Non-autonomous Area Coverage and Coordination of a Multi-agent System for Harbor Protection Applications

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Keywords: Harbor Protection, Optimal Control, Area Coverage, Multi-agent Robotic System.

Abstract: We propose optimal strategies to develop an automated layered defence system for protecting structured environments, illustrated here by a typical harbor. The harbor is modeled as a two-dimensional environment, with a cooperative set of autonomous mobile agents to be deployed as the defence system. Optimal deployment is measured in a well-known area coverage metric, that encodes agents' sensing performance and a weight measure defined on the harbor, which allows to represent a priori and time varying risk field. The time-varying risk field allows to model several interesting scenarios. In this work, we consider the case of area surveillance against moving targets or external threats penetrating through the perimeter. We present a feedback control law for the platoon of surveying agents that implies the emergency of locally optimal configurations, that adapt to time evolving harbor environments.

1 INTRODUCTION

Several applications in military and civilian domains, such as harbor patrolling (Simetti and Cresta, 2007; Kitowski, 2012; Miah et al., 2014a), perimeter surveillance (Pimenta et al., 2013; Zhang et al., 2013; Bishop, 2007), search and rescue missions (Hu et al., 2013; Allouche and Boukhtouta, 2010), and cooperative estimation (Spinello and Stilwell, 2014), have highly benefited from theoretical and technological advances in multi-agent robotic systems. On the theoretical side, this has resulted into developments of motion control algorithms for networked mobile agents, that have attracted considerable attention due to their capabilities to address, in part, various class of problems in the field of multi-agent systems, such as area coverage (Miah et al., 2015; Miah et al., 2014b; Leonard and Olshevsky, 2013; Cortes et al., 2004; Cortes et al., 2005; Pimenta et al., 2008; Caicedo-Nunez and Zefran, 2008; Kantaros et al., 2015; Zhong and Cassandras, 2011; Miah et al., 2017a; Miah et al., 2017b), locational optimization (Guruprasad and Ghose, 2013), target tracking (Yang et al., 2008), formation control (Fax and Murray, 2004; Marshall et al., 2004), environmental tracking and monitoring (Porfiri et al., 2007; Simic and Sastry, 2003), and coordinated decision and control algorithms (Batalin and Sukhatme, 2007; Beard et al., 2002; Ferrari et al., 2009; Shima et al., 2007).

In area coverage applications, a team of mobile agents spatially configure themselves over an area of interest to maximize a coverage metric that typically encodes agents' performance and a risk density that weights points in the area. This theoretical framework naturally allows to formulate the class of harbor protection problems considered in this work, which can be seen as a resource allocation problem. Specifically, given a set of cooperative mobile agents that can sense the environment, and an area (harbor) with a risk field defined on it to quantify the relative importance or susceptibility of different regions, one wants to determine an optimal spatial distribution of the agents to maximize a coverage performance index. In the celebrated algorithm proposed by Lloyd (Lloyd, 1982), it is shown that centroidal Voronoi tessellations, in which the agents converge to centroids of Voronoi cells, are optimal with respect to a coverage metric that encodes agents performance with respect to points in the area, and a scalar risk density function that is a field assigning a weight (that can be interpreted as importance assigned to) each point in the domain. Lloyd's algo-

Miah, S., Nguyen, B., Bourque, A. and Spinello, D.
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DOI: 10.5220/0006926804860492
In *Proceedings of 8th International Conference on Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH 2018)*, pages 486-492
ISBN: 978-989-758-323-0

rithm can be interpreted as an iterative search for fixed points of maps defined on the coverage metric (Liu et al., 2009). Considerable efforts in the literature of area coverage control have been devoted to developing this idea, and generalize it to different multi-agent system scenarios, with time-varying and/or space distributed network topology (that is, inter-agent communication structures). The majority of the work focuses on the case of time-invariant risk density function (eventually non-uniform), to which the classical Lloyd algorithm apply, and can, therefore, be used to generate optimal trajectories. However, there are several interesting scenarios that emerge and are implied by time-varying risk density (Cortes et al., 2002; Lekien and Leonard, 2009; Lee and Egerstedt, 2013; Lee et al., 2015), where the problem of optimality with non-autonomous performance index (coverage metric) has been addressed within some restrictive hypotheses. In recent work (Miah et al., 2015; Miah et al., 2017a) it was proposed a general feedback control law that ensures asymptotic local optimality with time varying environment, and therefore coverage metric. In technical terms, the proof of asymptotic optimality cannot be addressed by using classic LaSalle arguments, that apply to autonomous metrics, but it is required the use of Barbalat's Lemma which imposes stronger conditions. The general control feedback law has been applied to simulate area coverage scenarios with time varying environments, both considering an area surveillance case with infiltration of external targets, or an environmental monitoring problem with diffusion of some substance (i.e. oil) through the boundary.

In this paper we apply these ideas to model and simulate a harbor protection scenario with a realistic harbor geometry. The optimal deployment of a set of autonomous agents is time varying, to react to a change in the risk function defined on the harbor environment. The risk field function models in a distributed sense an a priori distribution of importance (static), superimposed to a time varying part that evolves due to the penetration of external threats through the boundary. Locally optimal solution correspond to the set of agents converging to the centroids of a generalized Voronoi partition of the environment, with the challenge of time varying centroids due to the evolving environment. The control law proposed in (Miah et al., 2017b) allows to react to the evolution of the environment, with agents tracking the trajectories of the centroids. Simulation results show the application of the theoretical framework, and reveal important aspects in terms of applicability.

2 HARBOR GEOMETRY AND SYSTEM DESCRIPTION

A high-level architecture of a harbor defence system is depicted in Fig. 1. A high value unit is located on the left side (area with patterns), introducing a non-uniformity in the risk field function defined in the domain. Sensors (sonars, for example) are mounted on the outermost trip-wire boundary. The purpose of the trip-wire sensors is to detect and track attackers while they are in the detect and track zones (see Fig. 1), therefore providing boundary conditions information to the system of agents deployed in the harbor. In the scenario illustrated in Fig. 1, four agents are deployed inside the area (inner reaction zone), with threats that may enter from several directions (underwater or sideways, for instance). We consider a decentralized architecture in which there is no central control unit, as opposed to the scenario presented in (Strode et al., 2009); therefore, in our case each agent locally performs relevant computations, based on local information and shared information with the rest of the platoon.

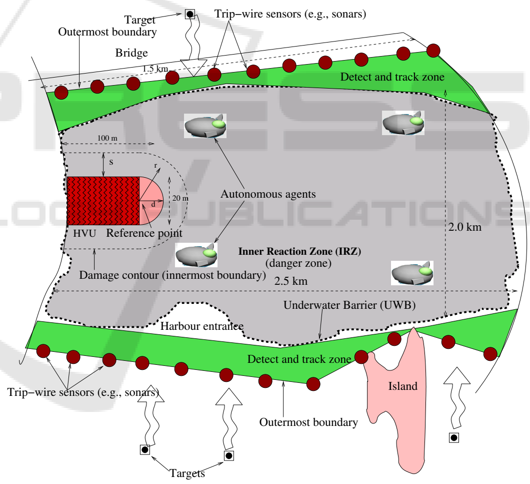


Figure 1: Harbor protection system architecture.

2.1 Modeling Agents

Formally, we consider the inner defence system to be a group of n homogeneous mobile agents where the motion of the ith, $i \in \{1, 2, \dots, n\} \equiv I$, agent is described by a simple integrator as $\dot{\mathbf{p}}^{[i]}(t) = \mathbf{u}^{[i]}(t)$, where $\mathbf{p}^{[i]}(t) = \left[x^{[i]}(t),\, y^{[i]}(t)\right]^T$ is the 2–D position and $\mathbf{u}^{[i]} = \left[u_x^{[i]}(t),\, u_y^{[i]}(t)\right]^T$ is its velocity vector input at time $t \geq 0$. The state of the agents' therefore collectively evolves as

$$\dot{\mathbf{p}}(t) = \mathbf{u}(t), \tag{1}$$

where $\mathbf{p}(t) = \left[\mathbf{p}^{[1]}(t), \ldots, \mathbf{p}^{[n]}(t)\right]^T \in \mathbb{R}^{2n}$ is the state of agents' group at time t and $\mathbf{u}(t) = \left[\mathbf{u}^{[1]}(t), \ldots, \mathbf{u}^{[n]}(t)\right]^T \in \mathbb{R}^{2n}$ the corresponding velocity vector.

2.2 Time–varying Risk Field

Let Ω be the inner harbor region. A time–varying density defined in Ω weights each point with a measure of risk. In area protection problems, the risk quantifies the relative importance of different regions in Ω, dictating how to distribute resources to protect the area. In this work, the risk density field is affected by the motion of m homogeneous mobile targets that at some point entered the inner harbor domain. Targets' kinematics is simply described by

$$\dot{\mathbf{s}}^{[j]} = \mathbf{v}^{[j]}, \tag{2}$$

where $\mathbf{s}^{[j]}(t)$ and $\mathbf{v}^{[j]}$ are the 2–D position and velocity vectors, respectively, for jth target, $j = 1, \ldots, m$. We assume that the risk density ϕ is twice differentiable in Ω, and consists of a time invariant component which can be considered as a priori independent of the targets, and of a time-varying component associated to the motion of the targets:

$$\phi(\mathbf{q}, t) = \bar{\phi}(\mathbf{q}) + \sum_{j=1}^{m} \phi_j(\mathbf{q}, t), \tag{3}$$

where $\mathbf{q} \in \Omega$ is a point in the area, $\bar{\phi}(\mathbf{q}) > 0$ represents the time-invariant density in the absence of any target, and $\phi_j(\mathbf{q}, t)$ is given by

$$\phi_j(\mathbf{q}, t) = \exp\left(-\frac{\left\|\mathbf{q} - \mathbf{s}^{[j]}(t)\right\|^2}{2\sigma^2}\right) \tag{4}$$

where we have adopted Gaussian functions centered at targets' positions, to reflect the assumption that each target contribution to the risk distribution ϕ is maximal at its own position, and it decreases with the relative distance from it. The choice of $\sigma > 0$ determines how narrow is the distribution of ϕ_j around $\mathbf{s}^{[j]}$. Note that the choice of the function ϕ_j is not unique, but it is informed by modeling considerations. Another suitable choice is the Lognormal function, which is often used in geological studies. The function $\bar{\phi}$ models the a priori risk, and intuitively is higher around the high value unit, so that protection agents allocate more resources in protecting the related area, rather than protecting more remote regions. The a priori risk field for this work is given by

$$\bar{\phi}(\mathbf{q}) = \exp\left(-\frac{1}{2}\left(\frac{(q_x - \bar{q}_x)^2}{\sigma_x^2} + \frac{(q_y - \bar{q}_y)^2}{\sigma_y^2}\right)\right) \tag{5}$$

where the highest risk is in the region centered around $(\bar{q}_x, \bar{q}_y)$, which is a point characterizing the high value area in the harbor.

2.3 Voronoi Tessellation

For optimal spatial placements and area coverage, agents partition the area to be covered using Voronoi tessellations (Okabe et al., 2000), where the optimality has to be intended as local since the objective function is in general nonconvex (Schwager et al., 2011). Following (Guruprasad and Ghose, 2013), the area Ω is partitioned in terms of Voronoi cells $\mathcal{V}(\mathbf{p}) = (\mathcal{V}_1(\mathbf{p}), \ldots, \mathcal{V}_n(\mathbf{p}))$, where the ith agent operates in the ith Voronoi cell, $\mathcal{V}_i(\mathbf{p})$, defined by

$$\mathcal{V}_i(\mathbf{p}) = \left\{\mathbf{q} \in \Omega \;:\; f(r_i) \geq f(r_j),\;\; \forall j \in I \setminus \{i\}\right\}, \tag{6}$$

$\forall i \in I$, $r_i = \|\mathbf{q} - \mathbf{p}^{[i]}\|$ is the Euclidean distance between the point $\mathbf{q} \in \Omega$ and the ith agent position, and $f(\cdot)$ is the agent's sensor performance function, which is differentiable in its argument. When f is the identity function, we have the classical Voronoi tessellation based on the Euclidean distance r_i^2. Here, we adopt the Gaussian form

$$f(r_i) = \alpha_i \exp\left(-\beta_i r_i^2\right) \tag{7}$$

which models sensors with degrading performance with the distance, modulated by the shape parameters α_i and β_i. Therefore, the generators of the Voronoi partition are the states $(\mathbf{p}^{[1]}, \ldots, \mathbf{p}^{[n]})$. For simplicity, $\mathcal{V}_i(\mathbf{p})$ will be denoted by $\mathcal{V}_i$ throughout the paper. Intuitively, $\mathcal{V}_i$ represents an area where each point is better sensed by the ith agent than to all other agents. The mass and the centroid of the ith Voronoi cell with respect to the density ϕ are respectively defined as $M_i(\mathcal{V}_i, t) = \int_{\mathcal{V}_i} \phi(\mathbf{q}, t) d\Omega$ and $\mathbf{C}_i(\mathcal{V}_i, t) = \frac{1}{M^i(\mathcal{V}_i, t)} \int_{\mathcal{V}_i} \mathbf{q}\phi(\mathbf{q}, t) d\Omega$.

3 HARBOR PROTECTION AS AN OPTIMAL CONTROL PROBLEM

3.1 Problem Formulation

Consider the time-varying density map $\phi : \Omega \times \mathbb{R}^+ \to \mathbb{R}^+$ as a density function representing the likelihood that some events take place over Ω at time t. This leads to a non-uniform time-varying distribution of agents, where more (less) agents are deployed with higher (lower) values of the measure $\phi(\mathbf{q}, t)$, $\forall \mathbf{q} \in \Omega$. Furthermore, we assume that the sensing performance function $f(r_i)$ is Lebesgue measurable, homogeneous, and that it is strictly decreasing with respect to the Euclidean distance r_i (Okabe et al., 2000). Motivated by the locational optimization problem (Okabe et al., 2000), we consider the total coverage metric

$$H(\mathbf{p}, \mathcal{V}, t) = \sum_{i=1}^{n} \int_{\mathcal{V}_i} f(r_i)\phi(\mathbf{q}, t)\mathrm{d}\Omega, \quad (8)$$

The model (8) encodes how rich the coverage in Ω is. In other words, higher H implies that the corresponding distribution of agents achieves better coverage of the area Ω. Hence, the problem can be stated as follows: Given the time-varying density function (3), find a distribution of agents such that the coverage H is maximized, *i.e.,*

$$\max_{\mathbf{p}} H(\mathbf{p}, \mathcal{V}, t), \text{ subject to (1) as } t \to \infty. \quad (9)$$

The coverage metric is in general non-convex, and therefore the equilibrium configurations $\mathbf{p}$ correspond to local maxima (Du et al., 2006).

3.2 Coordination Control of the Autonomous Robots

The coverage metric (8) encodes robots' performances and the risk field ϕ, which describes the environment to which the agents react to reach a suitable spatial distribution corresponding to a partition of the domain Ω in which each region is assigned to an agent. When the density ϕ is time-invariant, geometric center laws (Bullo et al., 2009, Ch. 5) on planar agents provide a well-established solution to the coverage optimization problem (9), where each agent asymptotically converge to its Voronoi centroid (or critical point). By considering a time-varying density, Voronoi centroids being time-varying no longer define the invariant set of the agents' trajectories. In this case, the solution of problem (9) relies on the definition of a non-autonomous feedback control law for the velocity vector $\mathbf{u}(t)$ so that agents track time-varying Voronoi centroids. A general non-autonomous feedback law for area coverage problems (9) has been proposed in (Miah et al., 2017b), with the proof that the trajectories generated by the feedback law maximize the coverage metric (8) by tracking time varying generalized Voronoi centroids. Therefore, the feedback law ensures that the group of agents can react to a time varying environment described by a smooth risk field ϕ, and optimally redistribute spatially with respect to the metric (8). The control is distributed in the sense that each feedback law depends only on local information; however, the computation requires the preliminary determination of a Voronoi partition of Ω, which in turn requires a shared knowledge of the state of the other mobile robots, as well as an estimation of the function ϕ. In a distributed scenario, this knowledge depends on information sharing among agents. Asymptotic convergence with respect to random communication failures has been studied in (Miah et al., 2015).

Since the system is non-autonomous, the technical proof of asymptotic convergence requires Barbalat's lemma, which poses a stronger condition that LaSalle's principle (uniform continuity of the second derivative of the Lyapunov function.). Here we just report the main result without repeating the proof, that can be consulted in detail in (Miah et al., 2017b).

Consider the following non-autonomous feedback control law

$$\mathbf{u}^{[i]}(t) = \frac{\left(\partial H/\partial \mathbf{p}^{[i]}\right)^T}{\left\|\partial H/\partial \mathbf{p}^{[i]}\right\|^2} \left(\kappa \left\|\tilde{\mathbf{c}}_i - \mathbf{p}^{[i]}(t)\right\|^2 - \frac{\partial H^i}{\partial t}\right), \quad (10)$$

with $\kappa > 0$. In view of (1), $\mathbf{u}^{[i]}$ is the velocity input for the ith agent, and $\tilde{\mathbf{c}}_i$ and H^i are defined by

$$\tilde{\phi}_i = -2\frac{\partial f(r_i)}{\partial r_i^2}\phi(\mathbf{q}, t) \quad (11)$$

$$\tilde{m}_i = \int_{\mathcal{V}_i} \tilde{\phi}_i \mathrm{d}\Omega \quad (12)$$

$$\tilde{\mathbf{c}}_i = \frac{1}{\tilde{m}_i}\int_{\mathcal{V}_i} \mathbf{q}\tilde{\phi}_i \mathrm{d}\Omega \quad (13)$$

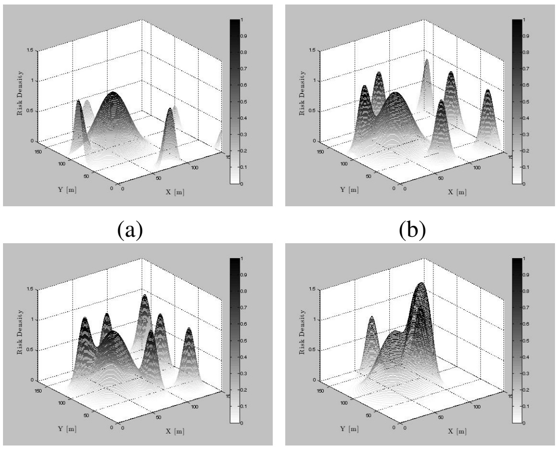
$$H^i = \int_{\mathcal{V}_i} f(r_i) \sum_{j=1}^{m} \mathbf{v}^{[j]} \cdot (\mathbf{q} - \mathbf{s}^{[j]})\phi_j(\mathbf{q}, t)\mathrm{d}\Omega \quad (14)$$

where m is the number of external targets in the ith Voronoi cell. The following Proposition establishes that the trajectories defined by (10) asymptotically maximize the non-autonomous coverage metric H.

Proposition 1. (From (Miah et al., 2017b)) *Consider the kinematic model of a team of mobile agents* (1)*, and the sensory performance function* $f(r_i)$ *to be strictly decreasing with respect to its argument. Then the feedback law* (10) *maximizes the non-autonomous coverage metric H defined in* (8).

4 SIMULATION

In this section we present simulations of harbor protection operations consisting of the autonomous deployment and operation of a platoon of mobile robots in a harbor environment, and their time varying redistribution to react to the motion of mobile targets penetrating the inner protection area.

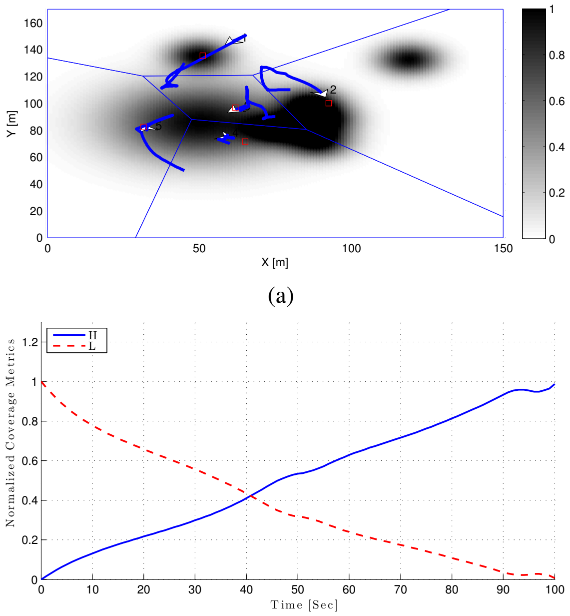


(a) (b)

(c) (d)

Figure 2: Evolution of density due to six mobile targets at time (a) 20 s, (b) 40 s, (c) 60 s, and (d) 100 s.

The non-autonomous optimal coverage of the agents' nonuniform deployment in the harbor is illustrated by simulating a 2D inner reaction zone of the harbor with its boundary vertices at (0, 0) m, (150, 0) m, (150, 170) m, and (0, 170) m. The high-valued unit is located at $(\bar{q}_x, \bar{q}_y) = (50, 85)$ m. Therefore the a priori risk field $\bar{\phi}(\mathbf{q})$, $\mathbf{q} \in \Omega$, around the high-valued unit is modeled using (5) with the shape parameters $\sigma_x = \sigma_y = 20$ m. The trajectories of six mobile targets ($m = 6$) affect the time-varying risk density $\phi(\mathbf{q}, t)$ defined in (3) with $\sigma = 7.5$ m. The evolution of time-varying targets' positions $\mathbf{s}^{[j]}(t)$ is described by the kinematic model of Newtonian accelerated particles, with initial speeds $\|\mathbf{v}^{[j]}\| = 0.5\,\mathrm{m\,s^{-1}}$, and constant accelerations $0.01\,\mathrm{m/s^2}$, for $j = 1, 2, \ldots, 6$. Initial positions of targets are (0, 70) m, (0, 85) m, (70, 180) m, (160, 100) m, (160, 0) m, and (170, 180) m. Figure 2 shows four snapshots of the evolving risk field by six mobile targets at time $t = 20$ s, $t = 40$ s, $t = 60$ s, and $t = 100$ s. As can be seen, the risk densities are the highest at the locations of targets and around the high value unit. Five agents ($n = 5$) are initially placed at positions (45, 130) m, (80, 110) m, (75, 90) m, (60, 70) m, and (45, 50) m. Each agent computes its Voronoi partition in the area Ω using the generalized Voronoi partitioning model (6) with the agents' sensory performance function defined by $f(r_i) = \alpha \exp(-\beta r_i^2)$, where $\alpha = 5$ and $\beta = 10^{-2}$. The performance metric used in this simulation study is the non-autonomous coverage metric defined in (8), which we aim to maximize. The performance of the proposed feedback control law (10) in maximizing the coverage metric, H, is summarized in Fig. 3 with feedback gain $\kappa = 0.3$. The figure visualizes the agents approaching time varying centroids, which



(a)

(b)

Figure 3: Performance of the proposed non-autonomous controller using target driven risk density: (a) Agents' optimal configuration at time $t = 100$ s (solid arrow: agent; diamond: centroid) and (b) coverage metric H and Lyapunov function versus time.

evolve due to the motion of targets in the domain. The vertical color bar in Fig. 3(a) represents the level of density in Ω with 0 (1) corresponding to lowest (highest) density for the time span of the simulation. The coverage metric (normalized), H, resulting from the proposed strategy is shown in Fig. 3(b) (solid blue line). The normalized coverage $H = 0$ when agents are initially placed in the harbor environment. As the mobile targets evolve, agents move towards their optimal configurations asymptotically yielding the normalized coverage metric $H = 1$. Note that the dashed red line represents the Lyapunov function (used to prove the proposed feedback law (10)) that is decreasing, as expected. Therefore, the feedback law (10) places all agents in the optimal configurations asymptotically yielding (local) maximum coverage.

5 SUMMARY AND CONCLUSIONS

We have applied a recently proposed non-autonomous state-feedback control law for maximizing non-autonomous area coverage metrics to an harbour protection problem. The non-autonomous feedback law applies to the general class of twice differentiable risk densities defined in closed domains. This feedback law allows the agents to solve the optimal control problem of time varying formations in area coverage problems with coordinated platoons of autonomous mobile robots, by maximizing a non-autonomous coverage that encodes the platoon's performance and an evolving environment. Simulation results modeled by a geometrically realistic harbor environment with non-uniform risk defined on the inner reaction zone illustrate the applicability and effectiveness of the optimal control framework to address harbor protection problems.

A fundamental assumption in this work is the mass particle kinematic model for the robots, which needs to be generalized to include the inertia a more sophisticated kinematics. Moreover, simulations are based on the assumption that all vehicles have a common knowledge of the environment (risk function ϕ) and the knowledge of the state of all the other robots in the platoon, so that Voronoi cells can be computed at every iteration. These assumptions need to be relaxed in order to adhere to realistic scenarios. Current work includes the application of reinforcement learning techniques to estimate the risk function ϕ. The effect of random drop of communication data packets to share information about robots' positions has been studied in (Miah et al., 2015).

REFERENCES

Allouche, M. K. and Boukhtouta, A. (2010). Multi-agent coordination by temporal plan fusion: Application to combat search and rescue. *Information Fusion*, 11(3, SI):220–232.

Batalin, M. and Sukhatme, G. (2007). The design and analysis of an efficient local algorithm for coverage and exploration based on sensor network deployment. *IEEE Trans. on Robotics*, 23(4):661–675.

Beard, R. W., McLain, T. W., Goodrich, M. A., and Anderson, E. P. (2002). Coordinated Target Assignment and Intercept for Unmanned Air Vehicles. *IEEE Trans. on Robotics and Automation*, 18:911–922.

Bishop, B. E. (2007). On the use of capability functions for cooperative objective coverage in robot swarms. In *Proceedings 2007 IEEE International Conference on Robotics and Automation*, pages 2306–2311.

Bullo, F., Cortes, J., and Martinez, S. (2009). *Distributed Control of Robotic Networks: A Mathematical Approach to Motion Coordination Algorithms*. Princeton University Press, Applied Mathematics Series.

Caicedo-Nunez, C. H. and Zefran, M. (2008). Performing coverage on nonconvex domains. In *Control Applications, 2008. CCA 2008. IEEE International Conference on*, pages 1019–1024.

Cortes, J., Martinez, S., and Bullo, F. (2005). Spatially-distributed coverage optimization and control with limited-range interactions. *ESAIM-Control Optimisation and Calculus of Variations*, 11(4):691–719.

Cortes, J., Martinez, S., Karatas, T., and Bullo, F. (2002). Coverage control for mobile sensing networks: Variations on a theme. In *10th Mediterranean Conference on Control and Automation*, volume 1.

Cortes, J., Martinez, S., Karatas, T., and Bullo, F. (2004). Coverage control for mobile sensing networks. *IEEE Trans. on Robotics and Automation*, 20(2):243–255.

Du, Q., Emelianenko, M., and Ju, L. (2006). Convergence of the Lloyd algorithm for computing centroidal voronoi tessellations. *SIAM Journal on Numerical Analysis*, 44(1):102–119.

Fax, J. A. and Murray, R. M. (2004). Information flow and cooperative control of vehicle formations. *IEEE Trans. on Automatic Control*, 49(9):1465–1476.

Ferrari, S., Fierro, R., Perteet, B., Cai, C., and Baumgartner, K. (2009). A geometric optimization approach to detecting and intercepting dynamic targets using a mobile sensor network. *SIAM Journal on Control and Optimization*, 48(1):292–320.

Guruprasad, K. R. and Ghose, D. (2013). Heterogeneous locational optimization using a generalized Voronoi partition. *International Journal of Control*, 86(6):977–993.

Hu, J., Xie, L., Lum, K.-Y., and Xu, J. (2013). Multiagent information fusion and cooperative control in target search. *IEEE Trans. on Control Systems Technology*, 21(4):1223–1235.

Kantaros, Y., Thanou, M., and Tzes, A. (2015). Distributed coverage control for concave areas by a heterogeneous robotswarm with visibility sensing constraints. *Automatica*, 53:195 – 207.

Kitowski, Z. (2012). Architecture of the Control System of an Unmanned Surface Vehicle in the Process of Harbour Protection. In Kitowski, Z and Garus, J and Szymak, P, editor, *Mechatronic Systems, Mechanics and Materials*, volume 180 of *Solid State Phenomena*, pages 20–26.

Lee, S., Diaz-Mercado, Y., and Egerstedt, M. (2015). Multirobot control using time-varying density functions. *Robotics, IEEE Trans. on*, 31(2):489–493.

Lee, S. G. and Egerstedt, M. (2013). Controlled coverage using time-varying density functions. In *IFAC Workshop on Distributed Estimation and Control in Networked Systems, Koblenz, Germany*.

Lekien, F. and Leonard, N. E. (2009). Nonuniform coverage and cartograms. *SIAM Journal of Control and Optimization*, 48(1):351–372.

Leonard, N. and Olshevsky, A. (2013). Nonuniform coverage control on the line. *IEEE Trans. on Automatic Control*, 58(11):2743–2755.

Liu, Y., Wang, W., Lévy, B., Sun, F., Yan, D.-M., Lu, L., and Yang, C. (2009). On Centroidal Voronoi Tessellation–Energy Smoothness and Fast Computation. *ACM Trans. Graph.*, 28(4):101:1–101:17.

Lloyd, S. (1982). Least squares quantization in PCM. *IEEE Trans. on Information Theory*, 28(2):129–137.

Marshall, J. A., Broucke, M. E., and Francis, B. A. (2004). Formations of vehicles in cyclic pursuit. *IEEE Trans. on Automatic Control*, 49(11):1963–1974.

Miah, M. S., Nguyen, B., Bourque, A., and Spinello, D. (2014a). Nonuniform deployment of autonomous agents in harbor-like environments. *Unmanned Systems*, 2(4):377–389.

Miah, M. S., Nguyen, B., Spinello, D., and Bourque, A. (2014b). Nonuniform coverage with time-varying diffusive density. In *ASME 2014 Dynamic Systems and Control Conference*, San Antonio, Texas, USA.

Miah, S., Fallah, M. M. H., and Spinello, D. (2017a). Nonautonomous coverage control with diffusive evolving density. *IEEE Transactions on Automatic Control*, 62(10):5262–5268.

Miah, S., Nguyen, B., Bourque, A., and Spinello, D. (2015). Nonuniform Coverage Control with Stochastic Intermittent Communication. *Automatic Control, IEEE Trans. on*, 60(7):1981–1986.

Miah, S., Panah, A. Y., Fallah, M. M. H., and Spinello, D. (2017b). Generalized non-autonomous metric optimization for area coverage problems with mobile autonomous agents. *Automatica*, 80:295 – 299.

Okabe, A., Boots, B., Sugihara, K., and Chiu, S. N. (2000). *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams*. John Wiley & Sons, LTM, second edition.

Pimenta, L. C. A., Kumar, V., Mesquita, R. C., and Pereira, G. A. S. (2008). Sensing and Coverage for a Network of Heterogeneous Robots. In *47th IEEE Conference on Decision and Control, 2008 (CDC 2008)*, pages 3947–3952.

Pimenta, L. C. A., Pereira, G. A. S., Goncalves, M. M., Michael, N., Turpin, M., and Kumar, V. (2013). Decentralized controllers for perimeter surveillance with teams of aerial robots. *Advanced Robotics*, 27(9, SI):697–709.

Porfiri, M., Roberson, D. G., and Stilwell, D. J. (2007). Tracking and formation control of multiple autonomous agents: A two-level consensus approach. *Automatica*, 43(8):1318–1328.

Schwager, M., Rus, D., and Slotine, J.-J. (2011). Unifying geometric, probabilistic, and potential field approaches to multi-robot deployment. *International Journal of Robotics Research*, 30(3):371–383.

Shima, T., Rasmussen, S. J., and Chandler, P. (2007). UAV team decision and control using efficient collaborative estimation. *Journal of Dynamical Systems, Measurements, and Control*, 129:609–619.

Simetti, E. and Cresta, M. (2007). Towards the use of a team of USVs for civilian harbour protection: the problem of intercepting detected menaces. In *IEEE OCEANS –Sydney*, pages 1–7, Sydney, NSW.

Simic, S. and Sastry, S. (2003). Distributed environmental monitoring using random sensor networks. In *Proceeding of the 2nd International Workshop on Information Processing in Sensor Networks*, pages 582–592, Palo Alto, CA.

Spinello, D. and Stilwell, D. J. (2014). Distributed full-state observers with limited communication and application to cooperative target localization. *Journal of Dynamic Systems, Measurement and Control - Trans. of the ASME*, 136(3).

Strode, C., Cecchi, D., and Calabro, V. (2009). Simulation of a system of systems to protect high value assets from surface and underwater terrorist attack. In *Undersea Defence Technology (UDT) Europe*, Cannes.

Yang, P., Freeman, R. A., and Lynch, K. M. (2008). Multi-Agent Coordination by Decentralized Estimation and Control. *IEEE Trans. on Automatic Control*, 53(11):2480–2496.

Zhang, G., Fricke, G. K., and Garg, D. P. (2013). Spill Detection and Perimeter Surveillance via Distributed Swarming Agents. *IEEE-ASME Trans. on Mechatronics*, 18(1):121–129.

Zhong, M. and Cassandras, C. (2011). Distributed coverage control and data collection with mobile sensor networks. *IEEE Trans. on Automatic Control*, 56(10):2445–2455.