Modelling and Optimization of the Operation of a Multiple Tank Water Pumping System

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Abstract: This paper studies the optimization of the operation of a water supply pumping system by means of standard solvers. The system consists of several tanks that supply the water to several districts in a town. Each tank can be filled from several wells through a hydraulic system that can be reconfigured by means of several valves. The automatic operation of the system tries to determine which valves and pumps must be active at each instant in order to minimise the operation cost, taking into account the tariff periods. The main constraints are the maximum and minimum volumes of the tanks. A mathematical model of the problem is proposed in order to formulate, in matrix form, the cost index and the constraints, to be able to use standard solvers as Mosek. A basic optimization problem is proposed, that only imposes the constraints of the finite volume of the tanks. The result of this basic problem tends to produce a large number of pump and valve commutations, that is not adequate in practice. To solve this problem, two alternatives are proposed: to include the number of commutations in the cost index, and to add as a constraint the maximum number of commutations. Both approaches lead to an adequate result, but the second one is easier to use, as the number of commutations can be fixed in an explicit way. An example of a real water supply system is analysed to demonstrate the validity of the approach, using Yalmip as parser and Mosek as solver.

1 INTRODUCTION

An important cost in the operation of water supply systems are the energetic cost associated to the pumping system. Water consumers receive the water from supply tanks that are filled up from wells. The operation cost depends on the instants during the day when the pumps are working, since the electric tariff varies along the day, following predefined tariffing periods.

The work (Ormsbee and Lansey, 1994) presents a review of different approaches to the optimization of water pumping systems, that depend on the configuration of the system and the mathematical models used. For the problem of filling the main tanks, that is the objective of this paper, the most commonly used model is the mass-balance one. In most of the works, the cost function takes into account the energy consumed, but not the tariffing periods when that energy is consumed. On the other hand, the decision variables are usually the fraction of operation time of each pump in some predefined intervals. Other indirect decision variables have also been proposed (as the tank volumes), but the relation to the cost index and to the final decision (the pumps and valves commands) is too complex for the case we are studying.

In a more recent work, (Bunn and Reynolds, 2009), different approaches are reviewed, focusing on real-time dynamic optimization technologies and data-mining techniques to improve energy efficiency. This work describes a commercial optimization software for water distribution systems that can solve the problem we are facing in this paper and other more complex ones, but the details about the algorithms used are not given.

Most of the recent works, as (Savic et al., 1997), (Powell and McCormick, 2004), (Martinez et al., 2007) , (Sotelo et al., 2002) or (Wegley et al., 2000) use complex models of the hydraulic systems, and are based on complex optimization algorithms, as genetic algorithms, particle swarm or simulated annealing. Those approaches have a high computer cost, especially for large systems.

In this paper a simple model is used in order to formulate an optimization problem that can be solved with standard solvers, with a low computer cost.

In (Pasha and Lansey, 2009), a linear programming approach is used for the optimization of the pumps operation, but is only applicable for a single

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tank system. In (Fang et al., 2010), the problem of optimization of pumps operation in a multi tank system is studied using complex genetic algorithms.

In (Ormsbee et al., 2009) three different explicit formulations of the optimal pump scheduling problem are presented. It takes into account the electric tariff, but the decision variables define the start and stop times of the different pumps in the system. The resulting optimization problem needs to be solved by non linear algorithms, genetic algorithms, or semi heuristic ad-hoc algorithms. This discrete explicit formulation is not directly applicable if the system has some valves that can reconfigure the network. In that case is not sufficient to decide which pumps are started or stopped at which time, but also the switching of the valves, that changes the resulting inlet flow to the tanks for a given pumps state. Furthermore, our objective is to use standard solvers to deal with the optimization problem. For that reason, in this paper, the decision variables are explicit, but consist of the combination of pumps and valves that must be active at each instant, from the set of possible combinations. The mathematical formulation proposed allows to use standard parsers (as Yalmip) and solvers (as Mosek or CBC), to solve the optimization problem.

In section II the problem of pumping system optimal operation is described. In section III the mathematical model of the problem is developed. In section IV the basic optimization problem, derived from the mathematical problem, is formulated. A more complex optimization problem is formulated in section V to reach more practical solutions, with a reduced number of pump commutations. Section VI shows the application to a real pumping system and section VII summarizes the main conclusions.

2 DESCRIPTION OF THE PROBLEM

This paper deals with the problem of optimizing the operation of a water pumping system a predefined structure, with several wells and pumps, several tanks and several ducts and valves. The objective of the optimization is the minimization of the overall operational cost. This cost is related to the individual pumping energetic cost of each well (kWh per cubic meter) and, especially, to the electric tariff that establish a different price depending on the time. The automatic control system must decide which valves and which pumps should be operated at every time along the day to minimize the cost while fulfilling some constraints.

The main constraint is to serve from each tank the required daily water flow. This flow is time varying and uncertain, but it can be predicted because it follows an approximate daily pattern. The other important constraint is due to the size of the tanks, each one having a maximum and minimum level that should not be violated.

Other secondary constraints that may apply include a maximum number of daily starts and stops of each pump, or a minimum running time once a pump is switched on. Other more complex restrictions could be related to the concentration of some pollutants in the different wells. For example, the nitrate concentration may be too high in a given well, so that, that water must be mixed with the one from another well to fulfil a concentration requirement. The constraint could then be a minimum mixing ratio. This last constraint could be time varying, as the concentration of the wells may change with time.

3 MATHEMATICAL MODELLING OF THE PROBLEM

First of all, the pumping system is assumed to have N_p pumps (one in each well), N_t tanks and N_v valves. The valves are used to connect or disconnect the pumps to the tanks. The total number of possible combinations is $2^{N_p+N_v}$, but not all the combinations are possible in a given system. Let us define as N_c the number of valid combinations of valves and pumps. In order to formulate mathematically the problem, a binary matrix, M_c, of size $N_c \times (N_p + N_v)$, is defined, where each row represents one of the valid combinations, and where the corresponding elements take the value 1 or 0 depending on the active or inactive state of the valves and the pumps in that combination.

Along the paper, a simple example will be used to illustrate the proposed approach. The considered pumping system has $N_p = 2$ wells (pumps), $N_v = 1$ valve and $N_t = 2$ tanks. The tank 1 can be filled from pump 1, no matter the state of the valve (with a flow of 200 m^3/h), and from pump 2 if the valve 2 is open (with an additional flow of 100 m^3/h), while the tank 2 can only be filled from pump 2 if the valve is closed (with a flow of 80 m^3/h).

Taking into account the physical limitations described above, the matrix that defines de $N_c = 6$ valid combinations of pumps and valves in this example is shown in table 1. The value X means that the resulting flows are the same if the valve is closed (0) or open (1).

For each combination of valves and pumps, there

Table 1: Possible combinations of simple example.

Comb	V	P_1	P_2
0	X	0	0
1	X	1	0
2	0	0	1
3	1	0	1
4	0	1	1
5	1	1	1

is a resulting outlet flow of each pump, and a resulting inlet flow for each tank. This can be expressed by a matrix of pump flows, F_P, that has as many columns as combinations, and one row per pump and a matrix of tank inlet flows, F_T, that has as many columns as combinations, and one row per tank, i.e. the size of matrices F_P and F_T are $(N_p \times N_c)$ and $(N_t \times N_c)$. In the proposed example, the resulting flow matrices could be (in m^3/h):

$$F_P = \begin{bmatrix} 0 & 200 & 0 & 0 & 200 & 200 \\ 0 & 0 & 80 & 100 & 80 & 100 \end{bmatrix} \quad (1)$$

$$F_T = \begin{bmatrix} 0 & 200 & 0 & 100 & 200 & 300 \\ 0 & 0 & 80 & 0 & 80 & 0 \end{bmatrix} \quad (2)$$

For each combination there is a matrix that relates the pump flows to the tank flows. For example, for combination 5, the tank flows can be expressed as a function of pump flows as

$$f_T = M_5 f_P = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 200 \\ 100 \end{bmatrix} = \begin{bmatrix} 300 \\ 0 \end{bmatrix} \quad (3)$$

The matrix that relates the flows is different for each combination of pumps and valves. In the example the matrices are:

$$M_0 = M_1 = M_2 = M_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M_3 = M_5 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Following the same procedure, a matrix P can be formed with the electric power consumption of each pump for each combination. The matrix has as many columns as combinations, and one row per pump, i.e. the size of matrix P is $(N_p \times N_c)$. In the proposed example, the power matrix (in kW) is:

$$P = \begin{bmatrix} 0 & 20 & 0 & 0 & 20 & 20 \\ 0 & 0 & 10 & 12 & 10 & 12 \end{bmatrix} \quad (4)$$

In order to operate the water supply facility, the automatic management system must decide which one of those N_c combinations has to be applied at each instant of time. The natural objective is to minimize the overall operation cost. In order to be able to formulate the optimization problem, a variable taking integer values from 0 to N_c could be defined. However, in order to formulate more easily the objective function as well as the constraints, a binary vector is proposed to define the applied combination as a function of time:

$$\delta(t) \in \{\delta_1, \ldots, \delta_{N_c}\} \quad (5)$$

with

$$\delta_i = [\underbrace{0 \cdots 0}_{i-1} \, 1 \, 0 \cdots 0]^T \quad (6)$$

Therefore, the elements of $\delta(t)$ can only be 0 or 1, and only one of the elements can be different from zero (i.e. the sum of the elements is 1).

With these definitions, the vector that gathers the inlet flows of the tanks at a given instant is simply the product

$$f_T(t) = F_T \delta(t),$$

the vector of output flow of the pumps is

$$f_P(t) = F_P \delta(t),$$

while the vector of power consumptions of the pumps is

$$p(t) = P\delta(t)$$

The matrix that relates the pump flows and the tank flows can be expressed as a function of δ:

$$M(\delta) = \begin{bmatrix} \delta^T & 0 & \cdots & 0 \\ 0 & \delta^T & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \delta^T \end{bmatrix}_{N_T \times N_T N_c} \cdot \begin{bmatrix} M_0(1,:) \\ M_1(1,:) \\ \vdots \\ M_{N_c}(1,:) \\ M_0(2,:) \\ \vdots \\ M_{N_c}(N_t,:) \end{bmatrix}_{N_T N_c \times N_p} \quad (7)$$

With that definition,

$$f_T(t) = M(\delta(t)) f_P(t) \quad (8)$$

In order to compute the overall cost, the electric tariff must be taken into account. The tariff can be expressed as a function that defines the price in $euro/kWh$ as a function of time, $T_i(t)$. Each pump could have a different tariff, hence, we define a row vector as:

$$T(t) = \begin{bmatrix} T_1(t) & \cdots & T_{N_p}(t) \end{bmatrix} \quad (9)$$

With this, the overall cost in a period of time can be expressed (in euros) as:

$$J = \frac{1}{3600} \int T(t) P \delta(t) dt \quad (10)$$

The equation of the tanks can be expressed as:

$$\dot{V}_j = f_{T,j}(t) - fo_j(t) \quad (11)$$

where $f_{T,j}(t)$ and $fo_j(t)$ are the inlet and outlet flow of tank j respectively. The equations of all the tanks can be joined in matrix form

$$\dot{V} = \begin{bmatrix} \dot{V}_1 \\ \cdots \\ \dot{V}_{N_t} \end{bmatrix} = \begin{bmatrix} f_{T,1}(t) \\ \cdots \\ fi_{T,N_t} \end{bmatrix} - \begin{bmatrix} fo_1(t) \\ \cdots \\ fo_{N_t} \end{bmatrix} = f_T(t) - fo(t) \tag{12}$$

If the future output flow of each tank, $fo(t)$, were known, then the volume could be calculated accurately. However, the future output flow is unknown and, hence, a prediction $\hat{f}o(t)$ must be used. On the other hand, the input flow can be expressed as a funcion of $\delta(t)$, therefore, the equation used to estimate the evolution of the volume of the tanks (in m^3/s) is

$$\dot{V} = \frac{1}{3600}\left(F_T \delta(t) - \hat{f}o(t)\right) \tag{13}$$

where the flows are in m^3/h.

In order to obtain a tractable formulation of the problem, the continuous time equations must be discretized. If we choose a constant discretizing period h, the vector funcions $T(t)$ and $\delta(t)$ would then be changed by vectors of discrete signals $T[k] = T(t = kh)$ and $\delta[k] = \delta(t = kh)$. $\delta(t)$ is assumed to maintain a constant value during interval h, i.e. $\delta(t) = \delta[k]$ for $k\,h <= t < (k+1)h$. Taking into account the units of the variables, the cost index (in euros) could be expressed as:

$$J = \frac{h}{3600} \sum T[k] P \delta[k] \tag{14}$$

The continuous time equation of the tanks must also be discretized at period h. The discretized equation can be expressed as:

$$V[k+1] = V[k] + \frac{h}{3600}\left(F_T \delta[k] - \hat{f}o[k]\right) \tag{15}$$

where $V[k] = V(t = k\ h)$ and $\hat{f}o[k] = \frac{1}{h}\int_{kh}^{(k+1)h} \hat{f}o(t)dt$.

The main constraints are the maximum and minimum limits of the tank volums, therefore, the constraints can be formulated as

$$V_{i,min} \leq V_i[k] \leq V_{i,max}, \quad i = 1, \cdots, N_t \tag{16}$$

or in matrix form

$$V_{min} \leq V[k] \leq V_{max} \tag{17}$$

4 BASIC OPTIMIZATION PROBLEM

The basic optimization problem consists of minimizing the cost of the energy while fulfilling the main constraints of maintaining the volumes of all the tanks inside their admissible range (between their minimum and maximum values). An additional constraint must be added in order to guarantee that the tanks finish the day with the same volume they have started. Otherwise, the solution will always finish the day with the tanks completely empty.

Taking into account the equations introduced in the previous sections, with the definition of vectors $\delta[k]$, $T[k]$, $V[k]$ and $\hat{f}_o[k]$, and taking a minimization horizon, $t_m = k_m h$, the minimization problem can be formulated as:

$$\begin{aligned} \min_{\delta} \quad & \tfrac{h}{3600}\textstyle\sum_{k=1}^{k_m} T[k]P\delta[k] \\ s.t. \quad & V_{min} \leq V(0) + \tfrac{h}{3600}\textstyle\sum_{j=1}^{k}\left(F_T\delta[j] - \hat{f}_o[j]\right) \leq V_{max} \\ & \textstyle\sum_{j=1}^{k_m}\left(F_T\delta[j] - \hat{f}_o[j]\right) \geq 0 \\ & \|\delta[k]\| = 1,\ \delta[k] \in \{0,\ 1\} \\ & k = 1, \ldots, k_m \end{aligned} \tag{18}$$

This problem can be expressed in matrix form in a more compact way by joining vectors $\delta[k]$ and $T[k]$ in some matrices of adequate sizes ($\Delta_{(k_m N_c \times 1)}$ and $T_{(1 \times k_m N_p)}$), and defining the following vectors and matrices:

$$\mathbf{\Delta} = \begin{bmatrix} \delta[1] \\ \vdots \\ \delta[k_m] \end{bmatrix}_{k_m N_c \times 1} \tag{19}$$

$$\mathbf{T} = \frac{\mathbf{h}}{\mathbf{3600}} \begin{bmatrix} T[1] & \cdots & T[k_m] \end{bmatrix}_{\mathbf{1 \times k_m N_p}} \tag{20}$$

$$\mathbf{P} = \begin{bmatrix} P & 0 & \cdots & 0 \\ 0 & P & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & P \end{bmatrix}_{\mathbf{k_m N_p \times k_m N_c}} \tag{21}$$

$$\mathbf{F_k} = \begin{bmatrix} \underbrace{F_T\ F_T\ \cdots\ F_T}_{k} & 0 & \cdots & 0 \end{bmatrix}_{\mathbf{N_t \times k_m N_c}} \tag{22}$$

$$\mathbf{I_k} = \begin{bmatrix} \underbrace{0 \cdots 0}_{(k-1)N_c} & \underbrace{1 \cdots 1}_{N_c} & 0 & \cdots & 0 \end{bmatrix} \tag{23}$$

$$\hat{F}_O[k] = \sum_{j=1}^{k} \hat{f}_o[j] \tag{24}$$

Using these definitions, the minimization problem can be formulated as:

$$\begin{aligned}
&\min_{\Delta} \quad \mathbf{T}\mathbf{P}\Delta \\
&s.t. \\
&\mathbf{F}_k\boldsymbol{\Delta} \geq 3600\frac{V_{min}-V(0)}{h}+\hat{F}_O[k],\ k=1,\ldots,k_m \\
&\mathbf{F}_k\boldsymbol{\Delta} \leq 3600\frac{V_{max}-V(0)}{h}+\hat{F}_O[k],\ k=1,\ldots,k_m \\
&\mathbf{F}_{k_m}\boldsymbol{\Delta} \geq \hat{F}_O[k_m] \\
&\mathbf{I}_k\boldsymbol{\Delta}=1,\ k=1,\ldots,k_m \\
&\boldsymbol{\Delta}[j]\in\{0,\ 1\}\quad j=1,\cdots,k_mN_c
\end{aligned} \tag{25}$$

The optimization problem (25) is a mixed integer one, where the decision variables are the elements of vector $\boldsymbol{\Delta}$ that can only take values 0 or 1. The number of decision variables is N_ck_m, and the number of constraints is $(2N_c+2N_t+1)k_m+N_t$. For a normal prediction horizon of 1 day, if a discretizing period of 1 minute is taken, the number of variables is $1440N_c$, and the number of constraints is $1440(2N_c+2N_t+1)+N_t$, that can be very large even for quite simple systems. Even if a longer perid of, for example, 5 minutes is chosen, the number of variables is still large ($288N_c$). Furthermore, the mixed integer optimization is highly demanding in computer resources, and the optimization should be run frequently (not only once a day), to use the most up to date output flow estimations. Therefore, in order to reduce the complexity, we propose to use two discretizing periods, a short one for the first hours, h, and a coarser one for the rest of the day, $h_M=Lh$. Defining the shorter time horizon as $t_m=k_mh$, and the total time horizon $t_M=t_m+(k_M-k_m)Lh$, the optimization problem could be expressed as:

$$\begin{aligned}
\min_{\Delta} \quad & \tfrac{h}{3600}\textstyle\sum_{k=1}^{k_m}T[k]P\delta[k]+\tfrac{Lh}{3600}\sum_{i=k_m+1}^{k_M}T[i]P\delta[i] \\
s.t. \quad & V_{min}\leq V(0)+\tfrac{h}{3600}\textstyle\sum_{j=1}^{k}\left(F_T\delta[j]-\hat{f}_o[j]\right)\leq V_{max} \\
& k=1,\ldots,k_m \\
& V_{min}\leq V(k_m)+\tfrac{Lh}{3600}\textstyle\sum_{i=k_m+1}^{k}\left(F_T\delta[i]-\hat{f}_o[i]\right)\leq V_{max} \\
& k=k_m+1,\ldots,k_M \\
& \|\delta[k]\|=1,\ \delta[k]\in\{0,\ 1\} \\
& k=1,\ldots,k_M
\end{aligned} \tag{26}$$

where the size of matrix Δ is now $(k_M)\times N_c$, and where

$$T[k]=\begin{cases} T(t=kh) & if \quad k\leq k_m \\ T(t=k_mh+(k-k_m)Lh) & if \quad k_m<k\leq k_M \end{cases} \tag{27}$$

and where

$$\hat{f}o[k]=\begin{cases} \frac{1}{h}\int_{kh}^{(k+1)h}\hat{f}o(t)dt & if \quad k\leq k_m \\ \frac{1}{Lh}\int_{k_mh+(k-k_m)Lh}^{k_mh+(k+1-k_m)Lh}\hat{f}o(t)dt & if \quad k_m<k\leq k_M \end{cases} \tag{28}$$

Using the same vectors and matrices defined previously ($\boldsymbol{\Delta}$, $\mathbf{P}$, $\mathbf{I_k}$), but of fize k_M instead k_m, and redefining matrix $\mathbf{T}$ as

$$\mathbf{T}=\frac{\mathbf{h}}{\mathbf{3600}}\left[\begin{array}{cc} T[1]\cdots T[k_m] & T[k_m+1]L\cdots T[k_M]L \end{array}\right]_{\mathbf{1\times k_MN_p}} \tag{29}$$

and redefining matrix $\mathbf{F_k}$ and $\hat{F}_O[k]$ for $k>k_m$ as

$$\mathbf{F_k}=\left[\begin{array}{cccc} \underbrace{F_T\ \cdots\ F_T}_{k_m} & \underbrace{LF_T\ \cdots\ LF_T}_{k-k_m} & 0 & \cdots & 0 \end{array}\right]_{\mathbf{N_t\times k_MN_c}} \tag{30}$$

$$\hat{F}_O[k]=\sum_{j=1}^{k_m}\hat{f}_o[j]+L\sum_{j=k_m+1}^{k}\hat{f}_o[j] \tag{31}$$

the optimization problem can be formulated as:

$$\begin{aligned}
&\min_{\Delta} \quad \mathbf{T}\mathbf{P}\Delta \\
&s.t. \\
&\mathbf{F}_k\boldsymbol{\Delta} \geq 3600\frac{V_{min}-V(0)}{h}+\hat{F}_O[k],\ k=1,\ldots,k_M \\
&\mathbf{F}_k\boldsymbol{\Delta} \leq 3600\frac{V_{max}-V(0)}{h}+\hat{F}_O[k],\ k=1,\ldots,k_M \\
&\mathbf{F}_{k_M}\boldsymbol{\Delta} \geq \hat{F}_O[k_M] \\
&\mathbf{I}_k\boldsymbol{\Delta}=1,\ k=1,\ldots,k_M \\
&\boldsymbol{\Delta}[j]\in\{0,\ 1\}\quad j=1,\cdots,k_MN_c
\end{aligned} \tag{32}$$

The number of decision variables in problem (32) is N_ck_M, and the number os constraints is $(2N_c+2N_t+1)k_M+N_t$. For example, choosing a short discretizing period $h=5$ minutes, $k_m=36$ (equivalent to 3 hours), and a long period $Lh=20$ minutes, with $k_M=99$ (equivalent to 1 day total horizon), for a system with $N_c=10$, and $N_t=3$, the number of variables is 990, while the number of constraints is 2676.

The optimization problem (32) depends on the prediction of the future flow demand $\hat{f}o[k]$, that is uncertain. The prediction can be improved for the short term as the output flow is measured on line. Therefore, in order to obtain more accurate results, the optimization must be run periodically, so that, a less uncertain flow prediction is used. For example, it could be run every hour. In that case, only the values of $\delta[k]$ obtained for the first hour would be applied, discarding the rest. This is a usual strategy in predictive control.

As we are enforcing a final volume equal than the initial one, the result of the optimization depends on the initial condition of the tanks, and the instant of time during the day when the optimization is carried out. In order to set a framework for comparison purposes, the optimization will be assumed to be carried out at the instant when the cheapest period ends, and the initial volumes are assumed to be $V(0) = 0.95V_{max}$, i.e. near the maximum. This conditions reinforce the idea of maximizing water availability in case a problem occurs with any pump.

One drawback of this simple optimization is that it tends to produce a large number of pumps and valves commutations.

5 ADVANCED OPTIMIZATION PROBLEM

The main drawback of the basic optimization problem (32) is that the optimal solution tends to imply a large amount of commutations, i.e. the pumps and valves would be stopping and starting every few minutes. This is a serious problem, since every start and stop implies an energy waste and a reduction in the working life of hydraulic components. Therefore, the optimization problem must be modified to achieve a low number of commutations for the proposal to be useful. There are two approaches for limiting the number of commutations:

- To include in the cost index a term that depends on the number of commutations. There are two difficulties: the formulation of the number of commutations, and to decide the weight of this term relative to the main cost index.
- To add some constraints to limit the number of commutations. The constraints could relate to the number of combination changes, or the number of commutations in each pump or in each valve.

5.1 Formulation of Number of Commutations

If we consider the number of commutations as the number of changes in $\delta[k]$ along the optimization period, we need an equation that expresses that number of changes as a function of matrix $\mathbf{\Delta}$. If the following matrices are defined:

$$I_N^+ = \begin{bmatrix} \overbrace{0 \cdots 0}^{N} & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \cdots & \ddots & 1 & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 1 \end{bmatrix}_{(k_M-1)N \times k_M N}$$

$$I_N^- = \begin{bmatrix} 1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 1 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & \underbrace{0 \cdots 0}_{N} \end{bmatrix}_{(k_M-1)N \times k_M N}$$

$$Y_N = I_N^+ - I_N^-$$

The number of changes in $\delta[k]$ can be expressed as:

$$\mathbf{\Delta}^T Y_{N_c}^T Y_{N_c} \mathbf{\Delta} = sum(abs(Y_{N_c} \mathbf{\Delta})) \quad (33)$$

Another possibility is to consider the number of commutations of the pumps. In order to obtain the expression of number of pump commutations, let us define matrix S_P as a matrix with 1 or 0 defining which pumps are active at each combination. In the case of our example this matrix would be:

$$S_P = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}^T \quad (34)$$

such that $S_p \delta[k]$ is a vector with values 1 or 0 depending on the state of each pump. Defining matrix S as

$$S = \begin{bmatrix} S_P & 0 & \cdots & 0 \\ 0 & S_P & 0 & \cdots 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & S_P \end{bmatrix}_{N_p k_M \times N_c k_M} \quad (35)$$

The overall number of pump commutations is

$$\mathbf{\Delta}^T S^T Y_{N_p}^T Y_{N_p} S \mathbf{\Delta} = sum(abs(Y_{N_p} S \mathbf{\Delta})) \quad (36)$$

5.2 Modification of Cost Index

The first alternative to reduce the number of commutations is to include the number of changes in the cost index. In this case the optimization problem can be formulated as:

$$\min_{\Delta} \ \mathbf{TP}\Delta + \alpha \Delta^{\mathbf{T}} \mathbf{Y}^{\mathbf{T}} \mathbf{Y} \Delta \quad (37)$$

$$s.t.$$

$$\mathbf{F}_k \mathbf{\Delta} \geq 3600 \frac{V_{min} - V(0)}{h} + \hat{F}_O[k], \ k = 1, \ldots, k_M$$

$$\mathbf{F}_k \mathbf{\Delta} \leq 3600 \frac{V_{max} - V(0)}{h} + \hat{F}_O[k], \ k = 1, \ldots, k_M$$

$$\mathbf{F}_{k_M} \mathbf{\Delta} \geq \hat{F}_O[k_M]$$

$$\mathbf{I}_k \mathbf{\Delta} = 1, \ k = 1, \ldots, k_M$$

$$\mathbf{\Delta}[j] \in \{0, 1\} \quad j = 1, \cdots, k_M N_c$$

where $Y = Y_{N_c}$ if the number of changes in $\delta[k]$ is to be minimized, and $Y = Y_{N_p} S$ if the number of pump commutations is to be considered. The weighting factor α must be chosen carefully to find a compromise between minimization of the cost and the number of commutations.

In order to add the expression of the number of changes to the cost index in Yalmip, such that the solvers can deal with it, instead of $\mathbf{\Delta}^T Y^T Y \mathbf{\Delta}$, the equivalent expression $sum(abs(Y\mathbf{\Delta}))$ is used.

5.3 Adding Constraints

The drawback of the previous approach is that the result is highly dependent on the weighting factor α used, and is not easy to predict the result in terms of number of commutations as a function of α. The second alternative tries to overcome this drawback. It consists of adding new constraints to limit the number of changes. The optimization problem would then be:

$$\begin{aligned}
&\min_{\Delta} \ \mathbf{TP}\Delta \qquad (38)\\
&s.t.\\
&\mathbf{F}_k\mathbf{\Delta} \geq 3600\frac{V_{min}-V(0)}{h} + \hat{F}_O[k],\ k=1,\ldots,k_M\\
&\mathbf{F}_k\mathbf{\Delta} \leq 3600\frac{V_{max}-V(0)}{h} + \hat{F}_O[k],\ k=1,\ldots,k_M\\
&\mathbf{F}_{k_M}\mathbf{\Delta} \geq \hat{F}_O[k_M]\\
&\mathbf{\Delta}^T Y^T Y \mathbf{\Delta} \leq c_{max}\\
&\mathbf{I}_k\mathbf{\Delta} = 1,\ k=1,\ldots,k_M\\
&\mathbf{\Delta}[j] \in \{0,\ 1\} \quad j=1,\cdots,k_M N_c
\end{aligned}$$

where again $Y = Y_{N_c}$ if the number of changes in $\delta[k]$ is to be minimized, and $Y = Y_{N_p}S$ if the number of pump commutations is to be considered. The constant c_{max} represents the maximum number of changes or pump commutations in the overall optimization period.

Again, in order to add the expression of the number of changes to the constraints in Yalmip, instead of $\mathbf{\Delta}^T Y^T Y \mathbf{\Delta}$, the equivalent expression $sum(abs(Y\mathbf{\Delta}))$ is used.

6 APPLICATION EXAMPLE

A real application example, obtained from a real water supply facility, will be used to illustrate the proposed approach. The considered pumping system has $N_p = 3$ wells (pumps), $N_v = 2$ valves and $N_t = 3$ tanks. The tank 3 can be filled from pump 1, no matter the state of the valves, or from pump 2, if the valve 2 is open, or from pump 3, also if the valve 2 is open. The tank 2 can only be filled from pump3 if the valve 2 is closed and the valve 1 open. Finally, the tank 1 can only be filled from pump 3 if both valves are closed. The pump 1 and 3 can work simultaneously, but pump 2 can only operate if the other two are stopped.

Taking into account the physical limitations described above, the matrix that defines de $N_c = 10$ valid combinations of pumps and valves is shown in table 2. Again, the X value represents that the flows do not depend on the state of the valve.

Table 2: Valid combinations in the application example.

Comb	V_1	V_2	P_1	P_2	P_3
0	X	X	0	0	0
1	0	0	0	0	1
2	1	0	0	0	1
3	X	1	0	0	1
4	X	1	0	1	0
5	X	X	1	0	0
6	0	0	1	0	1
7	1	0	1	0	1
8	X	1	1	0	1
9	X	1	1	1	0

For each combination of valves and pumps, there is a resulting outlet flow of each pump, and a resulting inlet flow for each tank, defined by the respective matrices. The resulting flow matrices are (in m^3/h):

$$F_P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 80 \\ 0 & 0 & 100 \\ 0 & 0 & 120 \\ 0 & 100 & 0 \\ 200 & 0 & 0 \\ 200 & 0 & 80 \\ 200 & 0 & 100 \\ 200 & 0 & 120 \\ 200 & 100 & 0 \end{bmatrix}^T \qquad (39)$$

$$F_T = \begin{bmatrix} 0 & 0 & 0 \\ 80 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 120 \\ 0 & 0 & 100 \\ 0 & 0 & 200 \\ 80 & 0 & 200 \\ 0 & 100 & 200 \\ 0 & 0 & 320 \\ 0 & 0 & 300 \end{bmatrix}^T \qquad (40)$$

For each combination there is a matrix that relates the pump flows to the tank flows. For example, for combination 6, the tank flows can be expressed as a function of pump flows as

$$f_T = M_6 f_P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 200 \\ 0 \\ 80 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ 200 \end{bmatrix} \tag{41}$$

The matrix that relates the flows is different for each combination of pumps and valves. In the example the matrices are:

$$M_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_1 = M_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_2 = M_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_3 = M_4 = M_5 = M_8 = M_9 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

The matrix P defines the electric power consumption of each pump for each combination. The matrix has as many columns as combinations, and one row per pump, i.e. the size of matrix P is $(N_p \times N_c)$. In the proposed example, the power matrix (in kW) is:

$$P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 13 \\ 0 & 0 & 11 \\ 0 & 0 & 7 \\ 0 & 5 & 0 \\ 12 & 0 & 0 \\ 12 & 0 & 13 \\ 12 & 0 & 11 \\ 12 & 0 & 7 \\ 12 & 5 & 0 \end{bmatrix}^T \tag{42}$$

In order to perform the calculations, the outlet flow of the tanks is estimated through a Fourier series from real data. The figure 1 shows the ouput flow of the three tanks for one day, starting at 8 a.m.

The simple optimization problem (32) has been solved with Matlab, using Yalmip as parser and Mosek solver.

As we are enforcing a final volume equal than the initial one, the result of the optimization depends on the initial condition of the tanks, and the instant of time during the day when the optimization is carried out. In order to set a framework for comparison purposes, the optimization will be assumed to be carried out at the instant when the cheapest period ends, and the initial volumes are assumed to be

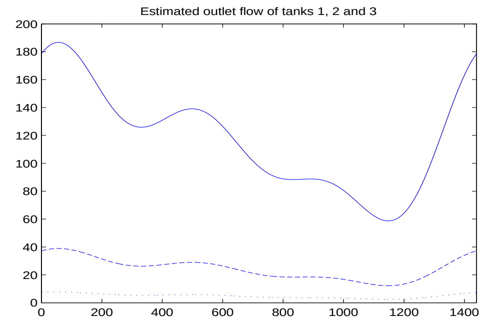

Figure 1: Estimated outlet flow of tanks 1,2 and 3, in m^3/h.

$V(0) = 0.95V_{max}$, i.e. near the maximum. This conditions reinforce the idea of maximizing water availability in case a problem occurs with any pump.

The figure 2 shows the result of the optimization with $h = 5$ minutes, $L = 4$, $k_m = 60$ and $k_M = 117$, i.e. 5 hours at a period of 5 minutes, and 19 hour at a period of 20 minutes. The optimum value is $J = 17.22$. The figure shows the volumes in percentage, the flow of the three pumps (with the timely price of the associated tariff), and the state of the valves. The drawback of this approach is that the number of commutations in the day is quite high (56 pump commutations and 28 valve commutations). The commutations are especially frequent during the first 5 hours when the period of discretization is smaller. If a period of $h = 5$ minutes is used for all the day, the number of commutations is 120 for the pumps and 68 for the valves, while the cost index is slightly lower $J = 17.04$.

If the number of changes in δ is considered in the cost index, with a weighting factor $\alpha = 0.02$, the result of the optimization (37) is shown in figure 3. The number of pump commutations is 31, and the number of valve changes is 14. The cost index is only slightly higher then the case where the number of changes is not included in the cost index, $J = 17.26$ (only the energy cost).

The result of the optimization with $\alpha = 0.05$ is shown in figure 4. The cost is similar, $J = 17.23$, but there are only 15 pump and 8 valve commutations. With $\alpha = 0.1$, the result is a higher cost ($J = 17.54$), but a really small number of commutations (10 pumps and 6 valves).

The resulting number of commutations is highly dependent on the weighting factor α. In order to fix a desired value, the second approach consists of adding as a constraint the number of commutations. The figure 5 shows the result of the optimization (38) when the number of changes in δ is constraint to 20. The cost index is $J = 17.12$ and the pump and valve commutations 18 and 9, respectively.

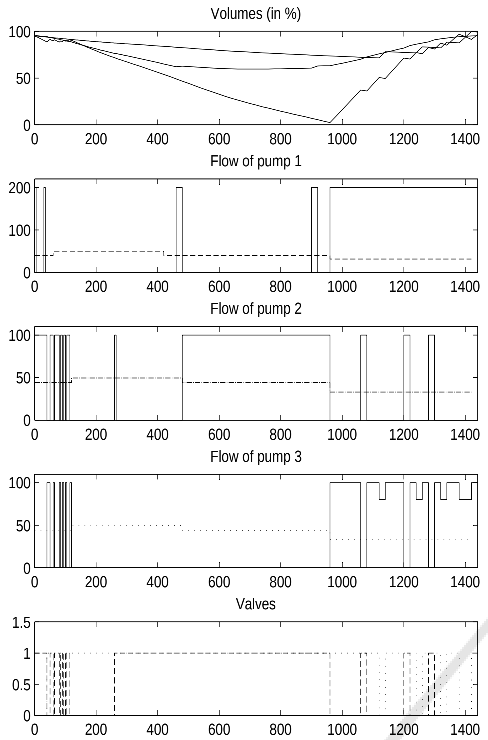

Figure 2: Optimization result for period=5 minutes, L=4. Volumes (in percentage), flow of pump 1, 2 and 3, and state of valves.

The figure 6 shows the result of the optimization when the number of changes in δ is constraint to 16. The cost index is $J = 17.1$ and the pump and valve commutations 15 and 9, respectively.

The results are similar to those obtained with the previous approach (in terms of cost index), but the constraint in the number of commutations is explicit.

7 CONCLUSIONS

In this paper we have studied the optimization of the operation of a water supply pumping system by means of standard solvers. The automatic operation of the system tries to determine which valves and pumps must be active at each instant of time in order to minimise the operation cost, taking into account the tariff periods. The main constraints are the maximum and minimum volumes of the tanks. A mathematical model of the problem is proposed in order to formulate, in matrix form, the cost index and the constraints, to be able to use standard solvers as Mosek or CBC. A basic optimization problem is proposed, that only imposes the constraints of the finite volume of the tanks.

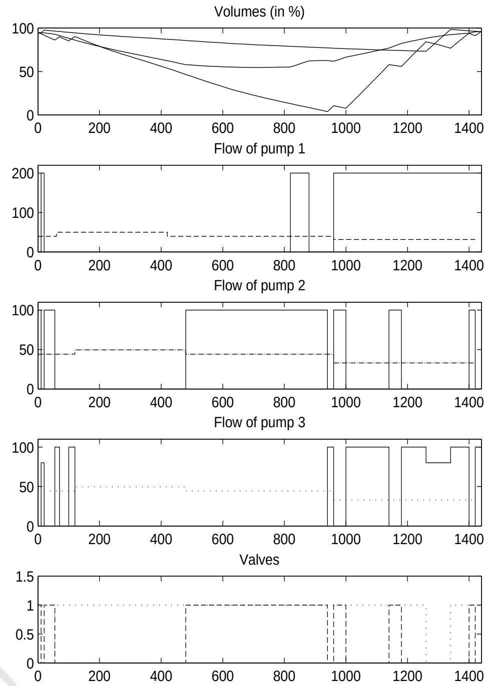

Figure 3: Optimization result for period=5 minutes, L=4. Volumes (in percentage), flow of pump 1, 2 and 3, and state of valves. Cost index with number of changes, $\alpha = 0.02$.

The result of this basic problem tends to produce a large number of pump and valve commutations, that is not adequate in practice. To solve this problem, two alternatives are proposed: to include the number of commutations in the cost index, and to add as a constraint the maximum number of commutations. Both approaches lead to an adequate result, but the second one is easier to use, as the number of commutations can be fixed in an explicit way, while in the other approach the final number of commutations depends on the weighting factor applied to the number of commutations in the cost index. An example of a real water supply system with 3 tanks, 3 pumps and 2 valves is analysed to demonstrate the validity of the approach, using Yalmip as parser and Mosek as solver. The basic optimization problem leads to a high number of commutations, as expected. Including the number of commutations in the cost index, or adding as a constraint the maximum number of commutations lead to a slight increase in the cost while the number of changes in the actuator states is maintained under a reasonable level.

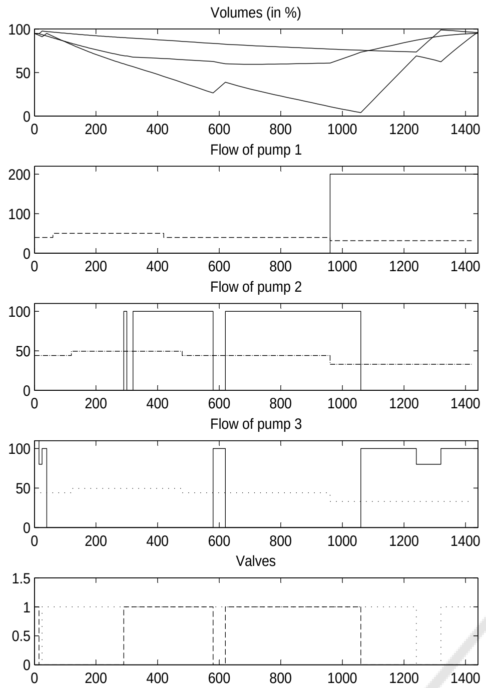

Figure 4: Optimization result for period=5 minutes, L=4. Volumes (in percentage), flow of pump 1, 2 and 3, and state of valves. Cost index with number of changes, $\alpha = 0.05$.

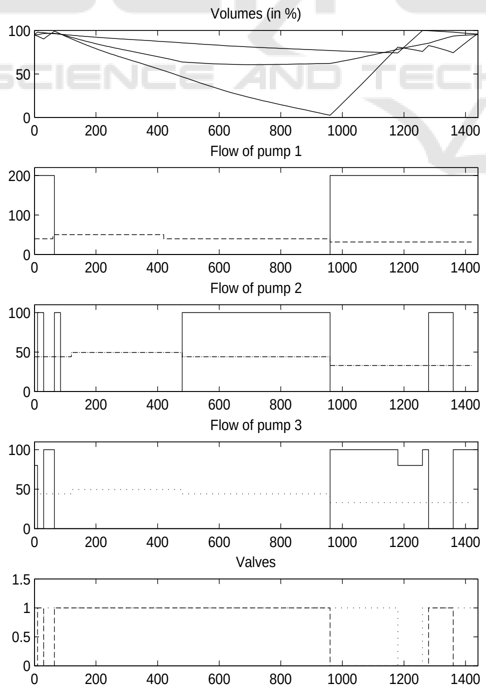

Figure 5: Optimization result for period=5 minutes, L=4. Volumes (in percentage), flow of pump 1, 2 and 3, and state of valves. Constraint in the number of changes, 20.

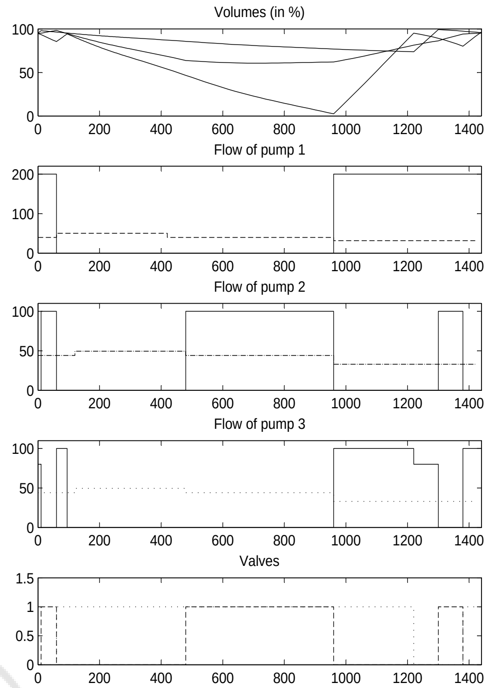

Figure 6: Optimization result for period=5 minutes, L=4. Volumes (in percentage), flow of pump 1, 2 and 3, and state of valves. Constraint in the number of changes, 16.

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