

Errors in Representation Translation in Solving Problems Related to Number Sense of Pre-Service Math Teachers

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Abstract: This research aims to describe errors in the representation translation ability in solving problems related to number sense of the pre-service math teachers. The representation translation process used the Lash model and focused on the representation translation of symbols, written language, and pictures. The research subject was chosen according to their ability, and the result was 3 people with low ability, 4 people with average ability, and 3 people with high ability. Based on the research result, the errors were mostly found in the representation translation of decimal symbols into written language representation and fraction symbols representation. It was easier for the pre-service math teachers to understand the representation translation of pictures into written language representation and symbol representation. It is recommended for further researches to study about the obstacles in representation of problems which are related to number sense.

1 INTRODUCTION

Number sense is important in learning mathematics. Number sense is related to intuitive feeling and the ability to be flexible when working with numbers (Howden, 1989; NCTM, 1989). Number sense is used as the basis in the concept of measurement, geometry, algebra, and data analysis (Purnima et al., 2014). Number sense is also used to develop mathematical ability at school (Cochran and Dagger, 2013). Furthermore, number sense also influences the performance in mathematics because people who do mathematic calculation through algorithm do not learn mathematics yet (Chattopadhyay et al., 2017). From the description above, number sense ability is important in building mathematic ability.

In many levels, number sense ability is very low. Number sense ability in Junior High School is very low in all grades (AK kaya, 2016). The number sense performance of students aged 12-13 years-old is weak (Purnima et al., 2014). According to Sa'dijah (2013), many Junior High School students have low number sense. Based on the research conducted on mathematics students, the result is that their number sense ability is low and they face difficulties in doing representation translation (Ali, 2014). Therefore, there is a lot of chance to review

the representation translation about number sense done by the pre-service math teachers.

A lot of researchers tried to analyze the errors in representation translation process. Pre-service teachers' ability in doing representation translation from mathematical notation (operation and brackets) into problem statement is low (Isik, 2012). The representation translation process from verbal into graphic is not easy because it needs more than one translation process, such as symbolic, schematic, equation, and numeric (Rahmawati et al., 2017). The ability to use mathematics flexibly in daily situation is being more emphasized, yet to solve a problem, representation translation from verbal description and interpreting it into other representation is needed (Usman, 2015). The difficulties in bridging these representations and changing one representation into other representation are the causes of the difficulties in mathematical representation (Yerushalmy, 1997). Therefore, further review on the errors in representation translation about number sense is needed.

Some researchers have used the representation translation model. Janvier model (1987) used the following representation translation: formulation, tables, verbal descriptions, graphs, and object. Lesh et al. model (1987) used the representation translation: verbal symbols, written symbols,

manipulatives, pictures, and real world situations, which can be used to solve mathematical problems. According to Pal (2014), real life experiences representation, concrete models and diagrams, oral language and symbols are needed to solve number sense problems. The representation translation about number sense problem in this research uses Lash et al. (1987) representation translation model which has been adapted: a) symbolic, b) written language, and c) pictures. Based on these descriptions, the aims of this research to describe errors in the representation translation ability in solving problems related to number sense of the pre-service math teachers.

2 METHOD

This research reveals the errors in representation translation of pre-service math teachers when solving problems related to number sense. The representation translation process is focused on the symbolic, written language, and pictures representation translation. The adapted Lash et al. (1987) representation translation model can be seen in the following Figure 1.

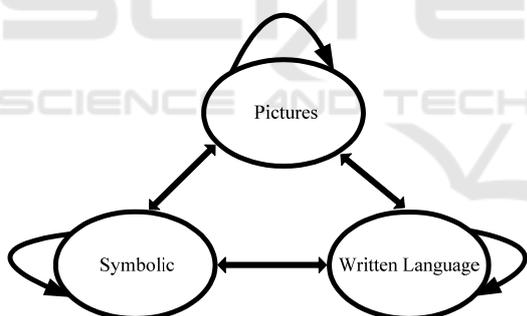


Figure 1: Representation Translation Model (Lash et al., 1987).

In particular, this research describes the errors in representation translation about *number sense* for pre-service math teachers. The researchers wanted to explain about the translation process from one representation into other representation. This research was conducted using qualitative data. Qualitative research can explain, interpret, and classify the obtained data (Yahia, 2006). This is descriptive explorative research because the researchers wanted to obtain detailed data naturally about the process of representation translation in number sense problems for the pre-service math teachers.

The subject of this research was 10 pre-service math teachers, which consisted of 3 people with low ability (X1, X2, and X3), 4 people with average ability (X4, X5, X6, and X7), and 3 people with high ability (X8, X9, and X10). These pre-service math teachers were given two problems in number sense. Their results were examined. If they did the representation translation correctly, their results would not be analyzed. However, if they did not do the representation translation correctly, their results would be analyzed. The steps of choosing the research subject can be seen in Figure 2 below.

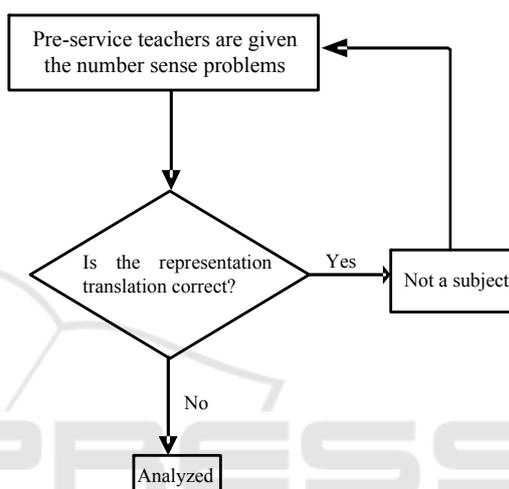


Figure 2: Choosing the Research Subject.

The instruments used in this research were: question sheets, recording tools, interview guidance, and field notes. The question sheet were given to find out the process of representation translation in number sense done by the pre-service math teachers. The question sheets were developed by the researchers with the help from advisors and then were validated by experts. There were two recording tools, those were picture and voice recorder. The interview was conducted to reveal the process of representation translation in number sense of the pre-service math teachers. The field notes were notes made by the researchers when observing the pre-service teachers solving the problems and answering the researchers' question in the interview.

The data analysis process was begun with examining the result of the pre-service teachers, recordings, field notes, and interview. The errors in representation translation made by the pre-service teachers were then analyzed. Qualitative analysis was used to observe their concept understanding that they had learned (Ghosh, 2015). The recordings were used to further examine the representation

translation errors. The research procedure was divided into several stages. In the preparation stage, we prepared the learning tools and conducted a pre-test. Based on this pre-test, we found that the representation translation process in number sense was still a problem for the pre-service math teachers in Malang. In the data collecting stage, a camera was used as a recording tool. To further understand about the representation translation process in number sense done by the pre-service teachers, an interview was conducted to the selected subjects after they finished solving the problems. These recordings were then transcript. The data analysis stage was done by reducing the data and illustrating diagrams of representation translation in number sense problems.

Instruments validation was made before conducting this research. The validation was made by 2 experts: 1 mathematics expert and 1 learning expert. The criteria of the instruments validation can be seen in Table 1.

Table 1. Validation Result Criteria (Akbar and Sriwiyana, 2011).

Standard Criteria	Category	Explanation
86% – 100%	Very valid	Can be used without revision
70% – 85%	Valid	Can be used with minor revision
60% – 69%	Not valid	Unuseable
0% – 59%	Very not valid	Cannot be used at all

3 RESULTS AND DISCUSSION

Based on the validation from validator 1, the score achieved was 84% and 88% from validator 2. Next, find the mean of the scores that was 86%. Thus, the instruments used were in very valid category, hence could be used without revision.

After the instruments were validated, they were ready to be used for this research. The researchers gave the question sheets to the pre-service math teachers. If the answer of the representation translation in number sense from the pre-service teachers was correct, it would not be used as the research subject. If the answer was incorrect, it would be used as research subject. There were two problems in the question sheet: a) 1 problem about decimal symbols representation that had to be translated into written language representation and

fraction symbols representation, and b) 1 problem about picture representation that had to be translated into fraction symbols representation and decimal symbols representation.

The description of the results of the pre-service math teachers who made errors in representation translation is as follows.

3.1 Result of Pre-Service Teacher X1

In the procedure of solving the first problem, pre-service teacher X1 could translate the decimal symbols representation into fraction symbols, but the final answer was incorrect. Pre-service teacher X1 made an error in translating the decimal symbols representation into written language and fraction symbols representation (Figure 3).

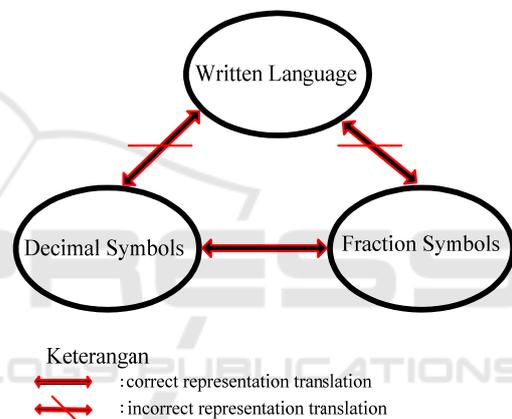


Figure 3: Representation Translation on the First Problem of Pre-Service Teacher X1.

The result of pre-service teacher X1 on the first problem (Write down the fraction along with the steps) can be seen in Figure 4 below.

Tuliskan dalam bentuk pecahan beserta cara pengerjaannya

0,252525... :

$$0,25 + 0,0025 + \dots$$

$$\frac{1}{4} + \frac{25}{10000} + \frac{25}{1000000} + \dots$$

$$\frac{1}{4} + \frac{1}{400} + \frac{1}{40000} + \dots$$

$$\frac{101}{400}$$

Figure 4: Answer on the First Problem of Pre-Service Teacher X1.

Pre-service teacher X1 did not solve the second problem, so the answer could not be analyzed.

3.2 Result of Pre-Service Teacher X2

On the first problem, pre-service teacher X2 made an error in translating the decimal symbols representation into written language representation and fraction symbols representation (Figure 5).

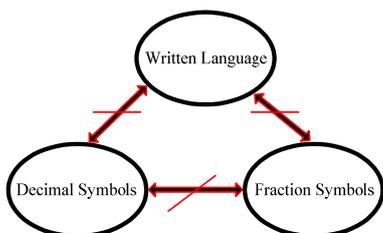


Figure 5: Representation Translation on the First Problem of Pre-Service Teacher X2.

The result of pre-service teacher X2 on the first problem can be seen in Figure 6.

Tuliskan dalam bentuk pecahan beserta cara pengerjaannya

$$0,25252525... = \frac{25}{100}$$

Figure 6: Answer on the First Problem of Pre-Service Teacher X2.

On the second problem, pre-service teacher X2 made an error in translating the pictures representation into written language representation, fraction symbols representation, and decimal symbols representation (Figure 7).

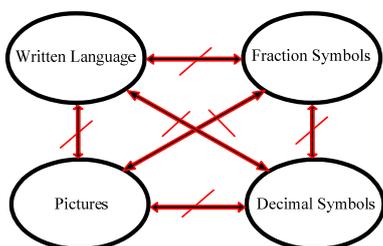


Figure 7: Representation Translation on the Second Problem of Pre-Service Teacher X2.

The result of pre-service teacher X2 on the second problem (Find the value of fraction and decimal represented by the shaded region of the

square below. Write down along with the steps!) can be seen in Figure 8.

Berapa nilai pecahan dan nilai desimal yang direpresentasikan oleh gambar yang diarsir pada persegi di bawah ini. Tuliskan beserta cara mengerjakannya!

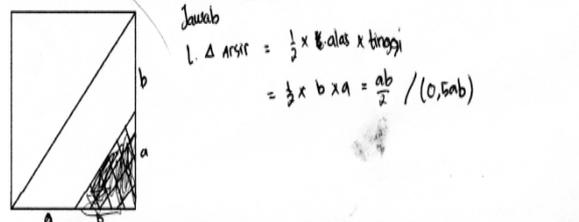


Figure 8: Answer on the Second Problem of Pre-Service Teacher X2.

3.3 Result of Pre-Service Teacher X3

On the first problem, pre-service teacher X3 made an error in translating the decimal symbols representation into written language and fraction symbols representation. Pre-service teacher X3 did not translate the written language representation but directly made incorrect representation translation of decimal symbols representation into fraction symbols representation instead (Figure 9).



Figure 9: Representation Translation on the First Problem of Pre-Service Teacher X3.

The result of pre-service teacher X3 on the first problem can be seen in Figure 10.

Tuliskan dalam bentuk pecahan beserta cara pengerjaannya

$$0,25252525... = \frac{25}{1000}$$

Figure 10: Answer on the First Problem of Pre-Service Teacher X3.

Pre-service teacher X3 did not solve the second problem, so the answer could not be analyzed.

3.4 Result of Pre-Service Teacher X4

On the first problem, pre-service teacher X4 did not translate the written language representation. Pre-service teacher X4 directly translated the decimal symbols representation into fraction symbols representation incorrectly (Figure 11).

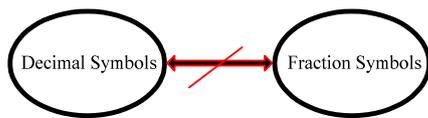


Figure 11: Representation Translation on the First Problem of Pre-service Teacher X4.

Figure 12 shows the result of pre-service teacher X4 on the first problem.

Tuliskan dalam bentuk pecahan beserta cara pengerjaannya
0,25252525...

$$\frac{2525 \ 2525 \dots}{1000000 \dots}$$

Figure 12: Answer on the First Problem of Pre-Service Teacher X4.

3.5 Result of Pre-Service Teacher X5

Pre-service teacher X5 did not solve the first problem, so the answer could not be analyzed. Meanwhile the answer of pre-service teacher X5 on the second problem can be seen in Figure 13.

Berapa nilai pecahan dan nilai desimal yang direpresentasikan oleh gambar yang diarsir pada persegi di bawah ini. Tuliskan beserta cara mengerjakannya!



Figure 13: Answer on the Second Problem of Pre-Service Teacher X5.

In Figure 14, we can see that on the second problem, pre-service teacher X5 did not translate the written language representation, but directly translate the pictures representation into fraction symbols representation and decimal symbols representation incorrectly.

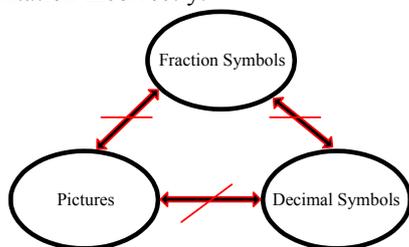


Figure 14: Representation Translation on the Second Problem of Pre-Service Teacher X5.

The result of the pre-service teachers for the first problem is: a) 4 people answered incorrectly (X1, X2, X3, and X4), b) 3 people did not answer (X5, X6, and X7), and c) 3 people answered correctly (X8, X9, and X10). There are 4 incorrect answer on the first problem which can be analyzed. The result of the pre-service teachers for the second problem is: a) 2 people answered incorrectly (X1 and X5), b) 2 people did not answer (X2 and X3), and c) 6 people answered correctly (X4, X6, X7, X8, X9, and X10). There are 2 incorrect answer on the second problem which can be analyzed.

In general, the representation translation on the first and the second problems of the pre-service math teachers can be seen in Table 2. Based on Table 2, for the first problem, 70% of the pre-service teachers were not able to translate the decimal symbols representation into written language representation and fraction symbols representation. On the second problem, the result was 40% of the pre-service teachers were not able to translate the pictures representation into written language representation, fraction symbols representation, and decimal symbols representation.

Table 2: Representation Translation of Pre-Service Math Teachers.

Name	Problem 1		Problem 2		
	a	b	a	b	c
X1	x	x	x	x	x
X2	x	x	x	x	x
X3	x	x	x	x	x
X4	x	x	√	√	√
X5	x	x	x	x	x
X6	x	x	√	√	√
X7	x	x	√	√	√
X8	√	√	√	√	√
X9	√	√	√	√	√
X10	√	√	√	√	√

Notes:

a = representation translation of written language

b = representation translation of fraction symbols

c = representation translation of decimal symbols

√ = pre-service teacher can do the representation translation correctly

x = pre-service teacher cannot do the representation translation correctly

The pre-service teachers could not do the representation correctly because they did not understand the problem well. This is in line with the

opinion from Hapsah, et al. (2017), which stated that the incorrect problem understanding will lead to incorrect representation. Errors in understanding the problem also cause the modelling process from one representation into other representation to be incorrect (Murniasih, 2016). After understanding the problem, the right strategy is needed to solve the problem (Risalah, et al., 2016).

The researchers conducted an interview with the pre-service teachers who made errors in doing the representation translation in number sense problems. The interview was not conducted on all of the pre-service teachers, but it was classified based on the characteristics of errors from the answers. From each group, one person was chosen to be interviewed. 3 subjects was chosen for the interview, namely X1, X2, and X5. A break was given in every interview so that the subjects could give the complete answer. This is in line with the opinion from Juairiyah et al. (2014). Based on the interview with subject X1 on the first problem, subject X1 was able to translate the decimal representation into fraction representation, but he found difficulties in finding the final answer. Subject X1 was confused because the addition was infinite, so he could not write down the final answer. Subject X1 stated that the denominators were distinct, thus he did not know how to add them. On the second problem, subject X1 stated that he was confused because the area was unknown; therefore, he could not find the final answer. Based on the interview with X2, on the first problem, subject X2 did not know the procedure; therefore, he directly gave the incorrect final answer. On the second problem, subject X2 stated that the area was unknown, so he could not find the final answer. Based on the interview with X5, on the first problem, subject X5 could not solve the problem because he forgot the steps. The researchers asked whether he had done similar problem before, and he answered that he had, but he could not recall the steps at all. On the second problem, subject X5 stated that he had difficulties with fractions. Based on the interviews, we can see that the learning for the pre-service teachers is still not effective. Ineffective learning is the cause of the students' inability to solve non-routine problems (Whittle and Pacaya, 2007; Heath, 2010; Wright, 2016).

4 CONCLUSIONS

Based on the result and discussion, the representation translation in number sense is still a

problem for the pre-service math teachers. The pre-service math teachers made more mistakes in translating the decimal symbols representation into written language representation and fraction symbols representation. On the other hand, they made less mistakes in translating the pictures representation into written language representation, fraction symbols representation and decimal symbols representation. Further research are recommended to study about the obstacles in representation.

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