

# Series-Parallel Optimization Model for Heat Exchanger Network

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**Abstract:** Based on the parallel heat exchanger network, a series-parallel optimization model of the heat exchanger network for industrial circulating water system is established. The flow rate and temperature of the heat exchangers in the network can be calculated automatically. The mathematical formulation exhibit a mix-integer nonlinear programming (MINLP) structure which can be solved by GAMS. A case study is done to compare the differences between the series-parallel model and the parallel model. The result shows that this series-parallel model can reduce fresh water and increase the outlet return temperature obviously.

## 1 INTRODUCTION

Cooling water system is widely used in factories to transform waste heat from industrial equipment. Conventional cooling water systems often use a parallel heat exchanger network. This will bring a huge amount of water flow rate and the return temperature to the cooling tower might be low if the network is arranged in parallel, which will cause a poor cooling performance according to the theory of Kim and Smith (Kim and Smith, 2001). In their work, the traditional parallel exchanger networks are changed into series types by applying water pinch technology so that the bottleneck problems were solved. However, if the exchangers are changed, the cooling network might be redesigned, thus the network will be a lack of flexibility. Therefore, several authors (Kim et al., 2001; Kim and Smith, 2003; Kuo and Smith, 1998) built mathematical optimization models to solve the network problem automatically. Xiao Feng et al. (Xiao et al., 2005) put forward an intermediate temperature in the water network design. In this way, the recirculating cooling water into or out of each cooler would be from or going to one of the three mains so that the water flow rate could be reduced and the return temperature could be increased. Ponce-Ortega et al. (Ponce-Ortega et al., 2007; Ponce-Ortega et al., 2010) put forward a mixed-integer nonlinear programming algorithm for the synthesis of cooling networks. This work was a development of the

intermediate main which contained several stages and the capital and utility cost was minimized.

The above papers mainly focused on reducing the system flow rate and changing the heat exchanger network structure. This paper proposes a new series-parallel method to solve the exchanger network problems. In this method, the water can be reused so that the total water flow rate will be reduced. And the outlet temperature can be increased, which will improve the cooling tower performance. The mathematical formulation exhibit a mix-integer nonlinear programming (MINLP) structure which can be solved by GAMS. A case study is done to compare flow rate and outlet temperature between the series-parallel model and the parallel model.

## 2 MATHEMATICAL MODEL

In the model formulation, suppose the maximum amount of heat exchangers is  $n$ . HE  $i$  and HE  $j$  represents for exchanger  $i$  and exchanger  $j$  respectively. In Figure 1, the circles represent the mixing point and the squares represent for splitting point. At the mixing point of each exchanger, water mixes by part of fresh water and part of reusing water from other exchangers. So the inlet mass flow rate for heat exchanger  $i$  can be shown as Eq. (1).

$$F(i) = \sum_{j=1, j \neq i}^n F_{in}(i, j) + F_{in}(i) \quad i, j \in 1, \dots, n \quad (1)$$

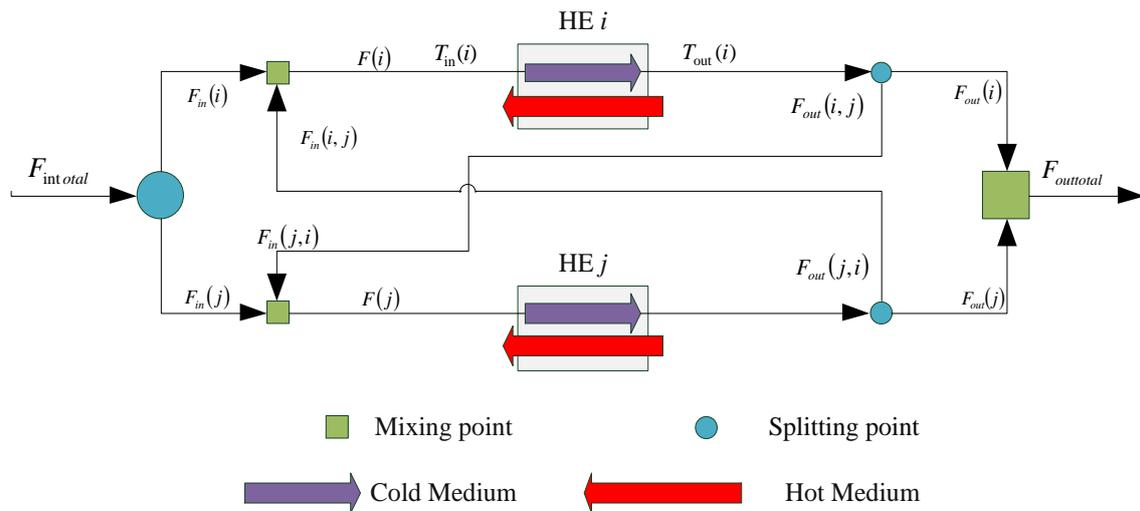


Figure 1: The structure of series-parallel exchanger network.

And at the splitting point of each exchanger, water splits into part of outlet water and part of reused water to other exchangers. The outlet mass flow rate is the same as the inlet mass flow rate which can be shown as Eq. (2).

$$F(i) = \sum_{j=1, j \neq i}^n F_{out}(i, j) + F_{out}(i) \quad i, j \in 1, \dots, n \quad (2)$$

For each exchanger, the mass flow rate from other exchangers equals the inlet mass flow rate:

$$F_{out}(j, i) = F_{in}(i, j) \quad i, j \in 1, \dots, n \quad (3)$$

The total inlet fresh water is:

$$F_{intotal} = \sum_{i=1}^n F_{in}(i) \quad (4)$$

The total outlet reused water is:

$$F_{outtotal} = \sum_{i=1}^n F_{out}(i) \quad (5)$$

Kim and Smith's (Xiao et al., 2005) work shows that because of different inlet temperature, some exchangers might not need fresh water or reused water. So on the right side of Eq. (1),  $F_{in}(i)$  and  $F_{in}(i, j)$  may not exist at the same time.  $B_{in}(i, j)$  and  $B_{out}(i, j)$  are composed of binary variables to limit reused water from exchanger  $i$  to  $j$ . And  $B_{in}(i)$ ,  $B_{out}(i)$  is to limit the inlet and outlet mass flow rate. For exchanger  $i$ , The upper and lower bounds of the mass flow constraints are shown as follows:

$$L_i B_{in}(i) \leq F(i) \leq U_i B_{in}(i) \quad i, j \in 1, \dots, n \quad (6)$$

$$L_i B_{out}(i) \leq F(i) \leq U_i B_{out}(i) \quad (7)$$

$$L_i B_{in}(i, j) \leq F_{in}(i, j) \leq U_i B_{in}(i, j) \quad (8)$$

$$L_i B_{out}(i, j) \leq F_{out}(i, j) \leq U_i B_{out}(i, j) \quad (9)$$

$L_i$  and  $U_i$  are the upper and lower bounds of the water streams respectively. This paper supposes each stream has the same upper and lower bounds. The number of reused streams should be limited to a certain amount. So the constraints are as follows:

$$\sum_{i=1}^n \sum_{j=1}^n B_{in}(i, j) \leq NB_{max} \quad (10)$$

$$\sum_{i=1}^n \sum_{j=1}^n B_{out}(i, j) \leq NB_{max} \quad (11)$$

$$B_{in}(i, j) = B_{out}(i, j) = 0 \quad i = j \quad (12)$$

Suppose the heat that the hot stream  $i$  exchangers is  $q(i)$ . The inlet temperature of cold stream  $i$  is  $t_{in}(i)$ , and the outlet temperature is  $t_{out}(i)$ . The heat balance is shown as Eq.(13).

$$q(i) = (t_{out}(i) - t_{in}(i)) F(i) CP \quad (13)$$

In Eq.(13)  $CP$  represents for heat capacity flow rate for the cold stream  $i$ . At the mixing point, the inlet temperature of cold stream  $i$  is the mixing temperature by fresh water and other exchangers' used water, so Eq.(14) needs to be satisfied.

$$F(i) t_{in}(i) = \sum_{j=1, j \neq i}^n F_{in}(i, j) t_{out}(j) + T_{in} F_{in}(i) \quad (14)$$

$T_{in}$  represents for the inlet temperature of the total network. At the splitting point, the temperature equals the outlet temperature of the cold stream. At the mixing point return to the cooling tower, in Eq.(15) outlet temperature  $T_{out}$  constraint is shown as follows.

$$T_{out} \sum_{i=1}^n F_{out}(i) = \sum_{i=1}^n t_{out}(i) F_{out}(i) \quad (15)$$

Ponce-Ortega et al. (Ponce-Ortega et al., 2010) discussed that the d-value between the inlet temperature of the hot stream and the outlet temperature of the cold stream should exceed a certain value. Thus, the inlet and outlet temperature of the cold stream cannot exceed a certain value. The temperature value is expressed as  $\Delta T$ . And the constraints are as follows.

$$T_{out}(i) \leq T_{HOT,in}(i) - \Delta T \quad (16)$$

$$T_{in}(i) \leq T_{in,max}(i) \quad (17)$$

In the series-parallel automated design method, the purpose is to reduce the fresh water.

$$OB = \min(F_{intotal}) \quad (18)$$

The program is shown above. The objective function is Eq.(18) to reduce cooling system flow rate, which can increase the outlet temperature simultaneously. Eq.(1) to Eq.(5) are equality constraints and Eq.(6) to Eq.(12) are inequality constraints and logical constraints to limit heat exchange cold medium flow rate. Eq.(13) to Eq.(17) are temperature constraints.  $B_{in}(i, j)$  and  $B_{out}(i, j)$  are binary variables while others are continuous variables. There are nonlinear constraints in Eq.(13)-(15). So the mathematical problem is a mix-integer nonlinear programming (MINLP). GAMS is an efficient mathematical tool for solving optimization problems. This article uses one of the solvers named DICOPT to solve this MINLP problem.

### 3 CASE STUDY

The case study is used to validate the mathematical problems and compare the traditional parallel heat exchange network and series-parallel model of this work. Suppose there are 4 hot streams in the cooling water system, the hot stream operation data are shown in the following Table.

Table 1: Hot stream data.

| Exchanger             | 1  | 2  | 3  | 4  |
|-----------------------|----|----|----|----|
| Inlet Temperature/°C  | 50 | 55 | 75 | 75 |
| Outlet Temperature/°C | 30 | 45 | 45 | 60 |
| CP(kW/°C)             | 15 | 80 | 20 | 40 |

Suppose each heat exchanger contains only one hot stream and one cold stream. According to the hot stream data in Table 1, suppose the temperature

difference  $\Delta T$  in Eq.(16) is 10°C. Then the cold medium data can be calculated in Table 2. The maximum inlet and outlet temperature and CP are shown as follows.

Table 2: Cold medium data.

| Heat Exchanger                | 1  | 2  | 3  | 4  |
|-------------------------------|----|----|----|----|
| Maximum Inlet Temperature/°C  | 20 | 35 | 35 | 50 |
| Maximum Outlet Temperature/°C | 40 | 45 | 65 | 65 |
| Maximum CP(kW/°C)             | 15 | 80 | 20 | 40 |

According to the model above, the exchanger network can be designed in a series-parallel method. The cold stream data is used to solve the model to optimize the minimum flow rate of the total system. The problem is solved by GAMS. And the results are shown as the following Figures and Table.

Figure 2 and Figure 3 shows the differences between the parallel model and the series-parallel model. In the parallel model, the CP is 155 kW/°C. Suppose the cooling water specific heat capacity is 4.18 kJ/(kg°C). So the total mass flow rate is 37.1kg/s. While in the series-model the mass flow rate is 12.4 kg/s. The outlet return temperature of the parallel is 34.8°C while that of the series-parallel model is 64.2°C. According to the theory of Kim and Smith (Kim and Smith, 2001), the higher the return temperature of the cooling tower is, the better the cooling tower performance will become.

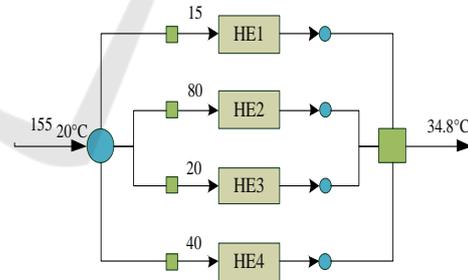


Figure 2: Parallel optimization model.

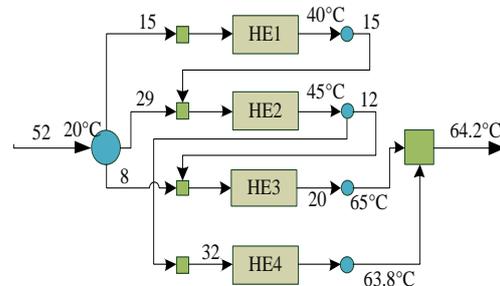


Figure 3: Series-parallel optimization model.

From the below Table 3, we can conclude that the series-parallel model can save large amount of fresh water and increase the outlet return temperature. From the perspective of economic and energy efficiency, series-parallel model can save costs and improve tower performance.

Table 3: Comparison of different models

| model           | Mass Flow Rate(kg/s) | Outlet Temperature/°C |
|-----------------|----------------------|-----------------------|
| parallel        | 37.1                 | 34.8                  |
| series-parallel | 12.4                 | 64.2                  |

## 4 CONCLUSIONS

This work builds a mixed-integer nonlinear programming (MINLP) algorithm and solves the model by GAMS. A case study explains that in the automated series-parallel model, the water flow rate can be reduced by almost 24.7kg/s and the temperature might be increased by 29.4°C. The improved optimization model can reduce operating costs and improve cooling tower performance which is important for improving the efficiency of the entire circulating water system.

## ACKNOWLEDGEMENT

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