Investigating the Use of Primes in Hashing for Volumetric Data

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Abstract: Traditional hashing methods used to store 3D volumetric data utilise large prime numbers. The intention of this is to achieve well-distributed hash addresses to minimise addressing collisions. Hashing is an attractive method to store 3D volumetric data, as it provides simple method to store, index and retrieve data. However, implementations fail to provide theoretical support as to why they utilise large primes which act to create a hash address through randomising key values. Neither is it specified what a "large" prime is. 3D data is inherently well-distributed as each coordinate in 3D space is already unique. It is thus investigated in this paper whether this randomisation through the use of large primes is necessary. The history of the use of primes for hashing 3D data is also investigated, as is whether their use has persisted due to habit rather than due to methodical investigation.

1 INTRODUCTION

Hashing is used to map data of arbitrary size to an addressing space, and is a popular method to store, retrieve and delete 3D volumetric data (Teschner et al., 2003) (Klingensmith et al., 2015) (Niener et al., 2013) (Eitz and Lixu, 2007) (Kähler et al., 2015). In much of the early literature discussing hashing (Knott, 1975) (Knuth, 1998), the common goal is to achieve distinct mapping addresses. However, Knuth states that "*it is theoretically impossible to define a hash function that creates truly random data from nonrandom data in files*" (Knuth, 1998). The birthday paradox (Good, 1950) highlights the difficulty in achieving distinct addresses: if a random function is selected to map 23 keys to a table of size 365, the probability that no two keys map to the same location is only 0.4927.

In hashing, the address calculation is generally achieved by a randomised scrambling of key values, with many methods using large primes to achieve this scrambling (Teschner et al., 2003) (Klingensmith et al., 2015) (Niener et al., 2013) (Eitz and Lixu, 2007) (Kähler et al., 2015). Hashing is an appropriate term, as a hash is "*a random jumble achieved by hashing*", where hashing means to cut or to chop (Knott, 1975).

Hash coding was first described in open literature by Arnold I. Dumey in the 1950s (Knuth, 1998) (Dumey, 1956) (although the idea of hashing appears to have been originated by H.P Luhn in an internal IBM memorandum in January 1953). Dumey discusses a real life example where the details of eight machines that are sold to the public must be stored. To ensure that the details of no two machines would be stored at the same address, originally the six digit identifiers of the machines were used. However, on inspection of the identifiers, it was found that the fourth digit of each of them was unique. It was therefore sufficient to store only the fourth digit as an identifier, and release the other five for other purposes. Although this analogy is rather extreme, Dumey explains that an examination of the item description may reveal built-in redundancy which can be used to reduce memory usage.

It is important to note that the choice of Hash function is dependent on the application. Some applications are focused on maintaining data integrity (i.e minimising the number of hashing collisions), while others are focused on execution speed (Eitz and Lixu, 2007) (speed of data retrieval). As well as minimising hashing collisions, another consideration is to provide a technique that allows for an ease of data insertion and removal, which is paramount when considering dynamic data.

A hashing algorithm should be selected so that the addresses are uniformly distributed (Teschner et al., 2003). An optimal solution is to provide a perfect hashing function, which allows the retrieval of data in a hash table with a single query (Jaeschke, 1981) (Sprugnoli, 1977). This would provide a hash with no addressing collisions. Intuitively, this could

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be achieved by providing a suitably large hash table. However, this is not always practical as volumetric data is often used in memory confined situations, such as for SLAM (Simultaneous Location And Mapping) on mobile and robotic platforms using real-time sensors (Klingensmith et al., 2015; Kähler et al., 2015). What is apparent however, is that 3D volumetric data inherently possesses well distributed voxel addresses.

Referring to hashing functions mapping different data to the same hash address, it has been stated that "...such collisions cannot be avoided in practice" (Kähler et al., 2015). It is true that collisions cannot be completely avoided, but it is investigated in this paper whether certain prime values can be chosen to reduce/minimise the number of collisions.

This paper not only compares the use of different prime values in 3D volumetric data, but also investigates whether large primes are necessary at all. The remainder of the paper is structured as follows:

- Section 2 - Related Work
- Section 3 - Hashing Parameters
- Section 4 - Tests administered
- Section 5 - Results

2 RELATED WORK

> Anyone who attempts to generate random numbers by deterministic means is, of course, living in a state of sin
>
> *John von Neumann*

To understand why the use of primes in the hashing of 3D data has persisted, an examination of the most important classical hashing methods is required. These methods are explored by Knuth (Knuth, 1998) in much detail, some of which are described below:

2.1 Division Hashing Method

For certain scenarios, it was found that basic hashing methods can provide adequate results by providing a simple mapping for a memory limited map. The division method is particularly simple (Knuth, 1998):

$$H(K) = K\%M \tag{1}$$

Where K is the identifier of the data to be hashed (or the Key), M is an arbitrary number and % is the modulus operator. Knuth emphasises the importance in choosing the value of M, and how this choice can introduce bias into the results. For example, if M is an even number, $H(K)$ will be even when K is even, and odd when K is odd, which would lead to substantial bias. Taking this and further considerations into account, he suggested choosing M to be a prime number such that $r^k \neq \pm(a\%M)$, where k and a are small numbers and r is the radix of the character set. Knuth states that using these prime numbers "*has been found to be quite satisfactory in most cases*", but does not provide results. Note that Knuth's recommendation of the use of primes relates only to the case specified above.

2.2 Multi-identifier Hashing Function

The Division Hashing Method above in Section 2.1, while effective for simple scenarios, is only of use for data with a single identifier. For data with multiple identifiers Knuth suggests utilising multiple independent hash functions - one per identifier - and then combining the results in some way. This idea was first introduced by J. L. Carter and M. N. Wegman and the equation is given below (Knuth, 1998; Carter and Wegman, 1979):

$$H(K) = (h_1(x_1) + h_2(x_2) + ... + h_l(x_l))\%N \tag{2}$$

where l is number of identifiers, h_j is an independent hash function, x_j is an identifier and N is the number of addresses available. This function is especially efficient when x_j is a single character because a LUT (look up table) can be utilised for h_j, which removes the need for multiplications.

2.3 XOR Hashing Function

Many implementations (Teschner et al., 2003) (Klingensmith et al., 2015) (Niener et al., 2013) (Eitz and Lixu, 2007)(Kähler et al., 2015) (Kähler et al., 2016) utilise an XOR hashing function to index 3D volumetric data into a hash table. This hash address is retrieved using Equation (3):

$$hash(x,y,z) = (P_1 * x \oplus P_2 * y \oplus P_3 * z)\%N \tag{3}$$

where $\oplus$ is the XOR operation, (P_1, P_2, P_3) are selected prime values, x, y, z are the 3D coordinates of the data in the volumetric structure that is to be stored and N is the size of the hash table. None of (Teschner et al., 2003) (Klingensmith et al., 2015) (Niener et al., 2013) (Eitz and Lixu, 2007) (Kähler et al., 2015) clarify why large primes are utilised. It is not stipulated in (Kähler et al., 2016) that (P_1, P_2, P_3), or the "hash coefficients" be primes.

In the case of hashing for 3D volumetric data, XOR hashing is the most commonly used, and is thus examined in further detail.

3 PARAMETERS IN XOR HASHING

There are many parameters that influence the results of the chosen hashing function. As discussed in section 1, a "successful" hash function is dependent on the application. For the purposes of this paper, the focus is on maintaining data integrity (minimising the number of voxels that are not stored in the hash table due to addressing collisions) while maintaining a reasonable data footprint. Regardless of the hashing function chosen, the following parameters can have an impact on the results of the function.

3.1 Use of Primes

The use of primes is common in hashing functions, with (Teschner et al., 2003) (Klingensmith et al., 2015) (Niener et al., 2013) (Eitz and Lixu, 2007) (Kähler et al., 2015) all using large primes. Knuth (Knuth, 1998) affirms that the first mention of using primes for hashing is by Dumey (Dumey, 1956) in 1956, where taking the remainder of dividing by a prime is discussed as a method of mapping data to addresses in a hash table. Dumey states: " *...divide this number by a number slightly less than the number of addressable locations (the writer prefers the nearest prime)*" indicating that the choice to use primes is a personal preference.

Many factors must to be taken into consideration when choosing constant/prime values in hashing functions (Knuth, 1998). As discussed in Section 2.1, Knuth found certain primes to be adequate when using the Division Hashing Method in Equation (1), but does not provide a blanket endorsement for using primes. Why the use of primes in hashing for 3D volumetric data persists may be understood with the help of a quote from John von Neumann: "*Young man, in mathematics you don't understand things. You just get used to them.*"

3.2 Hash Table Size

The size of the hash table relates to how many hash table addresses exist. The size of the hash table is of importance for a number of reasons. As discussed in Section 2.1, when the hash address is a modulus of the hash table size, the choice of hash table size can introduce biases. However, the focus of this paper is to investigate the influence of using primes in hashing, so the impact of altering the size of the hash table is not investigated at present.

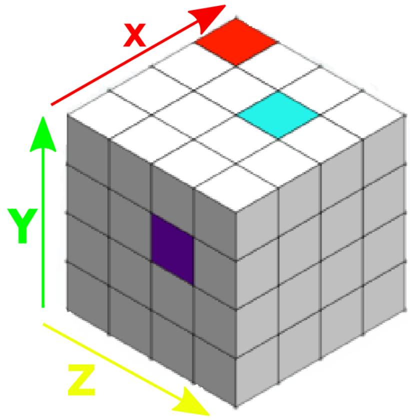


Figure 1: 4x4x4 voxel block.

3.3 Hash Table Entry Size

When H.P. Luhn first suggested hashing, he stressed the advantage of using hash table entries that store more than one element, as this reduces the number of external searches required to find the correct entry (Knuth, 1998). The hash table entry size equates to the number of voxels (and any other identifying data) that can be stored in a single hash table entry. The blocks sizes used in other implementations vary, with some not disclosing the size (Mirtich, 1997) (Teschner et al., 2003), and others using sizes of up to 512 voxels (8^3) (Kähler et al., 2016) (Kähler et al., 2015). This paper represents 64 voxels (4^3) per hash entry, this number is variable.

A reason for not wanting to represent a large number of voxels per hash entry is due to the sparse nature of much 3D data - the Dublin City Dataset (Laefer et al., 2015) was found to have an average occupancy of 1.36% (98.74% sparse), but with a large distribution of voxels. This is similar to the typical 93% sparsity found in the scenes examined in (Klingensmith et al., 2015). Representing 64 voxels per entry was found to be adequate, as if noise is present in the form of outliers, only a single entry would be allocated for that noise. The larger the hash entry, the more memory will be allocated to noisy voxels.

These 64 voxels represent a 4^3 block, like the one shown in Figure 1. When it is determined that a voxel is occupied and must be added to the hash table, it is first determined which block it belongs to. In a 64^3 voxel environment, there are 16^3 4^3 blocks. The identifier of the block is taken as the coordinates of the corner node of the block (i.e. the red voxel in Figure 1), discretised with respect to the size of the blocks. This is similar other implementations (Teschner et al., 2003) (Mirtich, 1997) (Kähler et al., 2015) and (Kähler et al., 2016).

4 COMPARISON OF PRIME VALUES FOR HASHING

To adequately determine whether large primes are required to minimise addressing collisions, various tests were administered on two datasets, the Stanford Bunny (Turk and Levoy, 2005) and the Liffey Tile from the Dublin City Dataset (Laefer et al., 2015). Parameters for the Stanford Bunny and the Liffey tile are shown in Table 1.

Further tests were then administered on larger, real world datasets which are detailed in Appendix A.

Table 1: Parameters for the Stanford Bunny and the Dublin City Dataset Liffey Tile.

	Bunny	Liffey Tile
Resolution	64^3	256^3
No occupied voxels	11070	234131
% Occupancy	4.2%	1.4%
Max x	63	255
Max y	62	255
Max z	49	153
Hash Table Size (No. Entries)	2040	128000
Hash Table Size (Bytes)	32kB	2MB

The following tests were carried out on the Stanford Bunny and the Liffey Tile:

- Comparison of Prime Values for XOR Hashing
- Comparison of the Ordering of Prime Values for XOR Hashing
- Comparison of Non Prime Values for XOR Hashing
- Determination of the influence of the choice of constant values on the distribution of addressing collisions

Upon completion of the above tests, the following were then administered on the larger, real world datasets:

- Determination of whether "Optimum values" exist for XOR Hashing

The following actions/events occur when testing various primes for XOR Hashing on the datasets previously mentioned:

- **Insertion.** Insertions into the Hash table are very simple to perform. Once it is determined that a voxel is occupied and should be inserted into the hash table, the hash address of the voxel block that the voxel belongs to is calculated using Equation 3. If an addressing collision does not occur, the bit corresponding to that voxel in the hash entry is set to one. If an addressing collisions occurs the voxel cannot be set. Dealing with collisions is discussed below.
- **Collisions.** While (Teschner et al., 2003) discusses the problem of mapping different data to the same hash address, and suggests increasing how many voxels are represented in each hash entry in an attempt to reduce collisions, it is not discussed how to deal with these collisions when they do occur. There are very simple methods in use to deal with these collisions. (Niener et al., 2013) deals with collisions through storing the data to be inserted in the next available sequential entry. A.D Lin, inspired by Luhn's early work (Knuth, 1998), suggested a technique which could accommodate collisions using "*degenerative addresses*". For example, if a collision occurs in entry 2748, the voxels would be placed in entry 274, from which overflows would be placed in entry 27 and so on.

  This paper does not implement collision resolution, as the benchmarking of different prime values is concerned with the percentage of addressing collisions. Resolving these collisions introduces additional overheads, so it is thus preferable to invest effort into reducing the initial number of collisions than to focus on accommodating these collisions. If a collision occurs an attempt is not made to accommodate the voxel which caused the collision.

5 RESULTS

The results for the tests detailed in Section 4 are outlined below.

5.1 Comparison of Prime Values for XOR Hashing

As discussed in section 3.1, many implementations of hashing for 3D volumetric data, such as (Teschner et al., 2003; Niener et al., 2013; Eitz and Lixu, 2007) and (Kähler et al., 2015) utilise specified large primes to calculate the hashed address. However, none of the aforementioned implementations provide evidence as to why they utilised those large primes. ((Klingensmith et al., 2015) does not specify the prime values, but uses "*arbitrary large primes*")

To determine why the primes in these implementations were chosen, various prime values were tested for both the Stanford Bunny (Turk and Levoy, 2005) and the Liffey Tile from the Dublin City Dataset (Laefer et al., 2015) for XOR Hashing. All prime values

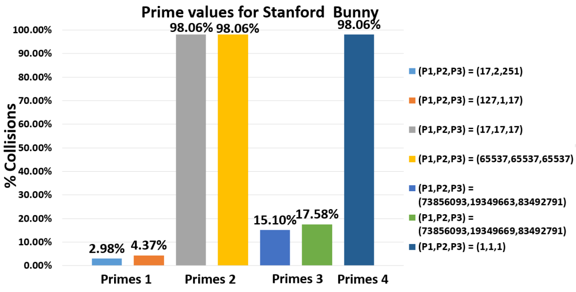


Figure 2: Comparison of Prime Values for the Stanford Bunny.

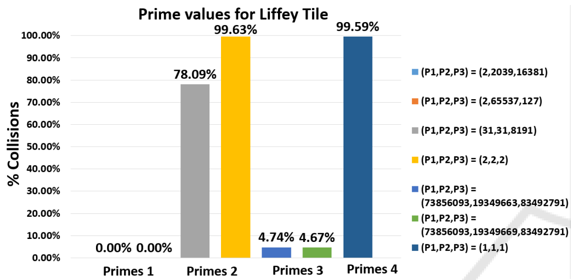


Figure 3: Comparison of Prime Values for the Liffey Tile.

less than 83492791, the largest of the primes suggested by (Teschner et al., 2003) were considered. 2040 and 128000 Hash Table Entries were used respectively. These table sizes were chosen as they equate to roughly half the total number of voxels in each of these datasets.

A subset of the results obtained from testing various primes for the Stanford Bunny and the Liffey Tile are shown in Figures 2 and 3 respectively. These results were chosen to demonstrate how the choice of primes can greatly influence the addressing collision rate. The following combinations are displayed:

1. Prime combinations that produced the lowest addressing collision rates (Primes 1)
2. Prime combinations that produced the highest addressing collision rates (Primes 2)
3. Prime combinations used in (Teschner et al., 2003; Eitz and Lixu, 2007; Niener et al., 2013) and (Kähler et al., 2015) (Primes 3)
4. All prime values set to one (Primes 4)

It was found that the primes used in (Teschner et al., 2003) and (Eitz and Lixu, 2007), and those used in (Niener et al., 2013) and (Kähler et al., 2015) achieved an addressing collision rate which was higher than other combinations of primes. It was found, as is evident from Figures 2 and 3, that many combinations of primes provided the minimum number of addressing collisions, i.e. 0% for the Liffey tile, and very low rates (2.98%) for the Bunny. While these values are still primes, they are not all "*large primes*" as stipulated in (Klingensmith et al., 2015).

These results indicate that "*large primes*" are not required to achieve optimal results for XOR hashing for volumetric data.

Table 2: Testing of various combinations of ordering of primes on the Stanford Bunny.

			Bunny
P_1	P_2	P_3	% Collisions
73856093	19349663	83492791	15.09
73856093	83492791	19349663	17.39
83492791	73856093	19349663	16.81
83492791	19349663	73856093	16.78
19349663	83492791	73856093	16.32
19349663	73856093	83492791	17.51

5.2 Comparison of the Ordering of Prime Values for XOR Hashing

Another observation is that none of (Teschner et al., 2003; Niener et al., 2013; Eitz and Lixu, 2007) or (Kähler et al., 2015) discuss the ordering of the prime utilised, i.e. which of the primes are assigned to (P_1, P_2, P_3) respectively.

To investigate whether varying the order of the primes would impact the addressing collision rate, the order of the primes from (Teschner et al., 2003) and (Eitz and Lixu, 2007) were varied for the Stanford Bunny. These results are shown below in Table 2.

While the collision rates did not vary greatly for the primes used in Table 2, other prime values produced greater variance. Testing ordering combinations of $(P_1, P_2, P_3) = (2, 17, 127)$ for the Stanford Bunny produced addressing collision rates of between 6.02% and 16.10%.

These results indicate that not only does the choice of primes impact the addressing collision rate for XOR Hashing, so too does the ordering of those primes.

5.3 Comparison of Non Prime Values for XOR Hashing

Upon the discovery in Section 5.2 that large primes are not necessary to optimally hash 3D volumetric data, it was investigated whether there is a need for the "*prime*" values used in XOR Hashing to be primes at all.

The same methodology as described in Section 5.2 was followed for this test - various non prime values were used for (P_1, P_2, P_3) from equation 3 for XOR

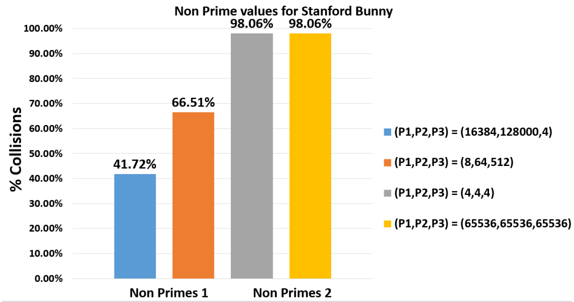


Figure 4: Comparing non prime values on the Stanford Bunny.

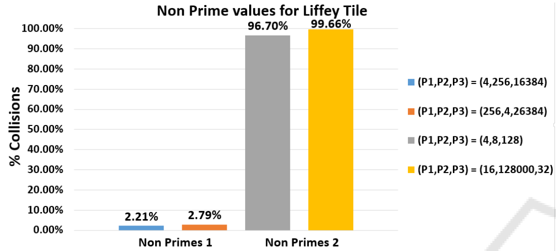


Figure 5: Comparing non prime values on the Dublin City Dataset Liffey Tile.

Hashing. These were again tested on the Stanford Bunny (Turk and Levoy, 2005) and the Liffey Tile from the Dublin City Dataset (Laefer et al., 2015). To ensure that the values tested were not primes, powers of 2 were used, up to 2^{17}. A subset of these results are shown in Figures 4 and 5. These results were chosen to demonstrate how the choice of non primes can greatly influence the addressing collision rate. The following combinations are displayed:

1. Non Prime combinations that produced the lowest addressing collision rates (Non Primes 1)
2. Prime combinations that produced the highest addressing collision rates (Non Primes 2)

Testing with non prime values for XOR Hashing provided mixed results. While addressing collision rates of $< 3\%$ were obtained using non prime values for the Liffey Tile, the lowest rate obtained for the Stanford Bunny was 41.72%.

While using non prime values for the Liffey Tile did not produce addressing collision rates as low as when using some prime combinations, it produced better results than using (P_1, P_2, P_3) from (Teschner et al., 2003; Eitz and Lixu, 2007), and (Niener et al., 2013; Kähler et al., 2015). However, non prime values for the Stanford Bunny did not produce addressing collision rates lower than when using (P_1, P_2, P_3) from (Teschner et al., 2003; Eitz and Lixu, 2007), and (Niener et al., 2013; Kähler et al., 2015).

This indicates that while non prime values for XOR Hashing can provide better results than using primes for some datasets, one cannot definitely say that using primes/non primes is favorable over using the other.

5.4 Determination of the Influence of the Choice of Constant Values on the Distribution of Addressing Collisions

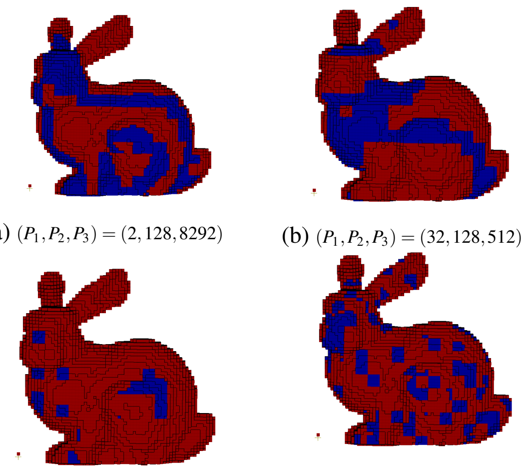
The distribution of collisions could be of concern depending on the application. For example, a large concentration of collisions in one place could lead to the loss of say an entire object in an indoor environment when using hashing for SLAM applications. This could be problematic for an indoor robot scanning a living room if collisions occurred for all the voxels associated with a chair. However, if the collisions are better distributed, only a number of addressing collisions may occur from a few objects. It is therefore important to determine if certain combinations of constant values can provide well distributed addressing collisions for a range of 3D volumetric data.

A visual representation for the collision rates using the constant values from Section 5.5 are shown in Figures 6 and 7 for both the Stanford Bunny (Turk and Levoy, 2005) and the Liffey Tile from the Dublin City Dataset (Laefer et al., 2015). These figures indicate the distribution of addressing collisions for both models, and how these distributions can be influenced by the constant values chosen for XOR Hashing. In Figures 6 and 7 red voxels represent voxels that are inserted into the hash table successfully. Blue voxels represent where an addressing collision occurred and thus the voxels could not be inserted.

What is apparent from these results is not only does the choice of constants for (P_1, P_2, P_3) significantly impact the addressing collision rate of XOR Hashing, it also impacts the distribution of collisions.

This is more apparent from Figure 6 than Figure 7 as the resolution is smaller (64^3 voxels v 256^3). In Figures 6a and 6b, the majority of collisions that occur are grouped together. However, in Figures 6c, and 6d, while the percentage of addressing collisions are lower, the collisions that do occur are for the most part well distributed across the model.

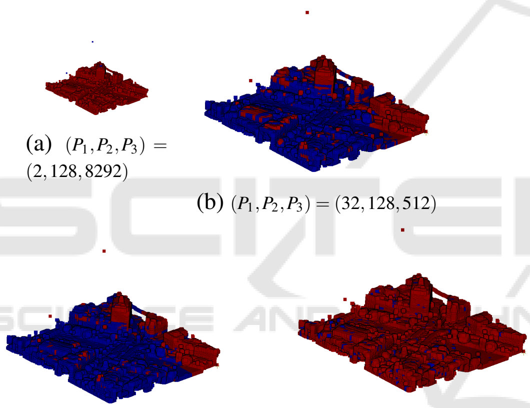
However, one cannot draw definitive conclusions from the distribution of collisions in Figure 6, as the same is not necessarily true in Figure 7. Figure 7c uses the same constants as in Figure 6c, but with a far higher addressing collision rate (81.27% v 2.98%), and thus cannot provide well distributed collisions for



(a) $(P_1, P_2, P_3) = (2, 128, 8292)$ (b) $(P_1, P_2, P_3) = (32, 128, 512)$

(c) $(P_1, P_2, P_3) = (17, 2, 251)$ (d) $(P_1, P_2, P_3) = (73856093, 19349663, 83492791)$

Figure 6: Comparing different combinations of constant values for XOR Hashing on the Stanford Bunny (Turk and Levoy, 2005).



(a) $(P_1, P_2, P_3) = (2, 128, 8292)$ (b) $(P_1, P_2, P_3) = (32, 128, 512)$

(c) $(P_1, P_2, P_3) = (17, 2, 251)$ (d) $(P_1, P_2, P_3) = (73856093, 19349663, 83492791)$

Figure 7: Comparing different combinations of constant values for XOR Hashing on the Liffey Tile from the Dublin City Dataset (Laefer et al., 2015).

the Liffey Tile in Figure 7c as the collision rate is so high.

While the constants used in Figures 6d and 7d do provide well distributed addressing collisions, there are constant value combinations that provide a lower addressing collision rate for the two models.

It is thus determined that while certain constant value combinations provide well distributed addressing collisions for the two models tested, these values do not necessarily provide the lowest possible addressing collision rate.

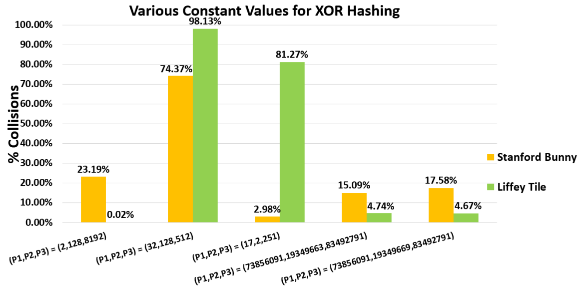


Figure 8: Comparing different constant values for XOR Hashing for the Stanford Bunny and the Liffey Tile from the Dublin City Dataset.

5.5 Determination of Whether "Optimum Values" Exist for XOR Hashing

In Section 5.2 it was shown that large primes are not necessary to minimise addressing collisions for XOR hashing - smaller primes produced less addressing collisions than the large primes recommended in (Teschner et al., 2003) and (Eitz and Lixu, 2007).

In Section 5.3 it was shown that for the Liffey Tile, small, non prime values can produce far fewer addressing collisions for XOR hashing when compared with the large primes recommended in (Teschner et al., 2003), (Eitz and Lixu, 2007), (Niener et al., 2013) and (Kähler et al., 2015). However, this did not hold true for the Stanford Bunny.

On the back of both of these findings, it was then investigated whether one set of constant values for (P_1, P_2, P_3) in XOR Hashing can provide optimal results regardless of the 3D volumetric data being considered. All of (Teschner et al., 2003; Klingensmith et al., 2015; Niener et al., 2013; Eitz and Lixu, 2007; Kähler et al., 2015) use large primes for XOR Hashing for 3D volumetric data, but do not consider, in detail, how the nature of the data could influence the choice of constants, be they primes or not.

To determine whether a set of values could be used in XOR hashing that would provide adequate results for a diverse range of 3D volumetric data, various combinations of primes and non primes were tested for the Stanford Bunny (Turk and Levoy, 2005) and the Liffey Tile of the Dublin City Dataset (Laefer et al., 2015). The results are shown below in Figure 8. These values were also tested on the whole Dublin City Dataset (Laefer et al., 2015) and the New York Dataset (of Commerce, 2014a), the results of which are shown in Figures 9 and 10 respectively in the form of a box-plot.

From Figure 8 it is clear that constant values that provide a low addressing collision rate for one

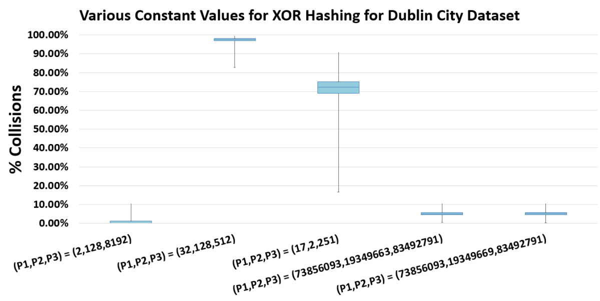


Figure 9: Comparing different constant values for XOR Hashing for the Dublin City Dataset.

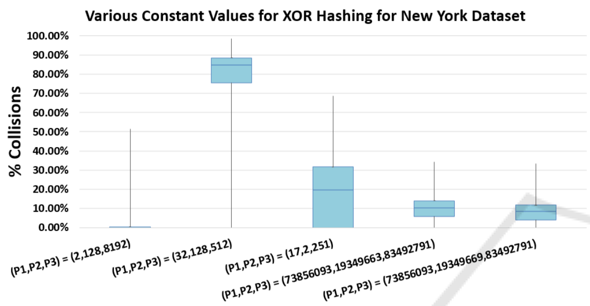


Figure 10: Comparing different constant values for XOR Hashing for the New York City Dataset.

model do not guarantee a low collision rate for another model. This is of importance as the distribution of voxels in models being considered is not always known before hashing.

Figures 9 and 10 indicate again that varying constant values cannot be guaranteed to provide a low addressing collision rate for a wide range of datasets. The results for $(P_1, P_2, P_3) = (2, 128, 8192)$ and $(P_1, P_2, P_3) = (73856093, 19349663, 83492791)$ in both Figures 9 and 10 are statistically significant as there is no overlap between the distributions. These results also indicate that the constant values recommended in (Teschner et al., 2003; Eitz and Lixu, 2007; Niener et al., 2013) and (Kähler et al., 2015) do not always provide the lowest possible addressing collision rate.

It was thus determined that there is no set of "optimum values" that can be used to optimally hash a wide range of 3D volumetric datasets using XOR Hashing, and it is unwise to advise the use of certain values over others without prior knowledge of the data to be hashed.

6 CONCLUSION

The following have been determined regarding the use of primes for the hashing of 3D volumetric data through the research conducted as part of this paper:

- Primes (large or otherwise) are not required in XOR Hashing for optimal results - non primes can provide equal and at times better addressing collision rates.
- The ordering of primes in XOR Hashing impacts the addressing collision rate.
- When primes are used in XOR hashing, larger primes do not necessarily produce optimal results. More specifically, the primes used in (Teschner et al., 2003; Klingensmith et al., 2015; Niener et al., 2013; Eitz and Lixu, 2007) and (Kähler et al., 2015) do not necessarily produce optimal results.
- The choice of constant values in XOR Hashing influences the distribution of addressing collisions. However, the constants that provide the highest distribution of addressing collisions do not necessarily provide the lowest percentage of addressing collisions.
- No one set of values, primes or otherwise, for XOR Hashing can be considered to provide optimal results due to the variance that exists across different 3D volumetric datasets.

Given the results above, it is the view of the authors of this paper that XOR hashing is not the optimal method to hash 3D volumetric data due to the variance that exists with 3D volumetric data, more specifically with dynamic 3D volumetric data. This is due to the need to predefine values for (P_1, P_2, P_3), at times without any prior knowledge of the distribution of the voxels in the data that needs to be hashed. A dynamic method, that could adapt to the distribution of voxels within models would provide more predictability while hashing 3D volumetric data.

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APPENDIX

A DATASETS

The following datasets were examined in the comparison of various hashing techniques as described in Section 4. The datasets include small scale synthetic models and publicly available large scale LiDAR datasets. The large scale models present a realistic representation of what would be processed by an embedded system in the real world, except on a much larger scale.

- **Stanford Bunny.** The Stanford Bunny is a widely used 3D test model developed by Greg Turk and Marc Levoy in 1994 at Stanford University (Turk and Levoy, 2005).
- **Dublin City Dataset.** The Dublin City Dataset is a collection of LiDAR scans of Dublin City (Laefer et al., 2015; Laefer et al., 2017).The scans are separated into tiles. Each of these tiles are 256^3 voxels, 100m on a side. The average occupancy of the Dublin City Dataset is 1.36%.
- **Liffey Tile from The Dublin City Dataset.** This tile was used in some tests. It was chosen as it is representative of the other tiles in the dataset, and is named the Liffey Tile as it shows the Liffey river flowing under the iconic O'Connell Bridge in Dublin's city centre.
- **New York Dataset.** The New York Dataset refers to the 2014 U.S. Geological Survey CMGP LiDAR: Post Sandy (New Jersey) (of Commerce, 2014a; of Commerce, 2014b). This dataset is also a collection of Lidar scans of New York and New Jersey. Each of these tiles are 64^3 voxels, 100m on a side, equating to $10,000m^3$. The average occupancy of the New York Dataset is 2.64%.