# Third-Order Elastic Moduli of the Dry and Water Saturated Rocks

Yanli Qu, Jinxia Liu*, Zhiwen Cui and Weiguo Lv
*Department of Acoustics and Microwave Physics, College of Physics, Jilin Univercity, Changchun, China.*
*Email: jinxia@jlu.edu.cn*

Keywords: Third-Order Elastic Moduli, water saturated rocks, acoustoelastic

Abstract: On the basis of acoustoelastic theory of elastic solid, on two different rocks acoustic velocities were measured in dry state and water saturated state under uniaxial stress. Using the least-squares method and error analysis, we obtained the third-order elastic moduli of dry rocks and saturated rocks. Then, by analogy with Biot theory, we introduced the seepage strain to modify the acoustoelastic theory in the porous media, and established an equivalent model of acoustoelastic theory in porous media. Using the above method, we acquired the high precision third-order elastic moduli of fluid-solid coupling. By comparing the experiment values with theoretical values of the acoustic velocities in rocks, the experiment values and theoretical values show good agreement, and the feasibility of the equivalent model has been proved.

## 1 INTRODUCTION

The acoustoelasticity is the acoustic velocity of elastic wave that changes with the stress (Pao et al., 1984). Before the 1980s, the acoustoelastic theory is mainly applied to metal media. Many scientists have studied the acoustoelastic theory. For example, Johnson and Shankland (1989), Meegan et al (1993), Winkler and Liu (1996) studied acoustoelastic effect. They found that the nonlinear effect of rock is much more obvious than other media, and proved the existence of acoustoelastic effect in rock through a large number of experiments. The acoustoelastic theory was extended to the porous rock with compatibility conditions of acceleration waves by Grinfeld and Norris (1996). Ba et al concluded the method of Grinfeld and Norrisby by including solid and fluid finite strains (Ba et al., 2013). Tian et al studied the acoustoelastic theory of fluid-saturated porous media in natural and initial coordinates (Tian, 2014).

Winkler and Liu measured third-order elastic moduli in a variety of dry rocks (Winkler and Liu, 1996), and obtained the theoretical results on the basis of the acoustoelastic theory of Thurston and Brugger (1964), the equations are shown in Table 1.They found that this theory describes the relation between acoustic velocities and stress. The porous medium has seven independent third-order elastic moduli, so there are certain limitations in laboratory measurement (Grinfeld and Norris, 1996). Currently, there are few experiments about measuring the values of third-order moduli in porous medium. Winkler and McGowan extended the work of Winkler and Liu to water-saturated rocks (Winkler and McGowan, 2004), and also obtained the theoretical results on the basis of the acoustoelasticity theory of Thurston and Brugger. Winkler and McGowan found obvious difference between theory and experiment. In this paper, we also carried out an experiment to measure the third-order elastic moduli of dry and saturated rocks under uniaxial stress. On the basis of the acoustoelastic theory of Winkler and Prioul, by analogy with Biot theory, we established the equivalent acoustoelastic model of water saturated rock. In addition, we compared the experimental and theoretical values of acoustic velocities.

## 2 PROCEDURE

The acoustoelastic formulas of dry rock under uniaxial stress in Table 1 (Thurston and Brugger, 1964),where $V_0$ is the acoustic velocity in the unstressed condition; $\rho$ is density; P is longitudinal wave; $S_{//}$ is shear wave with polarization direction parallel to stress; $S_{\perp}$ is shear wave with polarization direction vertical to stress; $V$ is the acoustic velocity of changing with stress. $E_{11}$ , $E_{22}$ and $E_{33}$ are strain

Qu, Y., Liu, J., Cui, Z. and Lv, W.
Third-Order Elastic Moduli of the Dry and Water Saturated Rocks.
In *Proceedings of the International Workshop on Environment and Geoscience (IWEG 2018)*, pages 428-433
ISBN: 978-989-758-342-1

components. The $C_{111}$, $C_{112}$, $C_{144}$ and $C_{155}$ are third-order elastic modulus, which are related to third-order elastic modulus $\nu_1$, $\nu_2$ and $\nu_3$: $C_{111} = \nu_1 + 6\nu_2 + 8\nu_3$, $C_{112} = \nu_1 + 2\nu_2$, $C_{144} = \nu_2$ and $C_{155} = \nu_2 + 2\nu_3$.

Table 1: The acoustoelastic formulas of dry rock under uniaxial stress.

| Mode | $\rho V^2 - \rho V_0^2$ |
|---|---|
| P(longitudinal) | $C_{111}E_{11} + C_{112}E_{33} + C_{112}E_{22}$ |
| $S_{//}$(shear, parallel to stress) | $C_{144}E_{22} + C_{155}E_{11} + C_{155}E_{33}$ |
| $S_{\perp}$(shear, vertical to stress) | $C_{144}E_{33} + C_{155}E_{22} + C_{155}E_{11}$ |

The experiment whose results are presented on this paper mainly measures the velocities of three types of waves under uniaxial stress by ultrasonic pulse transmission method. The three types of waves are longitudinal wave and two transverse waves, one with polarization direction vertical to stress and the second one, with polarization direction parallel to stress. The samples were selected from a quarry, and the rocks are defined as rock A and rock B. Rock A is yellow rust granite and rock B is granite 654.

The reference states (zero stress) of rocks are shown in Table 2. Normal state denotes the natural rocks under the indoor temperature. Dry state denotes the rocks dried in the constant temperature drying oven. Saturated state denotes the water saturated rocks.

The schematic diagram of sample under pressure is shown in Figure 1, which determines the direction of coordinate axis.

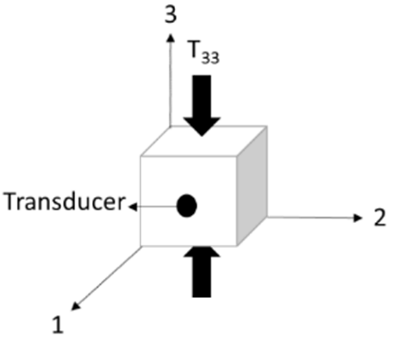

Figure 1: Schematic diagram of sample under pressure.

Rock A is fragile in water saturated state, so, it was loaded at 0-10MPa under dry state, and loaded at 0-8MPa under saturated state. Rock B was loaded at 0-10MPa under dry state and saturated state. The waveforms about rock A and rock B have similar patterns of change. Therefore, this paper only shows rock A's. The waveforms in dry state are shown in Figure 2. The waveforms in saturated state are shown in Figure 3. We can clearly see that the waveforms move forward with the increase of the stress.

The experimental measurements of acoustic velocities of rock A and rock B are shown in Figure 4 and Figure 5, respectively. It can be seen that the acoustic velocities in saturated rocks are faster than that of dry rocks. In the reference state (i.e., zero stress), the percentages of the increase of acoustic velocities are for rock A, P: 25.91%, $S_{//}$: 9.55%, $S_{\perp}$: 17.77%; for rock B, P: 22.61%, $S_{//}$: 7.93%, $S_{\perp}$: 9.86%.

Table 2. The reference state properties of rock A and rock B.

| | RockA | | | | RockB | | |
|---|---|---|---|---|---|---|---|
| | Normal | Dry | Saturated | | Normal | Dry | Saturated |
| Density(kg/m$^3$) | 2575.12 | 2574.58 | 2584.98 | | 2760.11 | 2755.56 | 2759.72 |
| P-wave(m/s) | 3623.1 | 3441.9 | 4333.7 | | 5036.2 | 4912.7 | 6023.4 |
| $S_{//}$-wave(m/s) | 2473.2 | 2323.9 | 2545.8 | | 3227.0 | 3133.1 | 3381.5 |
| $S_{\perp}$-wave(m/s) | 2322.4 | 2052.6 | 2417.3 | | 3142.1 | 3051.5 | 3352.3 |
| Porosity | 0.0104 | | | | 0.00416 | | |

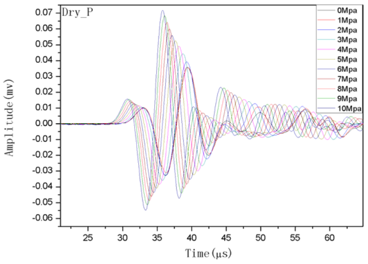

Figure 2a: P-waveform of dry rock A.

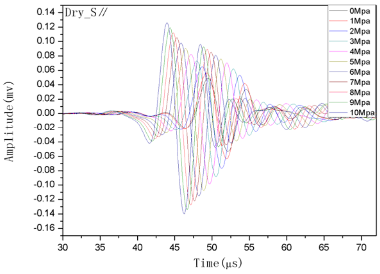

Figure 2b: S//-waveform of dry rock A.

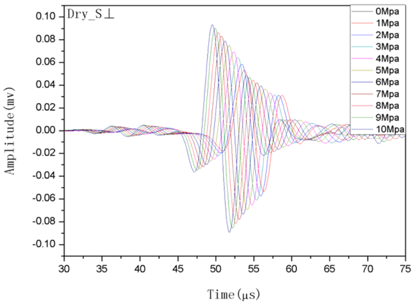

Figure 2c: S⊥-waveform of dry rock A.

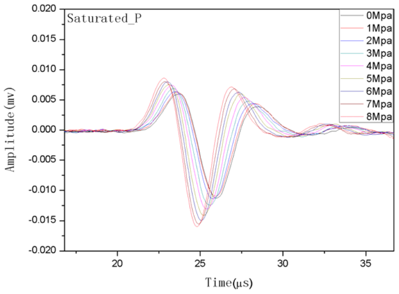

Figure 3a: P-waveform of saturated rock A.

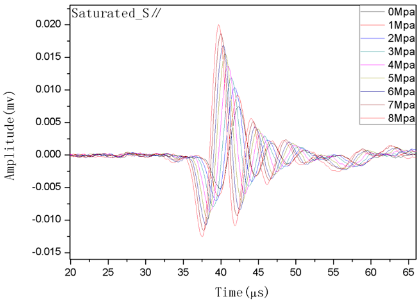

Figure 3b: S//-waveform of saturated rock A.

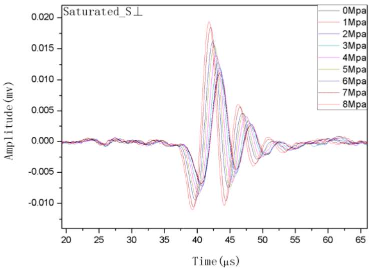

Figure 3c: S⊥-waveform of saturated rock A.

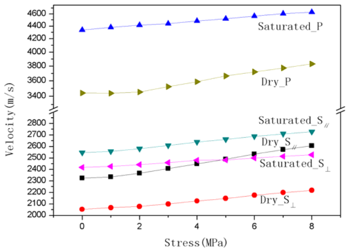

Figure 4: Experimental values of acoustic velocities in dry sample and saturated sample of rock A.

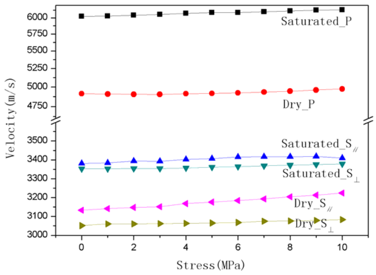

Figure 5: Experimental values of acoustic velocities in dry sample and saturated sample of rock B.

According to the formulas in Table 1, we can obtain multiple sets of third-order elastic moduli ( $V_1$ , $V_2$ and $V_3$ ) under different stress. Then, using $\chi^2 = \sum_{ij} \left[C_{ij}^{mes} - C_{ij}^{pred}\right]^2$ to determine the minimum deviation of third-order elastic moduli values, where $C_{ij}^{mes}$ is experimental value; $C_{ij}^{pred}$ is the predicted value based on acoustoelastic theory ( $C_{ij} = \rho V^2$ )(Prioul et al., 2004).The values of third-order elastic moduli( $V_1$ , $V_2$ and $V_3$ )were determined and are presented on as Table 3.

Referring to the paper of Prioul et al (Prioul et al., 2004), we analyzed the errors of the third-order elastic moduli. We assume that the third-order elastic moduli with $\chi^2_{\min}$ are $a_0 = \left[C_{111}, C_{112}, C_{123}\right]$, and the disturbance away from $a_0$ is $a$ . The increment of $\chi^2$ is $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ . We obtained $\Delta\chi^2 \leq 6.63$ . The results are shown in table 3. Comparing the values of third-order elastic moduli( $V_1$ , $V_2$ and $V_3$ ), we found that the values of $V_1$ in dry rocks and saturated rocks are quite different, and the changes of $V_2$ and $V_3$ are not obvious. We know that $V_2$ and $V_3$ are related to shear waves. In fact, the shear moduli of dry rocks and saturated rocks should be equivalent under ideal condition.

Table 3: The third-order elastic moduli of rocks.

| | State | Rock A | Rock B |
|---|---|---|---|
| $V_1$ , GPa | Dry | -28109 ± 45 | -7923.3 ± 59.5 |
| $V_1$ , GPa | Saturated | -81899 ± 170 | -72507 ± 500 |
| $V_2$ , GPa | Dry | -11849 ± 3 | -7702.2 ± 41.5 |
| $V_2$ , GPa | Saturated | -18907 ± 7.5 | -10043 ± 27.5 |
| $V_3$ , GPa | Dry | -2515.7 ± 7.8 | -2657.0 ± 6.6 |
| $V_3$ , GPa | Saturated | -2144.6 ± 13.9 | -1086.8 ± 30.4 |

# 3 THE EQUIVALENT MODEL OF ACOUSTOELASTIC THEORY OF WATER SATURATED MEDIA

Similar to Winkler and McGowan's work, we dealt with the experimental data based on the acoustoelastic theory of elastic solid. In fact, from the Biot theory (1972), the deformation of seepage strain is inevitable when the rock is immersed in water. Therefore, by analogy with the Biot theory, we proposed the seepage strain and established the formulas of acoustic velocities in saturated rocks. The equations are shown in table 4. $\rho_s$ is the density of water saturated rocks; $H$ , $h_{55}$ and $h_{66}$ are the third-order acoustoelastic moduli of fluid-solid coupling; $\xi^s$ is the relative seepage strain, ( $\zeta^s = \alpha e^s$ ; $\alpha = 1 - k_b / k_s$ , is the proportion of fluid content in body strain; $e^s$ , is total fluid strain; $k_b$ , is skeleton compression modulus; and $k_s$ , is the particle compression modulus (Dupuy and Stovas, 2014)).

Table 4: The equivalent model of acoustoelastic theory of water saturated rocks.

| Mode | $\rho_s V^2$ |
|---|---|
| P(longitudinal) | $\rho_s V_0^2 + C_{111}E_{11} + C_{112}E_{33} + C_{112}E_{22} + H \cdot \xi^s$ |
| S//(shear, parallel to stress) | $\rho_s V_0^2 + C_{144}E_{22} + C_{155}E_{11} + C_{155}E_{33} + h_{55} \cdot \xi^s$ |
| S⊥(shear, vertical to stress) | $\rho_s V_0^2 + C_{144}E_{33} + C_{155}E_{22} + C_{155}E_{11} + h_{66} \cdot \xi^s$ |

In a limited experimental environment, in order to verify the equivalent model of the acoustoelastic theory of porous media, here we assumed that the rocks are rigid. So the values of third-order elastic moduli( $V_1$ , $V_2$ and $V_3$ )in Table 4 are approximately equal to the values of dry rocks. Then, we used the values of third-order elastic moduli( $V_1$ , $V_2$ and $V_3$ )in Table 3 and formulas in Table 4, acquired the values of $H$ , $h_{55}$ and $h_{66}$ as Table 5.

Table 5: The third-order acoustoelastic moduli of fluid-solid coupling.

| | $H$ | $h_{55}$ | $h_{66}$ |
|---|---|---|---|
| Rock A, GPa | 1176.7 ± 8.6 | -8282.4 ± 46.5 | -3401.7 ± 40 |
| Rock B, GPa | 42970 ± 450 | -12450 ± 140 | -1682.8 ± 135 |

## 4 DISCUSSION

We measured the acoustic velocities in dry rocks and saturated rocks under uniaxial stress and obtained third-order moduli of dry rocks by the least squares method. Second, on the theory of acoustic elasticity of dry rock, we established the equivalent model of acoustoelastic theory of porous media, and acquired the values of the third-order elastic moduli of fluid-solid coupling. Third, we compared the experimental values and the theoretical values of acoustic velocities in dry rocks and saturated rocks. The comparisons between theoretical and experimental values of dry rocks are shown in Figure 6 and Figure 7, and the comparisons between the theoretical and experimental values of water saturated rocks are shown in Figure 8 and Figure 9.

Through the above results, one can find that the values of $V_1$ (third-order elastic modulus) in dry rocks and saturated rocks are quite different, however, the changes of $V_2$ and $V_3$ (third-order elastic modulus) are not obvious. It indicates that $V_1$ contains the contribution of fluid-solid coupling.

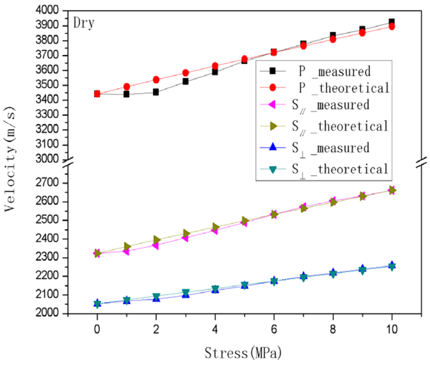

Figure 6: Comparison of experimental values and theoretical values of acoustic velocities in dry rock A.

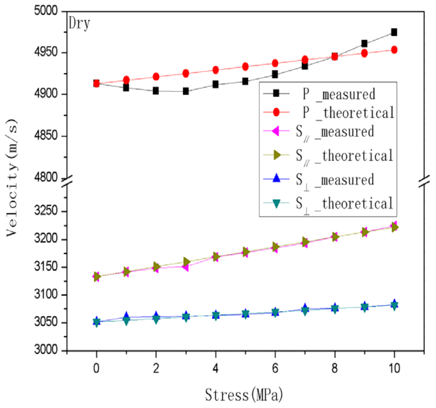

Figure 7: Comparison of experimental values and theoretical values of acoustic velocities in dry rock B.

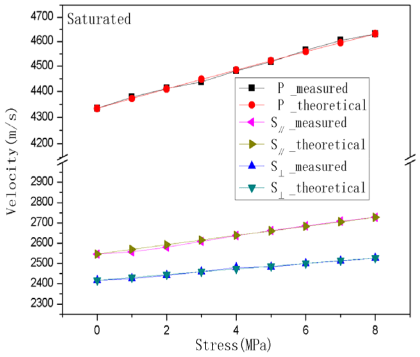

Figure 8: Comparison of experimental values and theoretical values of acoustic velocities in saturated rock A.

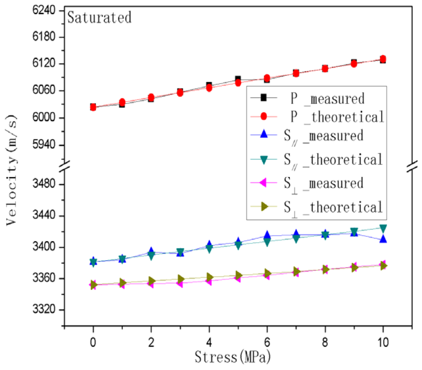

Figure 9: Comparison of experimental values and theoretical values of acoustic velocities in saturated rock B.

# 5 CONCLUSIONS

In this study, there are several conclusions: (1)The values of third-order moduli have differences in dry rocks and saturated rocks.(2)The acoustic velocities are increasing with the increase of stress in dry rocks and saturated rocks.(3)Through the comparisons between theories and experiments, the equivalent model of acoustoelastic theory of porous media can describe the relationship between acoustic velocitiesand stress. The feasibility of the equivalent model of acoustoelastic theory has been proved. It has a certain significance for the study of the acoustoelastic effectin porous media.

# ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (Grant No. 41474098), Natural Science Foundation of Jilin Province of China (Grant No.20180101282JC) and the State Key Laboratory of Acoustics, Institute of Acoustics, Chinese Academy of Sciences (Grant No.SKLA201608).

# REFERENCES

Ba J, Carcione J M, Cao H and Yao F C, Du Q Z 2013 Poro-acoustoelasticity of fluid-saturated rocks *Geophysical Prospecting* **61** 599

Biot M A 1972 Theory of finite deformations of porous solids *Indiana Univ.Math.J.* **21** 597

Dupuy B and Stovas A 2014 Influence of frequency and saturation on AVO attributes for patchy saturated rocks *Geophysics* **79** 19

Grinfeld M A and Norris N A 1996 Acoustoelasticity theory and applications for fluid-saturated porous media *J.Acoust.Soc.Am.* **100** 1368

Johnson P A and Shankland T J 1989 Nonlinear generation of elastic waves in granite and sandstone:Continuous wave and tral time obvservations *J.Geophys.Res.* **94** 17

Meegan G D, Johnson P A, Guyer R A and McCall K R 1993 Observations on nonlinear elastic wave behavior in sandstone *J.Acoust.Soc.Am.* **94** 3387

Pao Y H, Sachse W and Fukuoka H 1984 Acoustoelasticity and ultrasonic measurements of residual stresses *Physical Acoustics* 61

Prioul R, Bakulin A and Bakulin V 2004 Nonlinear rock physics model for estimation of 3D subsurface stress in anisotropic formations: Theory and laboratory verification *Geophysics* **69** 415

Thurston R N and Brugger K 1964 Third-order elastic constant and the velocity of small amplitude elastic waves in homogeneously stressed media *Phys.Rev.* **133** 1604

Tian J Y 2014 Acoustoelastic theory for fluid-saturated porous media *Acta Mechanica Solida Sinica* **27** 41

Winkler K W and Liu X 1996 Measurements of third-order elastic constants in rocks *J.Acoust.Soc.Am.* **100** 1392

Winkler K W and McGowan L 2004 Nonlinear acoustoelastic constants of dry and saturated rocks *J.Geophys.Res.* **109** 10204