A Stochastic Approach for Damage Modelling of Cast Alloys

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Abstract: The increasing interest of cast Aluminium alloys in structural application asks for appropriate simulation approaches. Besides the constitutive behaviour, damage properties play an important role for this material. The damage behaviour is significantly influenced by the microstructure. Due to the specific morphology of cast microstructure and the random spatial deviation of voids, a novel concept of material modelling is necessary. In this study, a concept for stochastic material characterisation and modelling in structural Finite Element simulations is introduced. Therefore, a test matrix for experimental tests is discussed. Based on the generated experimental data a stochastic evaluation is performed by a goodness-of-fit test. The achieved characterisation knowledge is used to introduce the concept for stochastic material modelling of Aluminium cast alloys.

1 INTRODUCTION

Since light weight demand raises in automotive application, cast Aluminium alloys gain more relevance for structural parts. Therefore, an accurate capturing of material behaviour is necessary for structural Finite Elemente (FE) simulations. Due to the casting process the material gets its characteristic morphology with certain small amounts of voids which is substantially influenced by process parameters, e.g. time dependent pressure distribution, cooling rate, flow behaviour. Furthermore, the quality of melted Aluminium is a sensitive parameter which depends on air exhibition duration and accuracy of designated mass fraction for each alloy component. The set of these parameters causes a material with varying microstructure which results in inhomogeneous mechanical properties. In conventional modelling approaches, e.g. (Gurson, 1978) or subsequent works (Tvergaard and Needleman, 1984) the material is considered as continuum with smoothed micromechanical behaviour. These models capture the damage behaviour by the evolution of void fractions within the material due to ductile fracture mechanisms. Furthermore, phenomenological approaches are proposed by (Wilkins, et al., 1980) and (Johnson and Cook, 1985) which describe the material damage by a triaxiality dependent fracture strain. The standard implementation of these models is as well smoothed and does not consider the spatial variation of material parameters.

In this paper, a phenomenological approach combined with a concept of stochastic consideration for cast material characterisation and modelling is introduced.

2 DUCTILE FRACTURE

Structural materials are characterised by several engineering parameters, e.g. yield stress and tensile strength. Furthermore, the damage behaviour is a characteristic of a material which is composed by the evolution of certain micromechanical phenomena. Essentially, two different failure modes can be observed at metallic materials which appear as brittle or ductile fracture. The characterisation of the failure mode is basically done by monitoring the plastic strain until fracture occurs. Aluminium die cast alloys show a ductile fracture behaviour with distinct plastic deformations (Fagerholt, et al., 2010; Muehlstaetter and Hartmann, 2016). The evolution of ductile fracture is based on micromechanical mechanisms including nucleation, growth and coalescence of micro voids within a material as depicted in Figure 1.

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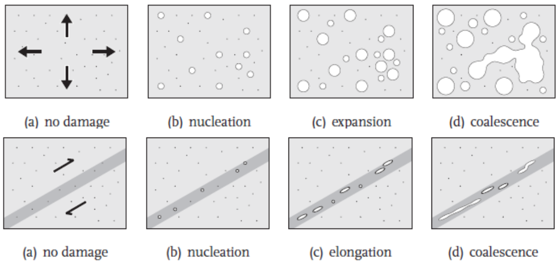

Figure 1: Evolution of material damage by different stress states (Engelen, 2005).

2.1 Solid Mechanics

A certain stress state, denoted by the stress tensor $\boldsymbol{\sigma}$, causes a characteristic damage mechanism after exceeding yield strength. Normal stresses (Figure 1 top) lead to expansion and coalescence of voids. Shear stresses (Figure 1, bottom) cause the formation of shear bands. Therefore, an arbitrary stress state needs to be evaluated regarding the damage effect of the material. A well-established method is proposed by (Bao and Wierzbicki, 2004) which consider the fraction strain as function of the stress triaxiality, defined as

$$\eta = \frac{\sigma_H}{\sigma_{Mises}}. \tag{1}$$

The triaxiality quantifies a stress state in terms of the multiaxiality by the ratio between the hydrostatic stress σ_H, given by

$$\sigma_H = \frac{1}{3} tr(\boldsymbol{\sigma}), \tag{2}$$

which is extracted from the stress tensor $\boldsymbol{\sigma}$ and the von Mises equivalent stress σ_{Mises}, calculated as

$$\sigma_{Mises} = \sqrt{\frac{3}{2} \sigma_{ij}^D \sigma_{ij}^D}, \tag{3}$$

composed by the stress deviator σ_{ij}^D in Einstein notation which is formulated as residual of the stress tensor with respect to the hydrostatic strain, therefore

$$\boldsymbol{\sigma}^D = \boldsymbol{\sigma} - \sigma_H \boldsymbol{E} \tag{4}$$

with the threedimensional unity matrix $\boldsymbol{E}$.

2.2 Characterisation of Damage Behaviour

For structural mechanics, numerical simulation software calculates displacements which appears due to external loads and boundaries. Therefore, strains are determined based on a defined constitutive law which leads to a stress field. In addition, the induced stresses cause certain micromechanical effects and the evolution of damage. As already outlined in the previous section, the damage mechanism depends on the stress state. State of the art damage modelling is based on the approach of (Bao and Wierzbicki, 2004), where a characteristic fracture strain is defined as function of the stress triaxiality η (see Figure 2).
Based on this approach a damage variable D is formulated as function of the plastic strain ε_{pl} and the fracture strain function $\varepsilon^f(\eta)$:

$$D = f\left(\varepsilon_{pl}, \varepsilon^f(\eta)\right) \tag{5}$$

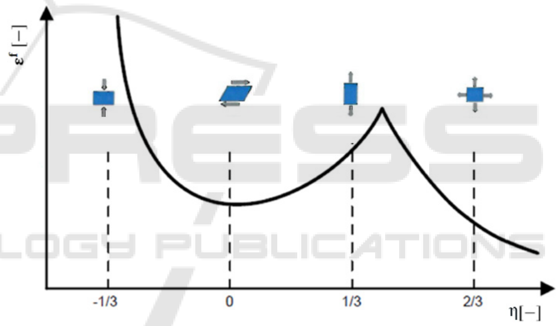

Figure 2: Characteristic metal fracture strain curve as function of the stress triaxiality (Bao and Wierzbicki, 2004).

Hence, for structural simulations including damage, additional material characterisation effort is necessary. Figure 3 shows a set of test coupons to capture different stress states which cause various damage mechanisms. Therefore, test geometries are manufactured and tested under tension load. The specific morphology within the gauge length, where fracture occurs, leads to triaxiality regimes of 0, for shear tensile and Merklein shear, to 0.33 for flat tensile tests and 0.57 for notched test. The negative triaxiality regime cannot be covered with this characterisation strategy.

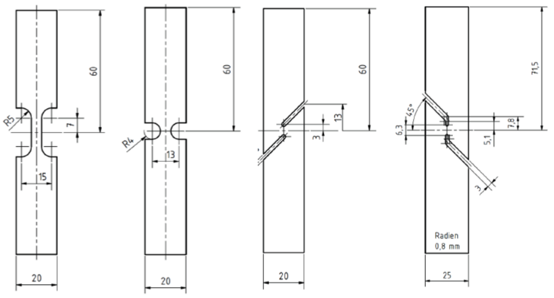

Figure 3: Test geometries for characterisation of the damage behaviour (from left: flat tensile, notched, shear tensile and Merklein shear).

2.3 Experimental Results

The intention of this study is to characterise a cast material which is produced by a stable and steady state die casting process. Therefore, the process runs a certain time to ensure constant process parameters, e.g. temperature field within the casting plant. The test specimens are extracted from casted hat profiles. For the experimental characterisation, a test matrix with at least 10 valid tests of each test geometry is defined. Furthermore, the spatial location of extraction with respect to the casting inlet is documented.

The output of these tests are force vs. displacement curves. Figure 4 shows the results of the flat tensile tests. The force-displacement curves show a significant deviation for a pure tension load with theoretical triaxiality of 0.33. A similar behaviour is observed for the notched tension tests (Muehlstaetter, 2015).

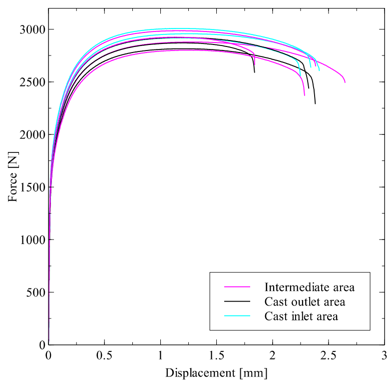

Figure 4: Experimental force vs. displacement curves of flat tensile tests for different regions within the casted part.

The test results of the Merklein tests are depicted in Figure 5. Deviation is also apparent in these results, although it is lower than in the triaxiality regimes of $\eta > 0.33$.

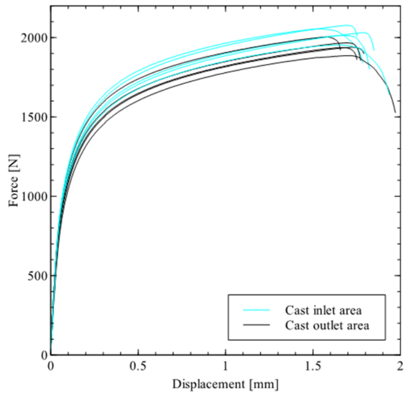

Figure 5: Experimental force vs. displacement curves of Merklein test geometry for different regions within the casted part.

3 STOCHASTIC EVALUATION

Based on the experimental data of the previous chapter an evaluation regarding the stochastic behaviour is performed.

3.1 Goodness-of-fit Test

In this section, a strategy for a stochastic consideration of the experimental data is introduced. The objective is to characterise the obtained scattering of the force-displacement curves. The scattering consists of the variation in the flow behaviour and the failure displacement. This study aims to consider only the failure displacement. Hence, discrete failure displacement values are extracted from the experiments.

By application of stochastic theory, the problem appears as issue of seeking a formulation of a potential underlying stochastic distribution. Therefore, goodness-of-fit tests are available (Schiefermayr and Weiß, 2014). These tests need an initial estimation of a stochastic distribution including their parameters, e.g. standard deviation for the normal distribution, as input. Based on the Cumulative Density Function (CDF) F of this stochastic distribution, the null hypothesis H_0 is tested against the alternative hypothesis H_1.

H_0 is defined as: The claim that the test data follows the estimated stochastic distribution is true; H_1 has the opposite meaning. Several goodness-of-fit tests are available in literature, e.g. (Schiefermayr and Weiß, 2014). The application of each test depends on the range of available data. In this study, the test data is relatively low for a stochastic consideration.

Therefore, the Kolmogorov-Smirnov (KS) test which delivers accurate results for low data is suitable.

This goodness-of-fit test is based on the steady distribution proposed by Kolmogorov with the CDF defined as

$$F_K(\varepsilon^f) = 1 + 2\sum_{k=1}^{\infty}(-1)^k e^{-2k^2{\varepsilon^f}^2} \quad (x \geq 0). \tag{6}$$

The random variable is the fracture strain ε^f from the experiments.

For the application of goodness-of-fit tests a significance level α needs to be defined. The significance level sets the probability, that the true null hypothesis is rejected accidentally. A common value for α is 5 %. Based on this parameter the condition for the rejection of the null hypothesis is formulated as

$$T \geq F_K^{-1}(1-\alpha)/\sqrt{n} \tag{7}$$

whereas F_K^{-1} is the Quantile of the Kolmogorov CDF and n is the amount of available test data for ε^f. The parameter T is based on the experimental data and the CDF of the estimated underlying distribution, defined as

$$T := sup\left|F_{emp}(\varepsilon^f) - F(\varepsilon^f)\right|. \tag{8}$$

This parameter determines the maximum deviation, or mathematically formulated as supremum between the empirical CDF $F_{emp}(\varepsilon^f)$, given by

$$F_{emp}(\varepsilon^f) = \frac{\varepsilon_i^f}{n}, \tag{9}$$

and the estimated CDF. ε_i^f in Equation 9 is the number of data which fulfil the condition $\leq \varepsilon^f$ and n is the total number of test data. The second term in Equation 8 is the CDF of the estimated stochastic distribution. In applied sciences, many processes succeed the normal distribution. Therefore, it is reasonable to choose this distribution for the first iteration of the KS test. The CDF of the normal distribution is defined as

$$F(\varepsilon^f) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\varepsilon^f} \exp\left(-\frac{(u-\mu)^2}{2\sigma^2}\right) du. \tag{10}$$

The normal distribution has 2 parameters, the expectation value μ and the variance σ which needs to be defined by application of moment estimators. Hence, μ and σ are calculated by the arithmetic mean value and the standard deviation.

The introduced test of this chapter is applied on the experimental data of the flat tensile test, depicted in Figure 4. The determination of the condition for the rejection of the null hypothesis from Equation 7 yields a value for T of 0.41. The determination of the parameter T leads to a value of 0.139. Thus, the condition of Equation 7 is not fulfilled, and the null hypothesis is true. Furthermore, the tested data follows the normal distribution.

If the first iteration leads to a rejection of the null hypothesis, an alternative distribution needs to be defined for the second KS test iteration. The concept of the KS test is depicted in Figure 6 in graphical representation. The empirical density function appears as step function in black line and the CDF of the normal distribution with estimated parameters in magenta line. The CDF of the normal distribution crosses every plateau of the empirical CDF, except at $\varepsilon^f = 17.5$. This is the visual interpretation of Equation 8 with the supremum between $F_{emp}(\varepsilon^f)$ and $F(\varepsilon^f)$.

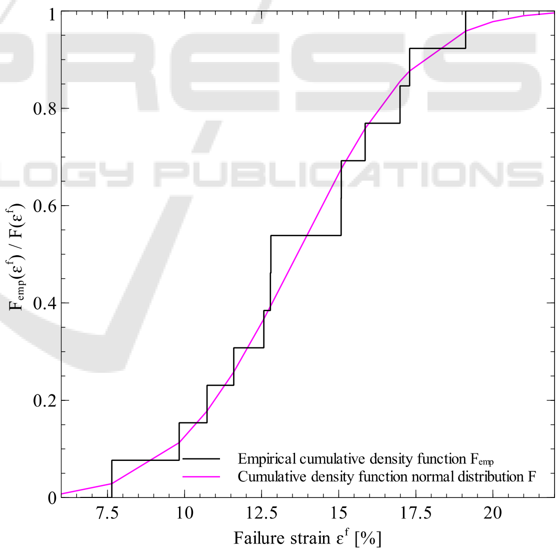

Figure 6: Visual representation of the Kolmogorov-Smirnov goodness-of-fit test with normal distribution and empirical CDF for tensile test data.

The application of this strategy for each test data set builds the concept for the development of an innovative approach for damage modelling. The fundament of this approach is the consideration of material damage proposed by (Bao and Wierzbicki, 2004) which is implemented in several Finite Element codes, e.g. LS-Dyna. In addition, the stochastic behaviour of a material is added by the concept of this study. Figure 7 depicts this combined approach.

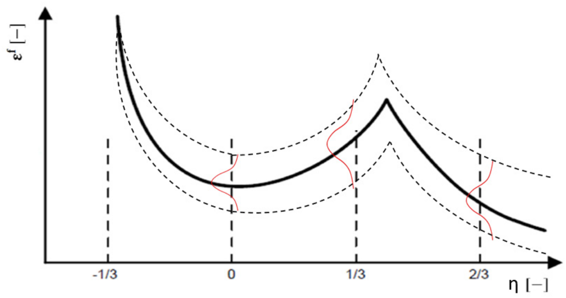

Figure 7: Schematic concept for the stochastic damage modelling (red and dashed line) based on the approach of (Bao and Wierzbicki, 2004) (black, solid line).

The black solid line represents the well-established approach of (Bao and Wierzbicki, 2004). The red Gauss' curve at $\eta = 0.33$ is introduced by the concept in this chapter. The remaining Gauss' curves are schematic for any probability function which needs to be establish by the KS test. Based on the probability functions, the dashed bounds are defined. This concept is intended as input for a random variable generator which delivers fraction strain curves as function of the triaxiality for each integration point in a simulation model. Hence, a random field of material property is generated initially for structural Finite Element simulations.

4 CONCLUSIONS

Damage modelling under consideration of the fracture strain as function of triaxiality is a well-established method. However, for cast Aluminium alloys the inhomogeneous material/damage behaviour is neglected. The introduced concept can overcome this drawback and builds a potential for more accurate capturing of material scatter of cast Aluminium alloys.

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REFERENCES

Bao, Y., Wierzbicki, T., 2004. *On fracture locus in the equivalent strain and stress triaxiality space.* s.l.:International Journal of Mechanical Sciences.

Engelen, R., 2005. *Plasitcity induced Damage in Metals: nonlocal modelling at finite strains.* s.l.:Habilitation thesis.

Fagerholt, E.; et al., 2010. *Experimental and numerical investigation of fracture in a cast aluminium alloy.* s.l.:International Journal of Solids and Structures.

Gurson, A. L., 1978. *Porous rigid-plastic materials containing rigid inclusions-yield function, plastic potential and void nucleation.* s.l.:The Physical Metallurgy of Fracture.

Johnson, G. R., Cook, W. H., 1985. *Fracture characteristics of three metals subjected to various strains, strain rates, terperatures and pressures.* s.l.:Engineering Fracture Mechanics.

Muehlstaetter, C., 2015. *Versagensmodellierung von Al-Gusswerkstoffen.* s.l.: Master Thesis, Upper Austrian University of applied sciences.

Muehlstaetter, C., Hartmann, M., 2016. *Material modelling of cast Aluminium by application of the Wilkins Damage Model.* s.l.:ECCOMAS Congress Proceedia.

Schiefermayr, K., Weiß, P., 2014. *Wahrscheinlichkeits-rechnung und Statistik.* s.l.:Course book, Upper Austrian University of applied sciences.

Tvergaard, V., Needleman, A., 1984. *An analysis of ductile rupture in notched bars.* s.l.:Journal of the Mechanics and Physics of Solids.

Wilkins, M. L., Streit, R. D., Reaugh, J. E., 1980. *Cumulative-strain-damage model of ductile fracture: Simulation and prediction of engineering fracture tests.* s.l.:s.n.