Size Effect of Bending Properties of Zirconia Ceramics Based on Cosserat Theory

Y Q Xu [1,*], M M Li [2] and X Huo [1]

[1] Department of Civil Engineering, Beijing jiaotong University, Beijing 100044, China.
[2] Beijing NO.6 Construction Engineering Quality Test Department CO.,LTD. No.1 Xinhua Road, Fengtai District, Beijing 10076,China.

Corresponding author and e-mail:Y Q Xu, yqxu@bjtu.edu.cn

Abstract. In order to study the size effect of zirconia ceramics flexural properties, based on the Cosserat generalized continuum theory, the zirconia ceramic flexural specimens with different heights were numerically simulated, and the intrinsic size parameters of zirconia ceramics were determined by comparing with experimental data. Compared with the classical continuum theory, it is verified that the calculated value of bending strength based on Cosserat theory is closer to the experimental results, which proves that Cosserat theory can explain the size effect of zirconia ceramics flexural strength. The equation of size effect of nominal bending strength and nominal bending stiffness is obtained by numerical regression based on the theoretical values of classical continuum.

1. Introduction

Zirconia ceramics have many advantages, such as high hardness, high strength, and corrosion resistance and wear resistance. They are widely used in the field of micro forming. The experimental results show that the flexural properties of zirconia ceramics increase with the decrease of specimen height, showing obvious size effect [1].

The classical continuum mechanics considers an object to be composed of a continuous distribution of particles without a geometric size. It is believed that the state of any point of matter depends only on the point or the history of the point, without the influence of other material points. Therefore, there are obvious limitations in describing the bending properties of zirconia ceramic specimens with different heights, which can't explain the bending property size effect caused by the change of beam height. From a statistical point of view, it is reasonable to assume that an object is made up of an ideal continuous medium when the macroscopic size of an object is much larger than the characteristic size of the material microstructures. But when the macroscopic dimension of the object is close to the characteristic size of the material microstructure, such assumption will cause a larger deviation.

In fact, any material is made up of material points with geometric dimensions. In 1909, French scientists Cosserat E. and F. two brothers, based on the classical continuum theory, put forward generalized continuum mechanics [2], which considered objects to be composed of continuous rigid particles of a certain size. Each particle can be both translational and rotatable. And the couple stress, rotational gradient and intrinsic size of the material are introduced into the basic equations. Therefore,

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Cosserat theory can explain the scale effect phenomenon of materials at different scales in a phenomenological way [3]. Many scholars have applied the Cosserat theory to explain the size effect of metal and polycrystal [4-7] in micro scale, but the size effect of the mechanical properties of brittle materials in the macro scale is very little. The study on the size effect of zirconia ceramic bending performance based on Cosserat theory is still blank. In this paper, the size effect of the bending properties of zirconia ceramics is studied by the comparison of the numerical simulation analysis and model test based on the Cosserat theory.

2. Basic equations of plane Cosserat theory

The basic unknown quantities of the plane Cosserat theory are: two line displacements u, v and one rotational displacement ω ; two normal stresses σ_x and σ_y, two shear stresses τ_{xy} and τ_{yx}, two couple stresses μ_x and μ_y; two normal strains ε_x and ε_y, two shear strains ε_{xy} and $\varepsilon_{yx}$and two even strains κ_x and κ_y.

2.1. Equilibrium differential equation

Compared with the classical elasticity theory, due to the consideration of couple stress, when the physical forces are not considered, the equilibrium differential equations of plane problems are as follows [2].

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}=0,\quad \frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}=0,\quad \tau_{xy}-\tau_{yx}+\frac{\partial\mu_x}{\partial x}+\frac{\partial\mu_y}{\partial y}=0 \tag{1}$$

From the above equilibrium, it is known that due to the existence of couple stress, shear stress τ_{xy} and τ_{yx} are no longer equal to each other.

2.2. Geometric equation

Considering the angle and curvature, the geometric equation is shown as follows:

$$\begin{aligned} \varepsilon_x&=\frac{\partial u}{\partial x}, & \varepsilon_{yx}&=\frac{\partial u}{\partial y}+\omega, & \kappa_x&=\frac{\partial\omega}{\partial x} \\ \varepsilon_y&=\frac{\partial v}{\partial y} & \varepsilon_{xy}&=\frac{\partial v}{\partial x}-\omega & \kappa_y&=\frac{\partial\omega}{\partial y} \end{aligned} \tag{2}$$

2.3. Physical equation

For ideal elastomers, the stress and deformation conform to Hooke's law.The relationship between normal stress and normal strain is the same classical elasticity theory. Its relationship is shown as follows on the plane stress condition:

$$\varepsilon_x=\frac{1}{E}(\sigma_x-\upsilon\sigma_y),\quad \varepsilon_y=\frac{1}{E}(\sigma_y-\upsilon\sigma_x) \tag{3}$$

Because the shear stress is no longer equal to each other, the relationship between shear stress and shear strain is different from that of classical elasticity theory. Based on the assumption of linear elasticity and isotropy, the relationship between shear stress and shear strain can be expressed as

$$\tau_{xy}=a\varepsilon_{xy}+b\varepsilon_{yx}\ ,\quad \tau_{yx}=b\varepsilon_{xy}+a\varepsilon_{yx} \tag{4}$$

In the formula, a and b are the undetermined constants, and half of the sum of the formula (4) two is

$$\frac{\tau_{xy}+\tau_{yx}}{2}=\frac{a+b}{2}\left(\varepsilon_{xy}+\varepsilon_{yx}\right)=\frac{a+b}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) \tag{5}$$

This is the normal symmetrical part of shear stress, which will cause shear deformation $\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}$, which is consistent with classical elasticity theory and its coefficient $\frac{a+b}{2}=G$ .G is the first shear modulus of the material. Half of the difference of type (4) two is

$$\frac{\tau_{xy}-\tau_{yx}}{2}=\frac{a-b}{2}\left(\varepsilon_{xy}-\varepsilon_{yx}\right)=\frac{b-a}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) \tag{6}$$

This is the antisymmetric part of the shear stress, which will cause the macroscopic angle $\omega_0=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$ and its coefficient $\frac{b-a}{2}=G_c$. G_c is the second shear modulus of the material. Therefore, for plane stress conditions, the relationship between shear stress and shear strain can be expressed as

$$\begin{cases}\tau_{xy}=(G-G_c)\varepsilon_{xy}+(G+G_c)\varepsilon_{yx}\\ \tau_{yx}=(G+G_c)\varepsilon_{xy}+(G-G_c)\varepsilon_{yx}\end{cases} \tag{7}$$

For the relationship between the couple stress and the even strain, in order to maintain the unity of dimension, the intrinsic size parameter of the material is introduced. The expression is given in the paper [8].

$$\mu_x=4Gl_c^{\,2}\kappa_x,\ \ \mu_y=4Gl_c^{\,2}\kappa_y \tag{8}$$

In the above equation, $4Gl_c^{\,2}$ is the bending stiffness of the material, in which l_c is the intrinsic size of the material, which depends on the characteristic length of the material microstructures, such as the modulus of elasticity and the Poisson's ratio, which are important parameters to measure the properties of the material, and are not affected by the macro size, load and constraint conditions of the structure. Although the intrinsic size is the dimension of the length, it is not equal to the particle size of the material, so it can’t be obtained directly through the test. The determination of intrinsic size parameters is the key to solve many mechanical problems by using Cosserat's basic equations and boundary conditions. However, up to now, there has not been a search for the study of the intrinsic parameter values of zirconia ceramics, there are no test standard and operation specification can be referred to get through experiment.

3. Determination of intrinsic dimensional parameters of zirconia ceramics

The Cosserat theory of plane problem contains fifteen unknown functions and fifteen differential equations, but its analytical solution is very difficult to obtain, and can only be solved by numerical method. Therefore, this paper adopts Cosserat finite element mode [9] numerical simulation and existing test data combination method to study the intrinsic parameters of zirconia ceramics.

3.1. Experimental data on size effect of bending strength of zirconia ceramics [1]

The samples were made of 6 kinds of sintered samples, the width of all was 4.7mm, and the height was 92μm, 191μm, 289μm, 378μm, 474μm, 568μm respectively. The three point bending test was carried out and the calculated span was 30mm.The formula for calculating the bending strength is

$$\sigma_f = \frac{3Fl}{2bh^2} \tag{9}$$

The σ_f is nominal bending strength, F is the ultimate load, l is the calculation span, h is the section height of the specimen, and b is the section width of the specimen. The bending strength of specimens at different heights tested is shown in Table 1 [1].

Table 1. Material parameters of zirconia ceramics.

h/μm	F/N	σ_f / MPa
92	0.88	998
191	3.39	890
289	6.80	780
378	9.40	630
474	12.67	540
568	18.20	540

It can be seen from Table 1 that the bending strength of zirconia ceramics decreases with the increase of specimen height, showing obvious size effect.

3.2. Bending failure criterion of brittle materials

Because the tensile capacity of brittle materials is far less than its compressive capacity, when the brittle material is bent, the damage occurs at the tensile fracture of the lower side fiber in the middle cross section, and destroy all of a sudden with no significant deformation. Therefore, according to the linear elastic failure characteristics of brittle materials, the maximum tensile stress failure criterion is adopted to simulate the bending specimens of zirconia ceramics.

The maximum tensile stress criterion holds that, no matter what stress state of the material is in, as long as the maximum tensile stress σ_1 of the element is reached to the strength σ_b of the material, the brittle fracture will be occurred. According to this criterion, the failure condition of material fracture is

$$\sigma_1 = \sigma_b \tag{10}$$

It can be seen from table 1, the bending strength of the specimen tends to be stable as the height of the specimen increase. Therefore, the bending strength obtained from the specimen with the height of 568 μm is regarded as the flexural tensile ultimate strength of zirconia ceramics, which is σ_b=540 MPa.

3.3. The determination of intrinsic dimensional parameters

Numerical analysis of zirconia bending specimens at different heights is carried out with different intrinsic sizes. The calculated values are compared with the experimental values, as shown in Figure 1.

As shown in Figure 1, when the intrinsic size is 0, the calculated value is the theoretical solution of the classical continuous medium. The deviation from the test results increases with the decrease of the specimen's height, and the deviation of the specimen is 84.8% when the specimen height is 92μm. When the intrinsic size is between 20~40μm, the deviation between calculated value that based on Cosserat theory and experimental value is relatively small. It can be seen that the intrinsic size of zirconia ceramics should be in the range of 20~40μm. In addition to the specimens with height of 92μm, the deviation of the numerical solution and the test value are basically the smallest, when the other height specimens are basically the same intrinsic size as 35μm. When the intrinsic size of the specimen with height of 92μm is 20μm, the deviation from the test value is the smallest, which may be because the height is smaller, the pores in sintering are easier to discharge, the internal porosity decreases, the sintering of the sample is more compact, and the internal defects of the material are less, resulting in the decrease of the intrinsic size.

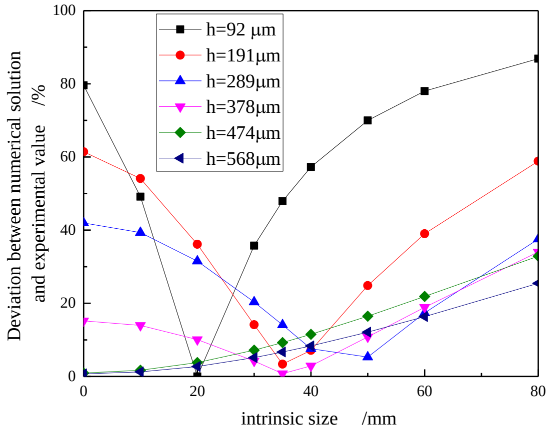

Figure 1. Deviation between numerical solution and experimental value with different intrinsic sizes.

The intrinsic dimension of zirconia ceramics is equal to 35μm, and the three point bending loading test is simulated numerically. The experimental values, classical continuum mechanics solutions and numerical solutions based on Cosserat theory are compared and analyzed, and the variation of nominal bending strength with the height of the specimen is shown in Figure 2.

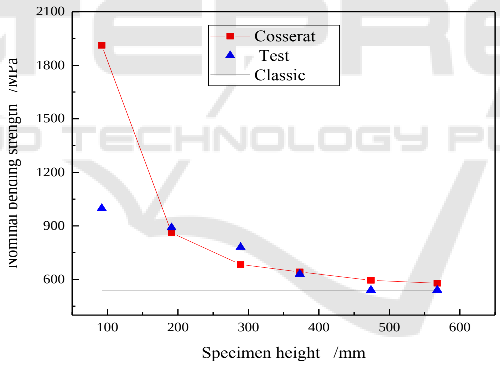

Figure 2. The variation of nominal bending strength with the height of the specimen.

It can be seen from Figure 2 that the bending strength of zirconia ceramics obtained by the classical continuum theory is constant, which is independent of the size of the specimen, and can't explain the size effect of the bending strength of the specimens at different heights. The calculation value of bending strength based on Cosserat theory and the test value are relatively closer, both are increased with the decrease of the specimen's height. When the specimen is high, the value of the two is close to the theoretical value of the classical continuous medium, which shows that the size effect of the bending strength is no longer obvious when the specimen is large enough.

4. The rule of size effect on the bending properties of zirconia ceramics

4.1. The rule of size effect of bending strength

In order to study the size effect of bending strength of zirconia ceramics, considering the influence of the span and width is less, the span and width of the specimen are kept unchanged, and the three point bending loading test is simulated with the Cosserat finite element model. When the relative dimension l_c/h is different, the ratio of the nominal bending strength σ_{Nb} to the classical theoretical solution σ_b is shown in Figure 3.

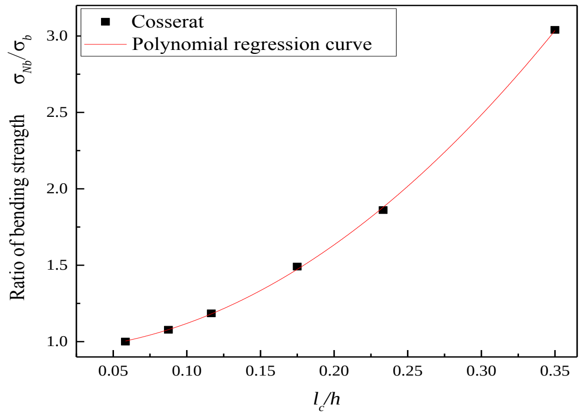

Figure 3. The dimensionless size effect law of bending strength.

The regression equation for the nominal bending strength σ_{Nb} of zirconia ceramicsis obtained by regression analysis of the numerical data:

$$\sigma_{Nb}=\sigma_b\left[1+0.1\frac{l_c}{h}+17\left(\frac{l_c}{h}\right)^2\right] \tag{11}$$

4.2. The rule of size effect of bending stiffness

For the three point bending beam, according to the classical continuum theory, the bending stiffness of the beam is as follows:

$$k_b=\frac{F}{\delta}=4Eb\left(\frac{h}{l}\right)^3 \tag{12}$$

The formula (12) shows that when the elastic modulus of the material, the width of the specimen and the high span ratio are certain, the bending stiffness of the beam calculated by the classical continuum theory is constant.

In order to study the size effect law of the bending stiffness of zirconia ceramics, the Cosserat finite element model is used to simulate the three point bending beam with a height span ratio of 0.01.When the relative size l_c/h takes different values, the ratio of the nominal bending stiffness k_{Nb} of the specimen to the classical theoretical solution k_b is shown in Figure 4.

As shown in Figure 4, the nominal bending stiffness of zirconia ceramics increases with the decrease of specimen height, and the bending stiffness exhibits obvious size effect. When the height of the specimen is far greater than the intrinsic size of the material, the nominal bending stiffness calculated by Cosserat theory is closer to the classical continuum theory, and the size effect is no longer significant. By regression to the numerical simulation data, the equation of the nominal bending stiffness k_{Nb} of zirconia ceramic with size is obtained as follows:

$$k_{Nb}=k_b\left[1+0.02\frac{l_c}{h}+18(\frac{l_c}{h})^2\right] \tag{13}$$

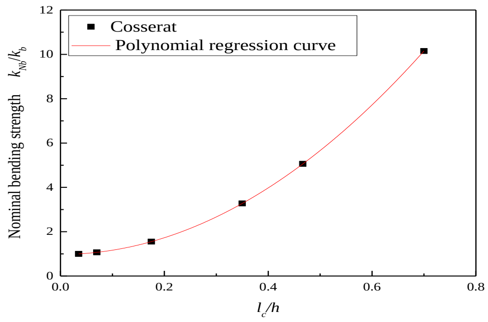

Figure 4. The dimensionless effect law of bending stiffness.

5. Conclusions

On the basis of Cosserat theory, the range of intrinsic size parameters of zirconia ceramics is determined by combining numerical simulation with flexural performance test data. The size effect law of the bending properties of zirconia ceramics is proposed. The conclusion is as follows:

1) The intrinsic size of zirconia ceramics is deter-mined to be 20~40μm by comparison of theory and experiment;

2) Considering the intrinsic dimensional parameters of the material, the bending strength of zirconia ceramic based on Cosserat theory is in good agreement with the experimental results, and it can explain the size effect of bending strength at different height beams;

3) The nominal bending strength and bending rigidity of zirconia ceramics increase with the decrease of beam height.

4) The nominal bending strength of zirconia ceramics and dimensionless size effect equation of nominal bending rigidity are obtained by numerical simulation regression analysis.

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