (17)$$
$$T_j^{on}(t-1) - T_{min,j}^{on} \cdot \gamma_j(t-1) \ge 0, \quad (18)$$

for all $t \in \mathcal{T}$ and $j \in \mathcal{J}$.
The limits for the number of time periods continuously off-line are expressed by

$$T_j^{off}(t-1) \ge 0, \quad (19)$$
$$T_j^{off}(t-1) - K \cdot T \cdot (1 - u_j(t-1)) \le 0, \quad (20)$$
$$T_j^{off}(t-1) - T_{min,j}^{off} \cdot \beta_j(t-1) \ge 0, \quad (21)$$

for all $t \in \mathcal{T}$ and $j \in \mathcal{J}$.
The load requirements are modelled by

$$\sum_{j=1}^{N} y_j(t) \cdot u_j(t) - D(t) \ge 0, \qquad \forall t \in \mathcal{T}. \quad (22)$$

The spinning reserve requirements are given by

$$\sum_{j=1}^{N} y_j^s(t) \cdot u_j(t) - R(t) \geq 0, \qquad \forall t \in \mathcal{T}, \qquad (23)$$

and

$$y_j(t) + y_j^s(t) - Ymin_j \cdot u_j(t) \geq 0, \qquad (24)$$

$$y_j(t) + y_j^s(t) - Ymax_j \cdot u_j(t) \leq 0, \qquad (25)$$

$$y_j^s(t) \geq 0, \qquad (26)$$

$$y_j^s(t) - \Delta_j^{up} \cdot (u_j(t) - \beta_j(t-1)) \leq 0, \qquad (27)$$

for all $t \in \mathcal{T}$ and $j \in \mathcal{J}$.

The Initial State Constraints. At the initial time $t = 0$ we have:

$$T_j^{on}(0) = T_{0,j}^{on} \quad \text{(given)}, \qquad (28)$$

$$T_j^{off}(0) = T_{0,j}^{off} \quad \text{(given)}, \qquad (29)$$

$$u_j(0) = \begin{cases} 0 & \text{if } T_{0,j}^{on} = 0 \\ 1 & \text{if } T_{0,j}^{on} > 0, \end{cases} \qquad (30)$$

$$y_j(0) = \begin{cases} 0 & \text{if } T_{0,j}^{on} = 0 \\ y_{0,j} \in [Ymin_j, Ymax_j] & \text{if } T_{0,j}^{on} > 0, \end{cases} \qquad (31)$$

3 UCP AS SWITCHING TIME OPTIMIZATION PROBLEM

This section presents a continuous–time optimal control formulation, based on the concept of the Switching Time Optimization Problem, see e.g. (Sager, 2005; Kaya and Noakes, 2003). This model uses only real-valued decision variables.

3.1 The Switching Time Optimization Problem

Let us consider the set of time points Ψ when the change of the unit status occurs. Ψ is either $\Psi_{n_{sw}} = \{\bar{\tau}_1, \bar{\tau}_2, ..., \bar{\tau}_{n_{sw}}\}$ a finite set of possible switching times. It should be noted for each $k \in \{1, 2, ..., n_{sw}\}$ $\bar{\tau}_k \in]\bar{\tau}_{k-1}, T]$ and $\bar{\tau}_0 = 0$. Then, we consider n_{sw} binary control functions

$$w_k : [\bar{\tau}_{k-1}, \bar{\tau}_k] \quad \longmapsto \{0, 1\}$$

defined by

$$w_k(t) =$$
$$\begin{cases} 0, & \text{if}(w_0(0) = 0 \wedge k \text{ odd}) \text{or}(w_0(0) = 1 \wedge k \text{ even}) \\ 1, & \text{if}(w_0(0) = 0 \wedge k \text{ even}) \text{or}(w_0(0) = 1 \wedge k \text{ odd}) \end{cases},$$
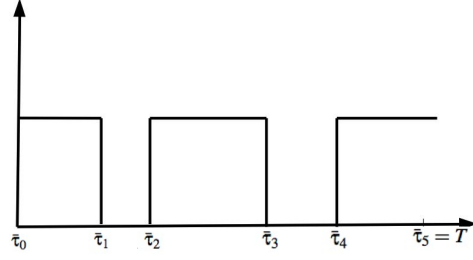$$t \in [\bar{\tau}_{k-1}, \bar{\tau}_k] \quad (32)$$

Figure 1: Possible realisation.

with $k = 1, 2, ..., n_{sw}$ and $0 = \bar{\tau}_0 \leq \bar{\tau}_1 \leq \bar{\tau}_2 \leq ... \leq \bar{\tau}_{n_{sw}} = T$. If we assume a finite number of switches n_{SW}, then the problem can be written in a multistage formulation

$$\min_{x_k, z_k, w_k, u_k, h} \sum_{k=1}^{n_{sw}} \Phi_k [x_k, z_k, w_k, u_k, p], \qquad (33)$$

where the h is the vector of stage lengths $h_k = \bar{\tau}_k - \bar{\tau}_{k-1}$, subject to the dynamic model stages control ($k = 1, 2, ..., n_{SW}$):

$$\dot{x}_k(t) = f_k(x_k(t), z_k(t), w_k(t), u_k(t), p) \qquad (34)$$

for $t \in [\bar{\tau}_{k-1}, \bar{\tau}_k]$, and path constraints:

$$0 \leq g_k(x_k(t), z_k(t), w_k(t), u_k(t), p), \qquad (35)$$

for $t \in [\bar{\tau}_{k-1}, \bar{\tau}_k]$.

The maximum number of switching times is $2 \times \lfloor \frac{T}{T_{min}^{off} + T_{min}^{on}} \rfloor$ where $\lfloor X \rfloor$ represents the nearest integer less than or equal to X. In Figure 1, an example of one possible realisation with $n_{sw} = 5$ is given. For example, we have the constraints concerning to the limits for the time continuously on-line expressed by

$$\bar{\tau}_k - \bar{\tau}_{k-1} \geq T_{min}^{on}, \text{ if } (\bar{\tau}_k < T) \wedge$$
$$[(w_0(0) = 0 \wedge k \text{ even}) \text{ or } (w_0(0) = 1 \wedge k \text{ odd})],$$

and the limits for the time continuously off-line are given by

$$\bar{\tau}_k - \bar{\tau}_{k-1} \geq T_{min}^{off}, \text{ if } (\bar{\tau}_k < T) \wedge$$
$$[(w_0(0) = 0 \wedge k \text{ odd}) \text{ or } (w_0(0) = 1 \wedge k \text{ even})].$$

3.2 The Unit Commitment Problem formulated as Switching Time Optimization Problem

Let us consider $\bar{\tau}_{1,j}, \bar{\tau}_{2,j}, ..., \bar{\tau}_{n_{sw,j},j}$ the possible switching times for unit j. Then, we consider $n_{sw,j}$ binary control functions

$$w_{k,j} : [\bar{\tau}_{k-1,j}, \bar{\tau}_{k,j}] \quad \longmapsto \{0, 1\}$$

defined as in equation (32). The objective function to be minimized is

$$\min_{y_j,\Delta_j,w_{k,j},h_{k,j}} \sum_{j=1}^{N} \sum_{k=1}^{n_{sw,j}} \int_{\bar{\tau}_{k-1,j}}^{\bar{\tau}_{k,j}} F_j(y_j(t)) \cdot w_{k,j}(t) \cdot dt +$$

$$\int_{\bar{\tau}_{k-1,j}}^{\bar{\tau}_{k,j}} S_j(t) \cdot w_{k,j}(t) \cdot dt + \int_{\bar{\tau}_{k-1,j}}^{\bar{\tau}_{k,j}} Sd_j \cdot (1 - w_{k,j}(t)) \cdot dt$$

$$- S_j(t) w_{0,j}(0) - Sd_j (1 - w_{0,j}(0)). \quad (36)$$

The unit output range limits are expressed by

$$y_j(t) - Ymin_j \cdot w_{k,j}(t) \geq 0, \quad (37)$$

$$y_j(t) - Ymax_j \cdot w_{k,j}(t) \leq 0, \quad (38)$$

for all $j \in \mathcal{J}$ and $t \in [\bar{\tau}_{k-1,j}, \bar{\tau}_{k,j}]$. It should be noted that the controls are all real-valued and satisfy

$$\delta_j(t) \in \left[-\Delta_j^{dn}, \Delta_j^{up} \right].$$

In addition, the power production and, for convenience, the unit status must also be defined for each time instant.

$$y_j(t) = \begin{cases} 0, & \text{if } w_{k,j}(t) = 0 \\ y_j(\bar{\tau}_{k-1,j}) + \int_{\bar{\tau}_{k-1,j}}^{t} \delta_j(s) ds, & \text{if } w_{k,j}(t) = 1 \end{cases},$$

for $j \in \mathcal{J}$ and $t \in [\bar{\tau}_{k-1,j}, \bar{\tau}_{k,j}]$. The limits for the time continuously on-line of each unit j given by inequations (18) are now expressed by

$$\bar{\tau}_{k,j} - \bar{\tau}_{k-1,j} \geq T_{min,j}^{on}, \text{if}(\bar{\tau}_{k,j} < T) \wedge$$

$$[(w_{0,j}(0) = 0 \wedge k \text{ even}) \text{or}(w_{0,j}(0) = 1 \wedge k \text{ odd})],$$

while the limits for the time continuously off-line of each unit j given in inequations (21), are now defined by

$$\bar{\tau}_{k,j} - \bar{\tau}_{k-1,j} \geq T_{min,j}^{off}, \text{ if } (\bar{\tau}_{k,j} < T) \wedge$$

$$[(w_{0,j}(0) = 0 \wedge k \text{ odd}) \text{ or } (w_{0,j}(0) = 1 \wedge k \text{ even})].$$

The load requirements are given by

$$\sum_{j=1}^{N} y_j(t) \cdot w_{k,j}(t) - D(t) \geq 0, t \in [\bar{\tau}_{k-1,j}, \bar{\tau}_{k,j}]. \quad (39)$$

The spinning reserve requirements are expressed by

$$\sum_{j=1}^{N} y_j^s(t) \cdot w_{k,j}(t) - R(t) \geq 0, t \in [\bar{\tau}_{k-1,j}, \bar{\tau}_{k,j}], \quad (40)$$

and

$$y_j(t) + y_j^s(t) - Ymin_j \cdot w_{k,j}(t) \geq 0, \quad (41)$$

$$y_j(t) + y_j^s(t) - Ymax_j \cdot w_{k,j}(t) \leq 0, \quad (42)$$

$$y_j^s(t) \geq 0, \quad (43)$$

for all $j \in \mathcal{J}, t \in [\bar{\tau}_{k-1,j}, \bar{\tau}_{k,j}]$.

4 CONCLUSIONS

The UCP, an intensively researched problem in the literature, is addressed in this paper. The problem is usually formulated using a mixed-integer nonlinear programming model. Here, the formulation of this problem using optimal control models is explored. Previous works on an optimal control approach to the UC problem, as far as we are aware of, are limited to the works in (Fontes et al., 2012; Fontes et al., 2014; Tuffaha and Gravdahl, 2016) that use a discrete-time optimal control model.

We propose here two formulations of the UCP as an Optimal Control model. First, we formulate it as a discrete time mixed-integer optimal control problem. Then, we propose a new optimal control switching time approach using a continuous-time optimal control model. An interesting feature of the continuous-time formulation is the fact that, contrary to the usual mixed-integer programming models in the literature, all decision variables are real-valued, which enables the use of more efficient optimization methods for its solution.

Additional advantages of the continuous-time optimal control formulation are the possibility of dealing more accurately with data provided with an irregular or fast-sampled time intervals, or even continuous-time varying (for instance, continuous-time varying demand data). The schedule can be changed and adapted according to the predictions and the fluctuations inherent to the renewable power supply. The reduction in unbalance implies, simultaneously, a reduction on expensive spinning reserve. This can not only account for significant cost savings, but also allows for better environmental characteristics.

Computational results will be reported in a future work.

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