

# Workforce Modelling in Support of Rejuvenation Objectives

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**Abstract:** This paper presents a method for measuring the effect of staffing policies toward objectives of workforce rejuvenation. It describes two deterministic models based on the application of rates of personnel flows to workforce segments. The first model works by solving a system of linear equations describing personnel flows to obtain the workforce's age profile at equilibrium. The second model, by iterating through successive future years, determines the age profile that will result from the set personnel flows. The dynamic model is necessary to identify shorter term effects of staffing policies.

## 1 BACKGROUND

This paper describes some elements of a study conducted for the Canadian Department of National Defence. The study was requested by the Chief of Staff for the Assistant Deputy Minister (Science and Technology). Among this office's responsibilities is the management of the Defence Scientific Service Occupational Classification – a subset of the Federal Public Service.

At the end of the month of June 2017, there were 616 Defence Scientists. This workforce's age distribution is shown in Figure 1.

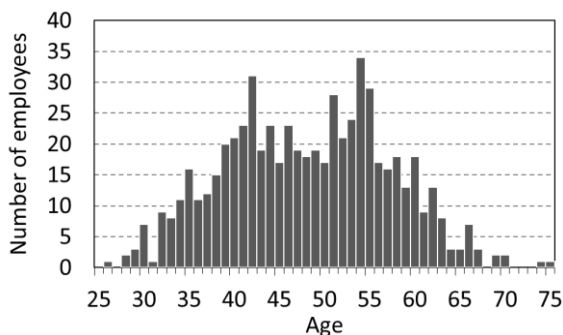


Figure 1: Workforce age distribution.

The study sponsors believed that this age profile was less than ideal, as it was thought to contain too high a proportion of employees that are either eligible, or close to eligible for retirement. Ideally,

Defence Scientists would acquire expertise over the course of a long career and pass it on to the next generation before retirement, through supervision and mentoring. With relatively few junior employees for each highly experienced employee approaching retirement, there was a fear that expertise was not going to be effectively transferred.

Federal Public Servants are eligible for an immediate annuity at the age of 65, or at 60 if they have served at least 30 years. For employees hired before 2013 the ages are respectively 60 and 55. Many Defence Scientists retire at the point of first eligibility, or soon after. For most current employees, this happens between the ages of 55 and 60. Otherwise, the amount of the pension still increases with the number of years of service, up to 35 years, leading some Public Servants to continue working past the date of their eligibility for an immediate annuity. Finally, some chose to continue to work beyond 35 years of service, despite their annuity no longer increasing as a proportion of their final salary.

A study of rejuvenation strategies was requested. The intent of this study was to identify policies that would result, over time, in a more balanced age distribution that would allow a better transfer of expertise from one generation of Defence Scientists to the next. In particular, the study aimed to predict the age distribution that could be expected if no corrective action was taken, and the range of possible outcomes from potential new staffing policies.

A related study of the Defence Scientist workforce

was described by Eles and Massel (2008), but that study focused on career progression, rather than rejuvenation. Past forecasts for other classifications of Department of National Defence employees have often been based on Discrete Event Simulation (Isbrandt and Zegers, 2006) (Erkelens et al., 2007). Instead, this paper presents deterministic models based on the application of rates of personnel flows to the entire workforce.

## 2 AGE AT THE TIME OF HIRE

The Defence Scientific Service Classification is broken down into levels, numbered from 1 to 8. The level of a Defence Scientist corresponds to his or her state of career progression, and is tied to a pay scale.

New hires are assigned a level according to an assessment of their education and prior work experience. The vast majority of hires are assigned levels between 2 to 6. Figure 2 shows hiring counts, by level and age, between 1 April 2008 and 30 June 2017.



Figure 2: Age and level of new hires.

New employees are hired on different dates throughout the year. To facilitate subsequent analysis, we will be tracking age at the time of hire as the age of the employee at the end of the fiscal year in which he or she was hired (31 March). For example, an employee hired in June, at the age of 50, and with a birthday in August, will be recorded as having been hired at the age of 51.

It is seen, in Figure 2, that the level assigned to new hires tends to increase with their age at the time of hire. This is because many older hires have acquired professional and academic experience warranting a higher level upon becoming Defence Scientists.

Public Service staffing competitions are always aimed at specified levels. Prospective employees will only be hired through competitions that target the

level that is commensurate with their previously acquired experience. Competitions targeted at lower classification levels then bring in less experienced (and thus younger) recruits than competitions targeting higher levels. Given that age discrimination is prohibited, younger employees cannot be directly targeted, but the age profile of the defence scientific workforce is indirectly a function of the levels targeted by staffing competitions.

The study described in this paper modelled the effects of changing the distribution of hiring across levels on the eventual workforce age profile.

Historically, as shown in Figure 2, approximately 15% of the recruits were hired at level 2, 40% at level 3, 28% at level 4, 8% at level 5 and 7% at level 6 (which does not add up to 100% due to rounding). At the same time, the mean age at hire was 31 at level 2, 35 at level 3, 46 at level 4, 55 at level 5, and 63 at level 6. Therefore, any shift of the hiring ratios toward junior levels would tend to lower the average hiring age. Table 1 shows six scenarios for different distributions of hires between the levels. These scenarios were selected in consultation with the study’s sponsor.

Table 1: Hiring scenarios to be modelled.

	Scenario					
	A	B	C	D	E	current
level 2	50%	25%	-	20%	20%	15%
level 3	50%	75%	100%	50%	40%	40%
level 4	-	-	-	30%	30%	28%
level 5	-	-	-	-	10%	8%
level 6	-	-	-	-	-	7%
mean age	32.8	33.9	35.1	37.4	39.3	41.2

The scenario denoted as *current* repeats the distribution observed in Figure 2. Scenario A, with 50% level 2 and 50% level 3 was thought to be the most extreme hiring regime that was feasible (new Ph.D. graduates automatically start at level 3, and were seen by study sponsors as an unavoidable recruitment pool). The other scenarios were selected as plausible regimes that gradually move towards the age profile of the current scenario. Table 1 also shows the mean hiring age that would result from these scenarios, assuming the age distribution of hires at each level remains unchanged from that observed in recent years.

### 3 ATTRITION

Along with the age at which new employees are hired, attrition behaviour is the other important determinant of a workforce's age profile. We have measured attrition rates among Defence Scientists as a function of age. Considering attrition as a function of age has previously been done in other modelling contexts (Doumic et al., 2016) (Foran and Straver, 2018).

In order to measure past attrition, we only had access to annual workforce snapshots broken down by age. Working from complete records of personnel flows (hires, departures, occupation transfers, etc.) would have been preferable, but such data was not available at the time. Working from annual snapshots means that we will only model attrition among employees present at the beginning of the year (thus excluding attrition among in-year hires), and will only model counts of net hires (only those that did not leave during the year when they were hired), instead of modelling all attrition and hires.

Let  $w_k[\alpha, \beta]$  be the number of employees whose ages are in the range  $[\alpha, \beta]$ , at the end of year  $k$ . A year earlier,  $w_{k-1}[\alpha - 1, \beta - 1]$  Defence Scientists had the potential to be among the  $w_k[\alpha, \beta]$  a year later, but some may have left during year  $k$  due to attrition.

By comparing workforce snapshots from successive years, we can count the number of employees who were present at the beginning of a given year, but who departed during the year. Let  $d_k[\alpha, \beta]$  be that count during year  $k$ , among employees whose ages would have been in the range  $[\alpha, \beta]$  at the end of year  $k$ . Note that this does not include the departures of new hires who left during the year when they were hired (those cannot be obtained from annual snapshots).

To obtain an annual attrition rate, we divide the count of departures by the headcount at the beginning of the year. The attrition rate, over year  $k$ , among employees who will reach an age in the range  $[\alpha, \beta]$  during that year is

$$A_k[\alpha, \beta] = \frac{d_k[\alpha, \beta]}{w_{k-1}[\alpha - 1, \beta - 1]} \quad (1)$$

Note that this rate does not fully describe all attrition, as it only applies to employees who are present at the beginning of the year. The new hires over the course of the year may also leave before the year's end, but are not factored into Equation (1). Additional data, beyond the annual workforce snapshots that we could access, would be necessary to obtain a rate that also considers in-year attrition

among new hires. The rate given by Equation (1) underestimates actual attrition, but is for the rate that will be required by our models.

Attrition rates tend to fluctuate from year to year. An attrition rate observed one year may not be representative of the long term trend, and so not ideally suited for modelling in support of long-term Human Resources Planning. We thus prefer multi-year attrition rates, which we compute by compounding the annual rates obtained from the annual workforce snapshots using Equation (1). The attrition rate observed over the multi-year period starting in year  $y_1$  and ending in year  $y_n$  is obtained by successively applying the annual rates as

$$A_{[y_1, y_n]}[\alpha, \beta] = 1 - \prod_{k=y_1}^{y_n} (1 - A_k[\alpha, \beta]) \quad (2)$$

The resulting multi-year rate can then be annualized to obtain the annual attrition rate that is representative of observed trends over the  $n$ -year period. We denote the annualized rate  $A[\alpha, \beta]$ , and obtain it as

$$A[\alpha, \beta] = 1 - (1 - A_{[y_1, y_n]}[\alpha, \beta])^{\frac{1}{n}} \quad (3)$$

An alternative would be to use an average, or weighted average of annual attrition rates, as done by Okazawa (2007). We prefer to use the annualized multi-year rate, as it more closely corresponds to a single rate that would have been in effect over the whole period. However, we have not investigated theoretical or empirical reasons for preferring this rate, over others, in the context of Workforce Modelling.

We estimated attrition rates using data from April 2008 to March 2017, for age ranges spanning five years, starting with ages 25 to 29, up to 64, and also for employees 65 and older. The age ranges were selected to ensure a sufficient number of person-years to derive representative rates. There were 133 person-years in the 25 to 29 range, 241 in the 65 and older range, and substantially more in the other segments. The resulting rates, based on the period from 1 April 2008 to 31 March 2017, are shown in Figure 3.

Attrition is higher among the youngest employees, who tend to have been recently recruited. It is then lower for several years. This pattern of higher attrition in the first years of service is typical in many workforces, as pointed out by Bartholomew et al. (1991). Finally, attrition increases greatly after employees reach the age of 55, corresponding to the

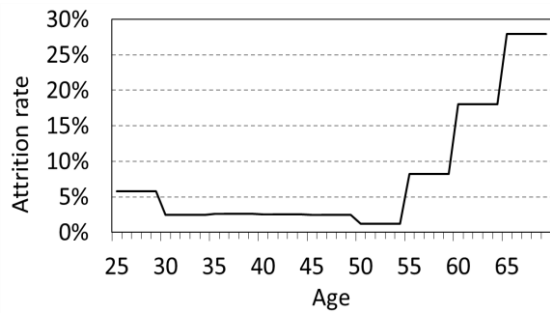


Figure 3: Annual attrition rates.

earliest eligibility for retirement with an immediate annuity, and in the years after, when all become similarly eligible. Many also attain the maximum number of pensionable years (35 for federal Public Servants).

Among Public Service classifications, Defence Scientists have comparatively low attrition. This is likely due to the fact that the specialised expertise of many Defence Scientists (combining advanced scientific expertise, and applications to the defence domain) is not as readily transferable in the wider labour market. In particular, many other Public Service classifications are found across government departments, and so it is common for personnel to progress in their career by moving from one department to the next (which counts as attrition, from an individual department's perspective). Defence Scientists are more likely to stay within the Department of National Defence.

## 4 EQUILIBRIUM MODEL

Now that we have the age distribution of Defence Scientists (shown in Figure 1), the hiring age distribution for selected scenarios (described in Table 1), and the expected attrition rate as a function of age (shown in Figure 3), we can model the workforce's demographic evolution. We start by looking at the eventual equilibrium that would be reached if hiring and attrition were to remain unchanged.

At equilibrium, the number of Defence Scientists remains unchanged from year to year. That is, each departing employee is replaced by the hiring of exactly one replacement. In equation terms,

$$r = \sum_{\alpha} w[\alpha] \cdot A[\alpha] \quad (4)$$

where  $r$  is the number of new employees to be hired each year,  $w[\alpha]$  is the number of employees of age  $\alpha$  at the beginning of the year, and  $A[\alpha]$  is the attrition

rate applicable to employees of age  $\alpha$ . The sum is over all ages present in the workforce.

Then, the hired employees are modelled as following the age distributions associated with the scenarios in Table 1. Let  $r'[\alpha]$  be the proportion of hires whose age will be  $\alpha$  at the end of the year. In each scenario,  $r'[\alpha]$  is the sum over all Defence Scientist Level, of the products of the proportion of the hires at each level (from Table 1), with the proportion of the historical hires at the respective levels whose age was  $\alpha$  (which can be observed in Figure 2).

Each year, employees age by one year, are subject to the attrition rate for their age band, and are joined by new hires according to the distribution given by the  $r'[\alpha]$  values. Thus, at equilibrium, when the workforces age profile is steady from year to year,

$$w[\alpha] = w[\alpha-1] \cdot (1-A[\alpha]) + r'[\alpha] \cdot r \quad (5)$$

Again, Equation (5) does not include in-year attrition among the new hires. The annual number of recruits,  $r$  is net of any in-year attrition, and  $A[\alpha]$  was defined in Section 3 as only applying to employees present at year-start. Also recall that the age,  $\alpha$ , is always the age taken at the end of the year (not at the time of hire or at the time of attrition).

Equation (5) defines a linear constraint on the age distribution for each  $\alpha$  (for this analysis, we have used ages from 25 to 80). In the resulting system of linear equations, the values of  $A[\alpha]$  and  $r'[\alpha]$  are determined from the historical record, and there is an unknown variable  $w[\alpha]$  for each  $\alpha$ . One more linear constraint is required to give the system a unique solution. It is the constraint that the total headcount be fixed at its current value, which we call  $w$  (it was 616, on 30 June 2017).

$$\sum_{\alpha} w[\alpha] = w \quad (6)$$

The system of linear equations defined by Equations (4), (5) and (6) can now be solved. The resulting equilibrium age distribution is shown in Figure 4 for the values of  $r'[\alpha]$  from the *current* scenario.

In figure 4, we see that the equilibrium age distribution follows a similar profile to the June 2017 age distribution, which is reproduced from Figure 1. Notice that the equilibrium age distribution is derived without using the initial state – the close resemblance between the latest distribution available and the equilibrium was thus somewhat surprising. The current age distribution is, thus, close to equilibrium



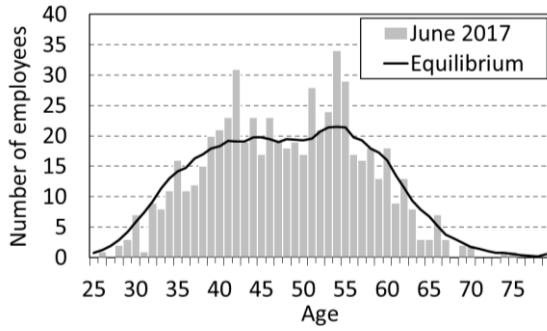


Figure 4: Equilibrium age distributions.

despite successive past periods of boom and bust in hiring.

To illustrate the impact of modifying the age distribution of hires on the equilibrium, Figure 5 includes the age profile at equilibrium that results from hiring as per Scenario A (defined in Table 1). Scenario A corresponds to the youngest age distribution that was deemed feasible, and so we can consider the resulting equilibrium age profile as the youngest that could realistically be achieved. We see that this equilibrium distribution is substantially younger than that obtained for the *current* scenario with a peak in the late 30s as opposed to the mid-50s.

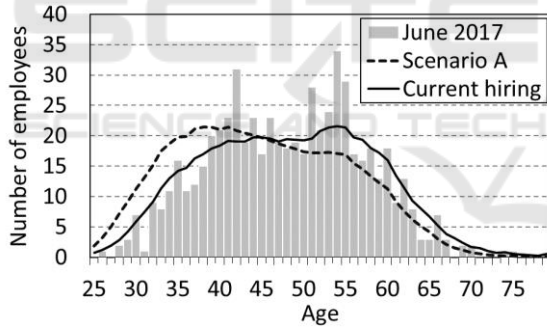


Figure 5: Scenario A equilibrium age distributions.

Table 2 shows how each of the hiring scenarios affects the eventual equilibrium mean age for Defence Scientists. The mean goes from 48.3 for the *current* hiring age profile, to 45.6 under scenario A.

As of 30 June 2017, the time of the latest available workforce snapshot preceding the study, the mean age of Defence Scientists in the Department of National Defence was 48.7. The *current* scenario thus leaves the mean age of Defence Scientists essentially unchanged, while the other scenarios eventually reduce it. Scenario A achieves the greatest reduction in mean age, reducing it by 3.1 years.

Table 2: Equilibrium average age for each hiring scenario.

	Scenario					
	A	B	C	D	E	current
Equilibrium mean age	45.6	46.4	47.1	47.7	48.3	48.8

## 5 DYNAMIC MODEL

The equilibrium age distributions derived above help to anticipate the eventual effects of proposed hiring policies, but do not say what their shorter term impact will be. Given that public service careers often span decades, while hiring policies are unlikely to survive that long, the shorter term effects of a hiring policy should be of interest. To look at these shorter term outcomes, a dynamic model is required.

Our dynamic model simply tracks the workforce composition that results from applying the previously used attrition rates by age, and replacing departing personnel with hires, while distributing the ages of hires according to the distributions from the previous scenarios. This is defined by Equation (7), which is like Equation (5), but with added indices to denote successive years:

$$w_k[\alpha] = w_{k-1}[\alpha-1] \cdot (1-A[\alpha]) + r'[\alpha] \cdot r_{k-1} \quad (7)$$

where,

$$r_{k-1} = \sum_{\alpha} w_{k-1}[\alpha] \cdot A[\alpha] \quad (8)$$

Equation (8) sets annual hiring to exactly make up for the year's attrition. It is identical to Equation (4), but with an index to denote the year. Figure 6 shows how the mean age of Defence Scientist would evolve, over 30 years, under the various hiring scenarios.

Each scenario converges differently toward its eventual equilibrium. For example, Scenario E starts with a slight decrease in the mean age over the first two years, followed by six years of increase, to reach 48.9 years. It then experiences 19 years of decrease, reaching a low of 48.1 years, before eventually converging to 48.3 years, as shown in Table 2. The trajectory of other scenarios reach peaks and troughs at different points in time on the way to convergence. After the 30 years shown in Figure 5, it appears that many scenarios will still fluctuate significantly before reaching equilibrium.

Figure 7 extends the horizon further into the future for scenario A, in order to show that the mean age

eventually does converge.

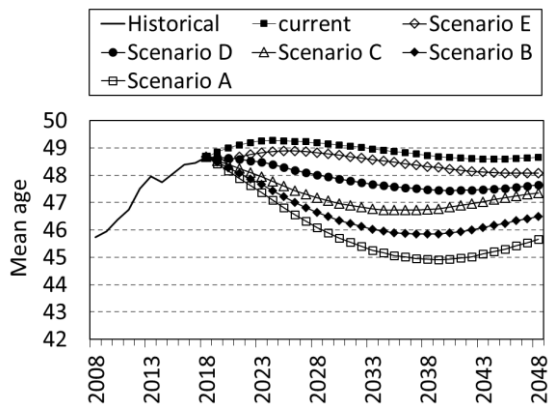


Figure 6: Mean age forecast.

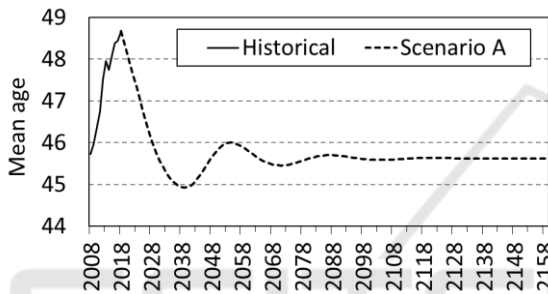


Figure 7: Longer term forecast for scenario A.

Figure 7 highlights the fact that although the dynamic model converges to the value identified by our equilibrium model, that convergence requires decades – longer than typical Human Resources Planning horizons. Therefore, in practice, the dynamic model that looks at fluctuations over coming years is necessary for meaningfully comparing hiring policies.

The oscillation observed on the way to convergence is something commonly observed in Workforce Modelling. In this case, the mean age of the workforce changes with the age distribution among hires, but it also changes with the number of hires (hires are generally younger, so more hiring results in a lowering of the mean age). But lowering of the mean age, itself, tends to reduce attrition in the following years, as attrition is highest among the oldest employees. This lower attrition results in fewer hires, and thus an ageing workforce. Which will itself eventually result in increased attrition.

These successive waves of lower attrition / less hiring / ageing, followed by higher attrition / more hiring / rejuvenation, continue in a feedback cycle that gradually tapers off, and eventually converges.

## 6 WORKFORCE GROWTH

So far, we have studied situations where the headcount was kept unchanged from year to year. However, growth or reduction of the workforce, if they were to occur, would lead to changes in the workforce’s age profile. To briefly investigate this, we consider the case of a modest annual growth rate of 2% in the number of employees.

In order to consider persistent growth or reduction of the headcount, Equation (8) must be replaced by

$$r = \sum_{\alpha} w_{k-1}[\alpha] \cdot A[\alpha] + (1+\varphi) \cdot \sum_{\alpha} w_{k-1}[\alpha] \quad (9)$$

where  $\varphi$  is the rate of change in the headcount. The first term of Equation (9) is as Equation (8), and accounts for the hires that are meant to replace departing employees. The second term accounts for the growth or reduction by adding a multiple of the total headcount. For a negative  $\varphi$ , corresponding to a shrinking workforce, Equation (9) only works if the rate of reduction is lower than attrition. Otherwise, layoffs are necessary.

Using Equation (9) for a 2% annual growth in headcount, along with the *current* scenario for the age distribution of new hires, we eventually get a reduction in the mean age of Defence Scientists of just over one year, as shown in Figure 8.

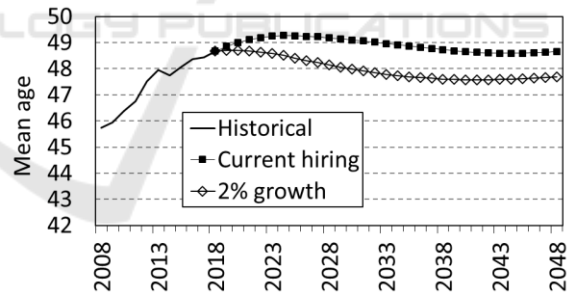


Figure 8: Mean age forecasts with 2% growth.

If incorporating a growth rate of 2% to the previously described equilibrium model, we obtain that the *current* scenario would then reach a mean age at equilibrium of 47.7. If combining scenario A (the one with youngest ages at hire) with the 2% growth rate, the mean age at equilibrium could fall to 45.0 (compared to the 45.6 without growth in Table 2).

However, the reduction in mean age achieved through headcount growth is only sustained as long as the workforce grows. The 2% growth rate used in our example implies a doubling of the headcount approximately every 35 years.

## 7 CONCLUSIONS

This paper presented two approaches to measuring the effect of changes in the age distribution of hires on the age distribution of a workforce. These methods can inform policy aimed at achieving workforce rejuvenation. The equilibrium method leads to an explicit solution for the eventual equilibrium age distribution, but this equilibrium can take a very long time to be reached. The dynamic method then allows us to chart the path taken from the present toward that equilibrium.

These methods can also be adapted to the analysis of other workforce demographic characteristics. For example, they were used by the author to investigate the impact of hiring policies on the proportion of women in Defence Scientific Services, in support of departmental objectives to increase their representation.

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