N4SID-VAR Method for Multivariable Discrete Linear Time-variant System Identification

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Abstract: In this paper, a method for multivariable discrete linear time-variant system identification is presented. This work is focused on slowly multivariable time-variant systems, so that it is possible to define time intervals, defined as windows, in which the system can be approximated by time-invariant models. In each window, a variation of N4SID that uses Markov parameters is applied and a state space model is estimated. For that reason the proposed method is defined as N4SID-VAR. After obtaining the models for all windows, the error between system model outputs are calculated and compared to the system outputs. The N4SID-VAR was tested with a time-variant multivariable benchmark and the results were accurate. The proposed method was also compared to the MOESP-VAR method and, for the tested benchmark, the N4SID-VAR was faster and more accurate than the MOESP-VAR algorithm.

1 INTRODUCTION

System identification consists in the search for a mathematical model that can describe the behavior of a dynamical system, from the observed input-output signals (Ljung, 1999), (Katayama, 2005). A significant part of activities and researches in system identification focuses on time-invariant dynamical systems. However, there are innumerable systems in nature that are multivariable with nonlinear and time-variant behavior. To deal with last problem, the time-variant systems can be approximated by linear time-invariant systems, as long as these systems vary slowly (Tamariz et al., 2005).

During the last two decades, subspace-based methods have been extensively studied to address the problem of identifying multivariable discrete linear time-invariant systems (Katayama, 2005). From that methods, the most popular are the MOESP (Verhaegen and Dewilde, 1992) and the N4SID (Overschee and Moor., 1994). Both methods have a mathematical support in linear matrix algebra.

The MOESP method is based on LQ decomposition of a matrix formed by input-output data, where: L is a lower triangular matrix and Q is an orthogonal matrix. From a block of the matrix L a singular value decomposition (SVD) is performed, from that it is possible to find out the system order and its observability matrix. With this last matrix it is possible to obtain the matrices C and A corresponding to the model in state space. The final step is to form a linear equation and apply the least squares method and estimate the matrices B and D of the model. The method is detailed in the section 4 of this paper.

Another subspace method called N4SID (Numerical Algorithms for Subspace State Space System Identification), the same way as the MOESP, is based on a LQ decomposition of data matrices. However, in this case this decomposition is interpreted as the oblique projection of future outputs in the subspace of the past inputs and outputs, towards the future inputs. From these projections the system states are estimated. With the states, inputs and outputs, the matrices A, B, C and D can be determined using a simple least squares method. This method and a variant proposed in (Clavijo, 2008) are presented in the section 5. A method inspired by MOESP and called MOESP-VAR, was introduced and developed in (Tamariz et al., 2005). The method is initialized by splitting the total input and output data into data groups that are associated with time intervals in which the system exhibits a slow change and can be approximated by a time-invariant system. The MOESP method is applied to the data of each interval, resulting in a linear time-invariant for the system in each of the time intervals. Following the same concept, the N4SID-VAR method is proposed in this article. The first step of the proposed method is to split the data

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into intervals, defined as time windows and then, to each window data a variation of the N4SID developed in (Clavijo, 2008) is applied.

In addition to the development of the algorithm, in this paper, a comparison between the MOESP-VAR and the N4SID-VAR is made using the same time window sizes and a slow time-variant benchmark. The computational time to process the algorithms is also evaluated.

The article is divided into eight sections. In the next section the state space system identification is presented. In the third section, the procedure to determine an extended state space model, which serves as the basis for the MOESP and N4SID methods, is detailed. In the following section the MOESP method is presented. The next section is focused on the variant of the N4SID method that uses the Markov parameters. In the sixth section the method proposed in this article is developed. In the seventh section, the results of the N4SID-VAR and MOESP-VAR methods to identify a time-variant benchmark are presented. Finally the last section, the conclusions of this article are exposed.

2 STATE SPACE SYSTEM IDENTIFICATION

The identification of discrete multivariable linear time-invariant systems using subspace methods allows to determine a causal and time invariant model, estimated only from the system inputs and outputs. The main advantage of this approach is that a multivariable system can be modeled, without any a priori assumption about the system order (state vector dimension). A discrete time-invariant model can be described by the following equation in the state space:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) + Du(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is defined as the system state at instant k, $A \in \mathbb{R}^{n\times n}$ is the state transition or system matrix, $B \in \mathbb{R}^{n\times m}$ is a matrix that relates the input $u(k) \in \mathbb{R}^m$ to the state, $C \in \mathbb{R}^{l\times n}$ is the matrix that relates the output $y(k) \in \mathbb{R}^l$ to the state and $D \in \mathbb{R}^{l\times m}$ is the matrix that relates the outputs to the inputs (Katayama, 2005).

3 EXTENDED STATE SPACE MODEL

In the same way that time-invariant systems can be represented by the equation (1), there are also other ways to represent the relation between the input, output and state vectors. In this section an extended model that is useful for the subspace methods is presented.

For a time instant t, it is defined that inputs before that instant are null. With this, from the equation (1), it is possible to substitute the relation between inputs, outputs and states between the instants t and $t+k-1$, where k is an integer, in the following way:

$$y_{t|k-1} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{k-1} \end{bmatrix} x(t) + \tag{2}$$

$$+ \begin{bmatrix} D & 0 & 0 & 0 \\ CB & D & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{K-2}B & \cdots & CB & D \end{bmatrix} u_{t|k-1}$$

where

$$y_{t|k-1} = \begin{bmatrix} y(t) \\ y(t+1) \\ \vdots \\ y(t+k-1) \end{bmatrix} \tag{3}$$

$$u_{t|k-1} = \begin{bmatrix} u(t) \\ u(t+1) \\ \vdots \\ u(t+k-1) \end{bmatrix} \tag{4}$$

the dimensions of the concatenated output vectors and inputs presented in equations 3 and 4, are respectively: $y_{t|k-1} \in \mathbb{R}^{kp\times 1}$ and $u_{t|k-1} \in \mathbb{R}^{km\times 1}$.

Rewriting the equation 2, can be obtain the following relation between the data matrices:

$$y_{t|k-1} = O_k x(t) + \Psi_k u_{t|k-1} \tag{5}$$

where O_k is a observability matrix, $\Psi_k \in \mathbb{R}^{kp\times km}$ is the Toeplitz matrix, as detailed in equations 6 and 7.

$$O_k = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{k-1} \end{bmatrix} \tag{6}$$

$$\Psi_k = \begin{bmatrix} D & 0 & 0 & 0 \\ CB & D & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{K-2}B & \cdots & CB & D \end{bmatrix} \quad (7)$$

If the vectors $u_{t|k-1}$, $y_{t|k-1}$ and $x(t)$ for $t = 0\ldots N-1$ are concatenated side by side, the following matrices can be defined:

$$U_{0|k-1} = \begin{bmatrix} u_{0|k-1} & u_{1|k} & \ldots & u_{N-1|k+N-2} \end{bmatrix} \in \mathbb{R}^{km\times N} \quad (8)$$

$$Y_{0|k-1} = \begin{bmatrix} y_{0|k-1} & y_{1|k} & \ldots & y_{N-1|k+N-2} \end{bmatrix} \in \mathbb{R}^{kp\times N} \quad (9)$$

$$X_{N-1} = \begin{bmatrix} x(0) & x(1) & \ldots & x(N-1) \end{bmatrix} \in \mathbb{R}^{n\times N} \quad (10)$$

where X_{N-1} is a state matrix. With the concatenated matrices the following extended model is written:

$$Y_{0|k-1} = O_k X_{N-1} + \Psi_k U_{0|k-1} \quad (11)$$

from the extended model the MOESP method is developed in the following section.

4 MOESP METHOD

The MOESP (Verhaegen and Dewilde, 1992) method is based on the LQ decomposition of a matrix formed by input and output data into two matrices: a matrix L, which is lower triangular, and a matrix Q, which is formed by linearly independent columns. The input and output data are concatenated in the Hankel matrices, presented in the equations 8 and 9.

Then an extended state space model is formed with the input and output Hankel matrices (11). With the extended model, $U_{0|k-1}$ and $Y_{0|k-1}$, is applied the LQ decomposition.

$$\begin{bmatrix} U_{0|k-1} \\ Y_{0|k-1} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \end{bmatrix} \quad (12)$$

where $L_{11} \in \mathbb{R}^{km\times km}$ e $L_{22} \in \mathbb{R}^{kp\times kp}$ are lower triangular matrices, $Q_1^T \in \mathbb{R}^{km\times N}$ e $Q_2^T \in \mathbb{R}^{kp\times N}$ are orthogonal and $L_{21} \in \mathbb{R}^{kp\times km}$.

From this decomposition, the equation (11) can be rewritten as shown below:

$$L_{21}Q_1^T + L_{22}Q_2^T = O_k X_{N-1} + \Psi_k L_{11} Q_1^T \quad (13)$$

Post-multiplying 13 by Q_2 yields

$$L_{22} = O_k X_{N-1} Q_2 \quad (14)$$

where $Q_1^T Q_2 = 0$, $Q_2^T Q_2 = I_{kp}$, due to the orthonormality between vectors that form these matrices. The observability matrix O_k can be obtained and the dimension of the system n after applying a SVD to $L_{22} \in \mathbb{R}^{kp\times kp}$.

Let SVD of L_{22} be given by

$$L_{22} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \quad (15)$$

where $U_1 \in \mathbb{R}^{kp\times n}$ and $U_2 \in \mathbb{R}^{kp\times(kp-n)}$. Then, from (15) and (14) it is possible to write:

$$O_k X_{N-1} Q_2 = U_1 \Sigma_1 V_1^T \quad (16)$$

so the extended observability matrix can be defined as follow

$$O_k = U_1 \Sigma_1^{1/2} \quad (17)$$

Define $O_{k\uparrow}$ as a matrix O_k with an offset of one row block, it is possible to write the following relation:

$$O_{k\uparrow} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \vdots \end{bmatrix} A = O_k A \quad (18)$$

After applying the pseudoinverse of O_k on both sides of the equation 18 , the matrix A is estimated.

$$A = O_k^{\dagger} O_{k\uparrow} \quad (19)$$

the matrix C is readily given by

$$C = O_k(1:p, 1:n) \quad (20)$$

The matrices B and D can be estimated using the following relationships: First taking advantage of the orthogonality between U_1 and U_2.

$$\begin{aligned} U_2^T L_{22} &= U_2^T U_1 \Sigma_1 V_1^T = 0 \\ U_2^T O_k &= U_2^T U_1 \Sigma_1^{1/2} = 0 \end{aligned} \quad (21)$$

Next step, multiplying both sides of equation (13) by U_2^T the following relationship is found:

$$U_2^T L_{21} L_{11}^{-1} = U_2^T \Psi_k \quad (22)$$

Splitting U_2^T in blocks with l columns defined as L_i and splitting the matrix $U_2^T L_{21} L_{11}^{-1}$ in blocks with m columns defined as M_i, so the following relationship is valid.

$$\begin{bmatrix} M_1 & M_2 & \ldots & M_k \end{bmatrix} = \begin{bmatrix} L_1 & L_2 & \ldots & L_k \end{bmatrix} \Psi_k \quad (23)$$

Afterwards, replacing Ψ_k, matrix defined in the equation 7, a linear relation 24 can be found. In fact 24 is a linear equation that has as variables the matrices B and D. Define $\tilde{L_i} = \begin{bmatrix} L_i & \dots & L_k \end{bmatrix} \in \mathbb{R}^{(kp-n)\times(k+1-i)p}, i = 2,\dots,k$, and substituting in equation 23 the following overdetermined linear system is obtained.

$$\begin{bmatrix} L_1 & \tilde{L_2}O_{k-1} \\ L_2 & \tilde{L_3}O_{k-2} \\ \vdots & \vdots \\ L_{k-1} & \tilde{L_k}O_1 \\ L_k & 0 \end{bmatrix} \begin{bmatrix} D \\ B \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_{k-1} \\ M_k \end{bmatrix} \quad (24)$$

A solution for the overdetermined system 24 can be found with the least squares method, resulting on estimates for the matrices B and D.

5 N4SID-VAR USING MARKOV PARAMETERS

In this section, another method to identify multivariable discrete time-invariant systems using subspace methods is presented. This method is called N4SID and was developed by Van Overschee and De Moor (Overschee and Moor., 1994). The algorithm is initialized by calculating the oblique projection of future outputs Y_f, on the vector W_p which is the concatenation of the inputs U_p and outputs Y_p passed in the direction of the future inputs U_f (Katayama, 2005). The classical N4SID estimates the system model using the least squares method, but in this work is considered a variation that was introduced and developed in (Clavijo, 2008). By definition:

$$W_P = \begin{bmatrix} U_P \\ Y_P \end{bmatrix} \quad (25)$$

and by the extended state space model presented in section 3.

$$Y_f = O_k X_f + \Psi_k U_f \quad (26)$$

Consider dthe following LQ decomposition:

$$\begin{bmatrix} U_f \\ U_p \\ Y_p \\ Y_f \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \\ Q_4^T \end{bmatrix} \quad (27)$$

where $L_{44} = 0$.

Then it is possible to rewrite the equation 27 as follows.

$$\begin{bmatrix} U_f \\ W_p \\ Y_f \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & 0 \end{bmatrix} \begin{bmatrix} \overline{Q}_1^T \\ \overline{Q}_2^T \\ \overline{Q}_3^T \end{bmatrix} \quad (28)$$

the relations between the matrices presented in equation 27 and 28, are as follows:

$$R_{11} = L_{11} \quad R_{21} = \begin{bmatrix} L_{21} \\ L_{31} \end{bmatrix} \qquad R_{22} = \begin{bmatrix} L_{22} & 0 \\ L_{32} & L_{33} \end{bmatrix}$$
$$R_{31} = L_{41} \quad R_{32} = \begin{bmatrix} L_{42} & L_{43} \end{bmatrix}$$

$$\overline{Q}_1^T = Q_1^T \quad \overline{Q}_2^T = \begin{bmatrix} Q_2^T \\ Q_3^T \end{bmatrix} \quad \overline{Q}_3^T = Q_4^T \quad W_p = \begin{bmatrix} U_p \\ Y_p \end{bmatrix}$$

From 28

$$\begin{aligned} U_f &= R_{11}\overline{Q}_1^T \Rightarrow \\ \Rightarrow \overline{Q}_1^T &= R_{11}^{-1}U_f \end{aligned} \quad (29)$$

The matrix $\overline{Q}_2^T$ can be written as:

$$\begin{aligned} W_p &= R_{21}\overline{Q}_1^T + R_{22}\overline{Q}_2^T \Rightarrow \\ \Rightarrow R_{22}\overline{Q}_2^T &= W_p - R_{21}\overline{Q}_1^T \Rightarrow \\ \Rightarrow \overline{Q}_2^T &= R_{22}^{\dagger}\left(W_p - R_{21}\overline{Q}_1^T\right) \end{aligned} \quad (30)$$

the matrix of future outputs Y_f, can be rewritten from 28

$$Y_f = R_{31}\overline{Q}_1^T + R_{32}\overline{Q}_2^T \quad (31)$$

Using the equations 29 and 30 in 31, the following relation is given for Y_f.

$$\begin{aligned} Y_f &= R_{31}R_{11}^{-1}U_f + R_{32}R_{22}^{\dagger}\left(W_p - R_{21}\overline{Q}_1^T\right) \\ &= R_{32}R_{22}^{\dagger}W_p + \left(R_{31} - R_{32}R_{22}^{\dagger}R_{21}\right)R_{11}^{-1}U_f \end{aligned} \quad (32)$$

The extended state space model 11, considering only future data, is given by:

$$Y_f = O_k X_f + \Psi_k U_f \quad (33)$$

Comparing 32 and 33, can be obtained two important relations to N4SID

$$R_{32}R_{22}^{\dagger}W_p = O_k X_f \quad (34)$$

$$\Psi_k = \left(R_{31} - R_{32}R_{22}^{\dagger}R_{21}\right)R_{11}^{-1} \quad (35)$$

applying the SVD to the equation 34

$$R_{32}R_{22}^{\dagger}W_p = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \quad (36)$$

where $U_1 \in \mathbb{R}^{kp\times n}$ and $U_2 \in \mathbb{R}^{kp\times(kp-n)}$. So from 34 and 36 we have:

$$O_k = U_1\Sigma_1^{1/2} \quad (37)$$

We also have the Toeplitz matrix $\Psi_k \in \mathbb{R}^{kp\times km}$ of the equation 35,

$$\Psi_k = \left(R_{31} - R_{32}R_{22}^{\dagger}R_{21}\right)R_{11}^{-1}$$

In the original N4SID method the future states matrix X_f is also obtained from the equations 34 and 36 and with the inputs, outputs and states, the matrices A, B, C and D from the state space model are estimated using the least squares method. Alternatively, the matrices can be estimated from the Toeplitz matrix Ψ_k, as proposed by (Clavijo, 2008) and detailed in the sequence.

The first column block of the matrix defined above represents the impulse responses, also called Markov parameters, given by:

$$G_{(k)} := \begin{cases} D & k=0 \\ CA^{k-1}B & k\neq 0 \end{cases}$$

$$\begin{bmatrix} D \\ CB \\ \vdots \\ CA^{k-1}B \end{bmatrix} = \begin{bmatrix} G_0 \\ G_1 \\ \vdots \\ G_k \end{bmatrix} \quad (38)$$

With these impulse responses, the following Hankel matrix can be formed

$$H_k = O_k\mathcal{C}_k = \begin{bmatrix} CB & CAB & CA^2B & \dots \\ CAB & CA^2B & CA^3B & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad (39)$$

The reachability matrix $\mathcal{C}_k \in \mathbb{R}^{nXkm}$ can be estimated using the observability matrix and the equation 39

$$\mathcal{C}_k = O_k^{\dagger}H_k = \begin{bmatrix} B & AB & A^2B & \dots & A^{k-1}B \end{bmatrix} \quad (40)$$

Finally, the matrix B is the first $n\times m$ block from $\mathcal{C}_k$ and the matrix D is G_0

$$\begin{aligned} B &= \mathcal{C}_k(1:n,1:m) \\ D &= G_0 \end{aligned} \quad (41)$$

From the observability matrix 37, the matrices A and C are estimated, the same way as is done in the MOESP method, according to the presented in the equations 19 and 20.

6 N4SID-VAR METHOD

In real life systems are not time-invariant, increasing the complexity of the identification problem. The system can be modeled in the state space, represented in equation 42, where the state space system matrices $A_{(k)}, B_{(k)}, C_{(k)}, D_{(k)}$ change over the time k.

$$\begin{cases} x(k+1) = A_{(k)}x(k) + B_{(k)}u(k) \\ y(k) = C_{(k)}x(k) + D_{(k)}u(k) \end{cases} \quad (42)$$

There is a version of MOESP for identify slowly time-variant systems, called MOESP-VAR (Tamariz et al., 2005). The principle of the method is as follows: given a time-variant system, time intervals, also called windows, are defined with a quantity of data (inputs and outputs). These windows are set so that the system does not undergo significant changes during each window. The MOESP method is applied to the data of each window, and with this the state space model matrices A, B, C and D that represent the system in that window are estimated. The process is repeated until all the data windows are modeled.

The study of subspace methods led to the development of a version of N4SID for the problem of identification of multivariable linear time-variant systems, named in this work as N4SID-VAR. The method works as follows: The first step is to define intervals or time windows (where system variations are slow). Each window contains a subset of input and output data. The next step is to apply the N4SID to each of the windows. With the algorithm the quadruple of matrices $A_{wj}, B_{wj}, C_{wj}, D_{wj}$ is estimated for each window.

The total number of windows j can be determined by dividing the total number of available data (inputs and outputs) N and the number of data per each time window N_w, as follows in the equation.

$$j = \frac{N}{N_w} \quad (43)$$

Each quadruple represents the state space model within a time interval, as defined in the equation 44.

$$A_{wj} = A_{(k)} \rightarrow 0 \leq k \leq jN_w - 1 \quad (44)$$

The subsets of input data $(U_{w1}, \dots, U_{wj})$ and outputs $(Y_{w1}, \dots, Y_{wj})$ of the system, in each of the windows are defined as:

$$\begin{aligned} U_{w1} &= [u(0) \quad u(1) \quad \dots \quad u(N_w-1)] \quad (45) \\ U_{w2} &= [u(N_w) \quad u(N_w+1) \quad \dots \quad u(2N_w-1)] \\ \vdots &= \vdots \\ U_{wj} &= [u(jN_w-N_w) \quad u(jN_w-N_w+1) \quad \dots \end{aligned}$$

$$\ldots \quad u(jN_w - 1)]$$

$$\begin{aligned} Y_{w1} &= [y(0) \quad y(1) \quad \ldots \quad y(N_w - 1)] \qquad (46) \\ Y_{w2} &= [y(N_w) \quad y(N_w + 1) \quad \ldots \quad y(2N_w - 1)] \\ \vdots &= \vdots \\ Y_{wj} &= [y(jN_w - N_w) \quad y(jN_w - N_w + 1) \quad \ldots \\ & \qquad \ldots \quad y(jN_w - 1)] \end{aligned}$$

The extended models for outputs and inputs (past and future) in each window are given by 47 and 48:

$$Y_{wj|p} = O_k X_{wj|p} + \Psi_k U_{wj|p} \qquad (47)$$

$$Y_{wj|f} = O_k X_{wj|f} + \Psi_k U_{wj|f} \qquad (48)$$

From these extended models it is possible to start the N4SID-VAR algorithm using the variation presented in the previous section. The flowchart of the algorithm is presented in the figure 1.

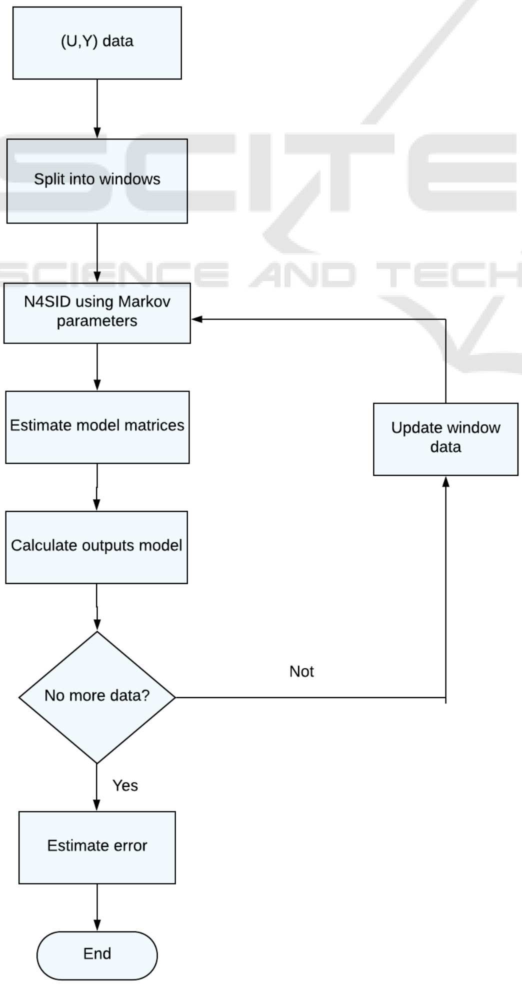

Figure 1: Steps of N4SID-VAR algorithm.

7 RESULTS

In order to test the proposed N4SID-VAR algorithm, a benchmark from the reference (Tamariz et al., 2005) was selected. In this benchmark, the matrix A is slowly time-variant.

$$A_k = \begin{bmatrix} a(k) & b(k) \\ 1 & -1 \end{bmatrix} \qquad (49)$$

where

$$a(k) = -\frac{1}{2}(k/2500)^2 + \frac{1}{2}(k/2500)$$

$$\begin{aligned} b(k) &= -\frac{1}{16}(k/2500)^4 - \frac{1}{8}(k/2500)^3 \\ & \quad -\frac{13}{16}(k/2500)^2 - \frac{3}{4}(k/2500) - \frac{1}{2} \end{aligned}$$

The matrices B_k, C_k and D_k are constant over the time

$$B_k = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}; \qquad C_k = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$$

$$D_k = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix} \qquad (50)$$

The system was excited by a two-dimensional white noise with $N = 1000$ samples. In addition, a noise equivalent to the 30% of the output generated after excitation of the system with the input was added. This was done to verify the robustness of the algorithms when applied to noisy data. The variation of the parameters $a(k)$ and $b(k)$ is plotted in the figure 2.

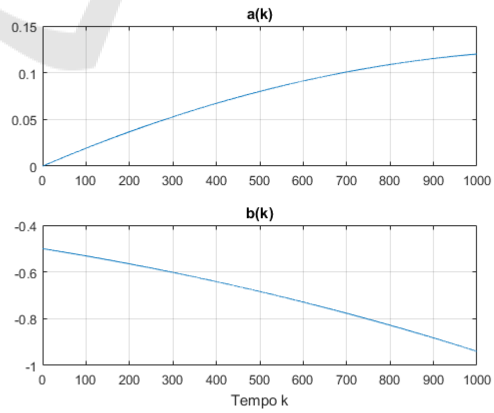

Figure 2: Variation of A_k elements.

At first a size of each data window $N_w = 50$ (inputs and outputs) was defined. In the time interval for each window the system varies slowly, and can be approximated by a time-invariant model. The MOESP-VAR and N4SID-VAR, were executed 2000 times.

For each of the runs, a new set of white noise inputs was generated, always following the same mean and covariance characteristics. This is done to ensure greater consistency in the comparison between the two methods. In the figures 3 and 4 the system and model outputs are presented for each of the methods for one of the executions.

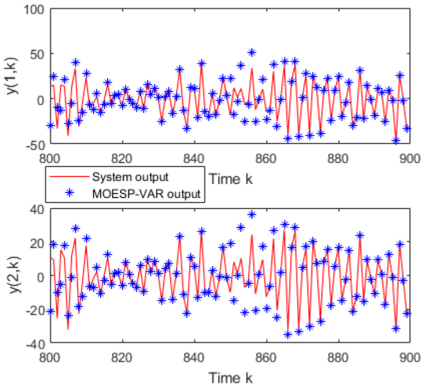

Figure 3: In the solid red line the system outputs are shown and in blue stars the outputs of the model obtained with the MOESP-VAR are shown for one of the executions with a window of $N_w = 50$ data.

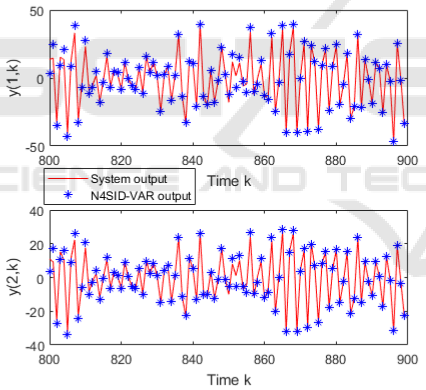

Figure 4: In the solid red line the system outputs are shown and in blue stars the outputs of the model obtained with the proposed method N4SID-VAR are shown, for one of the executions with window of $N_w = 50$ data.

To compare the quality of the methods, the mean square error was calculated between the actual and estimated outputs, which is defined by the equation 51, the variable m is the number of runs of each algorithm. The mean of the results of the mean square error calculation for the two methods after 2000 runs are shown in the table 1.

$$e = \frac{\sum_{r=1}^{m} \frac{1}{2N} \sum_{i=1}^{N} (y_{r(i)} - \hat{y}_{r(i)})^2}{m} \tag{51}$$

In the figure 5 the mean squared errors are shown after 2000 runs, for the following data window sizes: $N_w = 50, 60, 70, 80, 90, 100, 150, 200, 250, 300, 350, 400, 500$. Another graph showing more about the performance of N4SID-VAR is figure 6, where windows in the range of $N_w = 50, 60, 70, 80, 90, 100$, are presented in detail.

Table 1: Mean square error.

$N_w = 50$	e
N4SID-VAR	1.1521
MOESP-VAR	2.8896

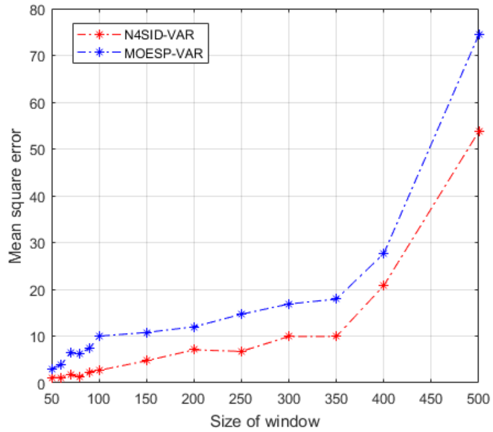

Figure 5: Mean squared errors for the N4SID-VAR and MOESP-VAR methods, for windows varying between 50 and 500.

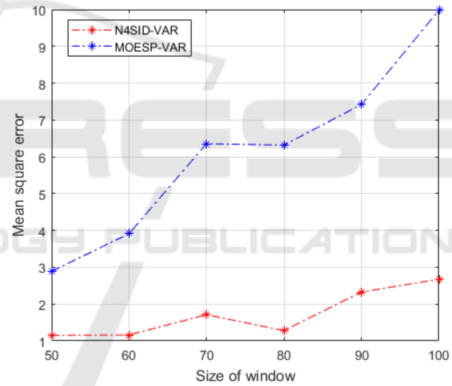

Figure 6: Detail of mean square errors for windows ranging from 50 to 100.

The mean square error obtained, presented in the table 1, with the N4SID-VAR was lower than with the MOESP-VAR. For all window sizes evaluated, the proposed method presents the smallest mean square error. One intuition about this is that the algorithm is based on a version of N4SID that does not use the method of least squares, one only has to find the Toeplitz matrix formed with the Markov parameters. In the case of MOESP, the least squares method is part of the algorithm to estimate the matrix B and D of the model.

Thus, the small number of data in a window causes the MOESP algorithm to have a less accurate result, an important observation is that, for both algorithms, the error tends to increase with window size, because, in this way, the system or benchmark presents larger variations within the same window and its representation by a system invariant in time becomes less exact.

The average execution time T_m for all the executions for the window size 50 was calculated. The N4SID-VAR method was 5 times faster than MOESP-VAR, in other words the time compilation was 5 times smaller if compared to the MOESP-VAR execution time. The results are displayed in the table 2. This result is also expected due to the lower need for matrix inversions in the N4SID method.

Table 2: Average runtime.

$N_w = 50$	$T_m(segundos)$
N4SID-VAR	2.87
MOESP-VAR	14.35

8 CONCLUSIONS

From the results presented in this article it is concluded that the deterministic method N4SID-VAR has a good performance based on the results, a smaller quadratic error in comparison to MOESP-VAR. The algorithm also presents a shorter execution time because it uses the impulse responses (Markov parameters) in the estimation of the matrices B and D and not the least squares method.

For future work, it is possible to study the accuracy of the proposed algorithm for higher order systems. In addition, other evolutionary heuristic techniques to identify time-variant systems developed in (Giesbrecht and Bottura, 2015) and (Robles and Giesbrecht, 2017) can be compared with the proposed method.

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