Research on the Controllability of Urban Road Network

Chuanci Cai and Xuan Zhang
School of Traffic and Transportation, Beijing Jiaotong University, Beijing100044, China
{16120778, 15120925}@bjtu.edu.cn

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Abstract: In order to study the factors influencing the controllability of urban road network, the paper calculates the network topology features of the real urban road networks and the simulated urban road networks, such as average degree, 2-core, clustering coefficient, heterogeneity, organic ratio and meshedness coefficient. According to the PBH judgment theorem and the minimal control input theorem, the minimal number of control inputs and the network controllability are obtained. Then the correlation between network controllability and topological characteristics is analyzed. It is found that heterogeneity, 2-core and average degree have a great influence on controlling the urban road network. The results show that it is of great significance to balance the function, strengthen the connectivity and regularity of network to ensure the orderly operation of urban road network.

1 INTRODUCTION

Urban traffic system is a typical open complex giant system. It is composed of road network, traffic flow and management control system. The topological structure of road network directly affects the characteristics of traffic flow dynamics. Numerous studies show that most urban road networks are scale-free networks and exhibit small-world characteristics (Lämmer et al., 2006; Porta et al., 2006; Jiang, 2007). Changes in the network structure will affect the operation of the network (Arrowsmith et al., 2005; Kwangho et al., 2005).

Motter et al. (2008) study the metabolic networks of single-celled organisms, where disturbances caused by genetic or epigenetic defects can lead to unfeasible strains. By knocking out specific genes, the consequences of these defects can be alleviated and the ability of the strain to grow can be restored; Sahasrabudhe and Motter (2011) study food networks. As we know, human or natural forces may lead to the subsequent extinction of many species. The study shows that a significant proportion of these extinctions can be prevented by targeted inhibition of specific species in the system; These findings have similarities in the power grid. Equipment failure, damage or operational errors can lead to widespread blackouts, but proper power release can greatly reduce subsequent failures (Anghel et al. 2006).

Therefore, to alleviate traffic congestion and ensure the orderly operation of urban road network, it is necessary to study the relationship between the controllability of urban road network and the network topology features.

2 THEOREM

2.1 PBH judgment theorem

The necessary and sufficient condition for a continuous linear time invariant system to be strictly controlled is that the system matrix satisfies the following rank condition:

$$\mathrm{rank}[\lambda I_N - \mathrm{A}, \mathrm{B}] = \mathrm{N} \tag{1}$$

λ is the eigenvalue of the coupling matrix A.

2.2 Theorem of minimal control input

According to the PBH judgment theorem, B is the input matrix, so we can get the minimum number of inputs: $\mathrm{N_D} = \min\{\mathrm{rank(B)}\}$. Using the inequality relation of the matrix rank, the algebraic multiplicity $\delta(\lambda_i)$ of the eigenvalue λ_i is the occurrence times of $(\lambda - \lambda_i)$ in the factorization of $\mathrm{P_A}(\lambda) = \det(\lambda \mathrm{I_N} - \mathrm{A})$. The geometric multiplicity $\mu(\lambda_i)$ is the dimension of the corresponding Characteristic

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subspace $V_{\lambda_i} = \{\alpha | A\alpha = \lambda_i \alpha\}$. Therefore, we obtain the geometric multiplicity $\mu(\lambda_i) = N - \mathrm{rank}(\lambda_i I_N - A)$ and the minimal number of control inputs is calculated as follows:

$$N_D = \max_i \{\mu(\lambda_i)\} \quad (2)$$

Through this theorem, we can use the eigenvalues of the system coupling matrix A to determine the controllability of the original system. Moreover, by using the PBH theorem, the minimal number of control inputs and the corresponding driving nodes for the system to be strictly controlled is theoretically obtained (Yuan, 2014).

2.3 Minimal control inputs of sparse network

The geometric multiplicity of zero eigenvalue is $\mu(0) = N - \mathrm{rank}(A)$, and we can get the minimal number of inputs in sparse networks:

$$N_D = \max\{1, N - \mathrm{rank}(A)\} \quad (3)$$

The real road network and simulated road network in this paper are regarded as sparse networks. The calculation method of minimal control input number adopts the formula (3).

2.4 Controllability

The controllability of the network refers to the proportion of the drive nodes in the network:

$$n_D = N_D / N \quad (4)$$

The controllability of the network reflects controllable difficulty of the network from the demand of the control inputs. The smaller the n_D is, the easier it is to control the network. Otherwise, the larger the n_D is, the harder it is to control the network.

3 TOPOLOGY FEATURES

Average degree $\langle k_i \rangle$ is the average of the degree k_i of all nodes i in the network. The calculation method is shown in equation (5), where a_{ji} denotes the exit of node i.

$$\langle k_i \rangle = \frac{1}{N}\sum_{i=1}^{N} k_i = \frac{1}{N}\sum_{i,j=1}^{N} a_{ji} \quad (5)$$

Clustering coefficient C_i refers to the ratio of the actual number of edges and the total possible number of edges between k nodes.

$$C_i = \frac{E_i}{(k_i(k_i-1))/2} = \frac{2E_i}{k_i(k_i-1)} \quad (6)$$

Heterogeneity *H* can characterize the uniformity of node distribution. When H = 1, the distribution of network degree is the most uniform. When H = 0.5, the distribution of network degree is the most disorder (Wu et al., 2007). The calculation method is as shown in formula (7) .

$$H = \frac{-\sum_{i=1}^{N} q(k_i) \ln q(k_i)}{\ln N} \quad (7)$$

Meshedness coefficient M_3 is used to measure the network topology (Courtat et al., 2010). e is the number of edges in the network, v is the number of nodes in the network, $\overline{N}(2)$ is the number of nodes with a degree of 2 in the network.

$$M_3 = \frac{e - v + 1}{2 * v * \left(1 - \overline{N}(2)\right) - 5} \quad (8)$$

Organic ratio γ_N is used to determine whether a city has been planned, as shown in formula (9). Where N (j) denotes the number of nodes with degree j in the network.

$$\gamma_N = \frac{N(1) + N(3)}{\sum_{j \neq 2} N(j)} \quad (9)$$

k-core refers to the union of k-shells whose $k_s \geq k$ (Wang et al., 2012).

4 CORRELATION ANALYSIS

4.1 Real urban road network

The road network coordinates of Beijing, Berlin, Manchester, London, Chicago and Singapore are extracted from the Open Street Map. Use Python programming to generate each city's road network. The roads in the network are regarded as edges and intersections as nodes. The road networks generation map for the six cities are shown in Figure 1.

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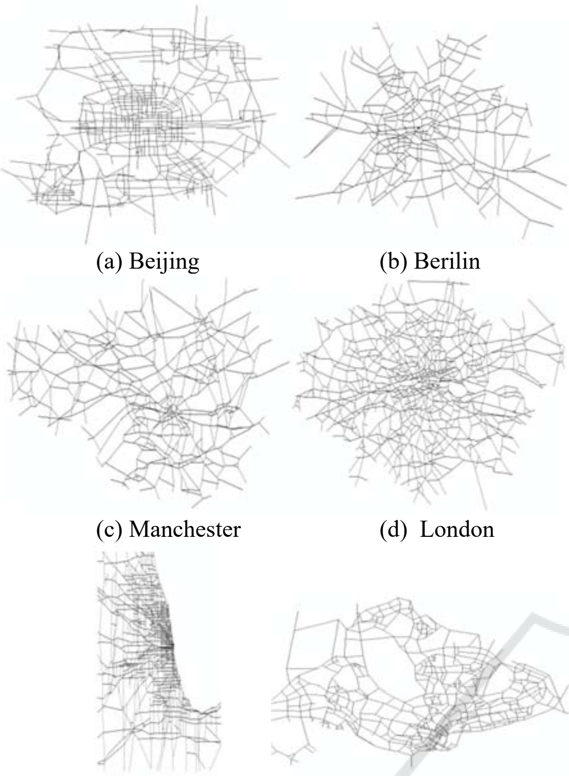

(a) Beijing (b) Berilin

(c) Manchester (d) London

(e) Chicago (f) Singapore

Figure 1: Urban road network.

The controllability and the network topology features such as average degree, 2-core, clustering coefficient, heterogeneity, meshedness coefficient and organic ratio of the above six cities are calculated, the results are shown in Table 1. Analyze the correlation between controllability and network topology features, the Pearson correlation coefficients between controllability and the above topological features are -0.574821, -0.702653 0.262591, -0.202581, -0.102783, -0.802577 respectively. It can be found that the controllability of the network has a great negative correlation with the average degree, 2-core and heterogeneity.

4.2 Simulated urban road network

Through the analysis of the real road network, it can be found that the controllability has a great correlation with some network topological features. In the following, we will use simulated urban road network to verify the above results.

The research on network evolution mechanism is an important means to explore the network formation mechanism. It mainly involves five kinds of events in the evolution of networks: adding nodes, adding edges, reconnecting, removing edges, and removing nodes. Researches on small-world network model and scale-free network model in this respect are of groundbreaking significance. Studies have shown that as the network size increases, the controllability of the network becomes smaller. Therefore, this paper starts with the evolution of road network and studies the influence of topology characteristics on the network controllability as the network size changes. Mainly include the urban road network evolution model (Barthelemy and Flammini, 2007) and the urban road network model based on β-Skeleton structure (Osaragi and Hiraga, 2011).

4.2.1 Road network based on the evolution model

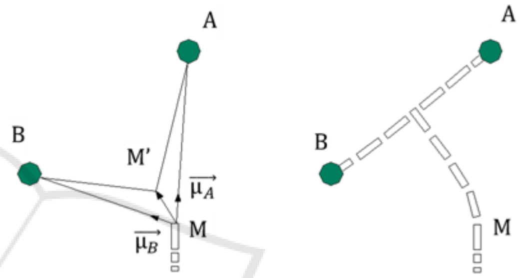

Figure 2: The optimal principle of urban road network evolution model.

At a given stage of the road network evolution, two nodes A and B still need to be connected to the network. At any time step, each new node can trigger the construction of a single new portion of road of fixed length. In order to maximally reduce their distance to the network, both A and B would select the closest points M_1 and M_2 in the network as initial points of the new portions of roads to be built. If M_1 and M_2 are distinct, segments of roads are added along the straight lines M_1A and M_2B. If $M_2 = M_1 = M$, it is not economically reasonable to build two different segments of roads and in this case only one single portion MM' of road is allowed. The main assumption here is that the best choice is to build it and maximize the reduction of the cumulative distance Δ from M to A and B, as shown in Figure 2.

$$\Delta = [d(M,A) + d(M,B)] - [d(M',A) + d(M',B)] \tag{10}$$

The maximization of Δ is done under the constraint $|MM'| = \text{const.} \ll 1$.

Table 1: The topology features of urban road network.

city\parameter	$\langle k_i \rangle$	2-core	C_i	γ_N	M_3	H	n_D
Manchester	3.2538	0.8914	0.1080	0.5138	0.2884	0.9922	0.0245
Berlin	3.1689	0.8733	0.0773	0.4955	0.3017	0.9914	0.0178
Singapore	3.2148	0.9065	0.0603	0.5359	0.3082	0.994	0.0105
Beijing	3.1069	0.8265	0.0528	0.3781	0.2798	0.9893	0.0222
London	3.2456	0.9287	0.0902	0.5837	0.3147	0.9948	0.0132
Chicago	3.3705	0.9223	0.0770	0.4186	0.3477	0.9950	0.0089

When the growth rate is taken as 0.2 and the distribution of nodes is uniform, the network controllability analysis under different network size is carried out. Through correlation analysis, it can be found that the controllability of the road network generated by the evolution model shows a large negative correlation with the average degree, 2-core and the heterogeneity. The Pearson correlation coefficients were -0.758669, -0.941210, -0.904039, respectively.

Table 2: Network topology features of evolutionary network.

Size	$\langle k_i \rangle$	2-core	H	n_D
50	1.9758	0.400	0.96888	0.1400
80	1.9958	0.625	0.96842	0.1375
100	1.9983	0.700	0.96618	0.1300
150	2.0002	0.800	0.96577	0.1267
180	2.0390	0.833	0.96530	0.1222
200	2.1602	0.875	0.96529	0.1200

4.2.2 Road network based on the β-skeleton

$\beta -$ skeleton structure: given a point distribution p_i $(i = 1,2,\dots,n)$ in a two-dimensional plane, and randomly connect these points to form a side to create a geometric map. Assuming that there are two arcs passing through any point p_1 and p_2, the size of the intersecting region E of the two arcs increases as the parameter β increases. It is determined whether or not a third point is included in the area. If there is a third point, the line segment between the points p_1 and p_2 is not the edge of the network and needs to be deleted. If there is no third point, Then the line segment between points p_1 and p_2 exists as a side of the network.

The form of the network with different parameters β is shown in Table 3. When $\beta = 0$, the shape of the network is Delaunay triangulation graph; when $\beta \in (0,1]$, the shape of the network is Gabriel graph; when $\beta \in (1,2]$, the shape of the network is the Relative neighborhood graph; when $\beta > 2$, the shape of the network is the spanning tree graph .

Because when $\beta \in [1, 1.5]$, the simulated urban road network model based on β-Skeleton structure has the maximum similarity with the real urban road network. So this paper carries out the network controllability analysis under $\beta = 1.5$. Through correlation analysis, it can be found that the controllability of the road network generated by the β-Skeleton structure also shows a large negative correlation with the average degree、2-core and heterogeneity. The Pearson correlation coefficients were -0.990535, -0.945245, -0.966129 respectively.

Table 3: The definition of β-skeleton.

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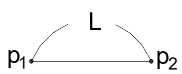
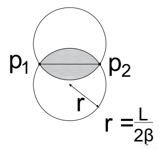
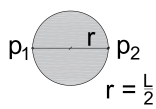
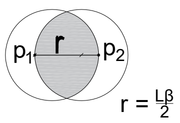
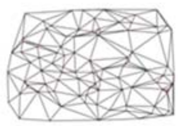
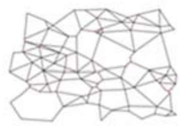
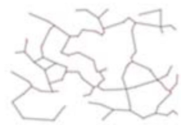
Definition				
Network				

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Table 4: Network topology features of $\beta - \mathrm{Skeleton}$ network.

Size	$\langle k_i \rangle$	2-core	H	n_D
50	2.6000	0.9400	0.9849	0.0400
80	2.7250	0.9500	0.9898	0.0250
100	2.7400	0.9500	0.9895	0.0200
150	2.8267	0.9667	0.9923	0.0133
180	2.8556	0.9722	0.9927	0.0111
200	2.8800	0.9750	0.9926	0.0050

5 CONCLUSION

Based on the theory of complex network, this paper uses the PBH judgment theorem and the minimal control input theorem to analyze the network topology features of real and simulated road network. It is found that network average degree, 2-core and heterogeneity are strongly related to the controllability of the network. Increasing the average degree will improve the synchronization ability of the network as well as the range of propagation. 2-core can help to distinguish the truly important core nodes in the actual network. Reasonable control of network heterogeneity can improve the network anti-congestion ability. Therefore, it is of great significance to control the orderly operation of urban road network by rationally configuring the number of intersections and the number of road intersections. It is also of great importance to control the size of the network and the capacity of the road and strengthen the connectivity of the network.

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