# Effect of Dispersive Reflectivity on the Stability of Gap Solitons in Dual-core Bragg Gratings with Cubic-quintic Nonlinearity

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Abstract: We consider dynamical stability of quiescent gap solitons in coupled Bragg gratings with dispersive reflectivity in a cubic-quintic nonlinear medium. It is found that there exist two disjoint families of quiescent solitons within the bandgap, namely Type 1 and Type 2 solitons. Also, in each family, there exist symmetric and asymmetric solitons. It is found that dispersive reflectivity has a stabilizing effect on asymmetric solitons. In addition, the symmetric Type 2 solitons are always unstable.

## 1 INTRODUCTION

The periodic variation of the refractive index in a fiber Bragg grating (FBG) results in the formation of a bandgap in the transmission spectrum of FBGs, where linear waves cannot propagate. As a result of the coupling between forward- and backward-propagating waves, FBGs possess a strong effective dispersion (Russell, 1991). At high intensities the induced dispersion and nonlinearity in FBGs may be balanced and gives rise to gap solitons (GSs). GSs may have any velocity in between zero and speed of light in a medium (Christadoulides and Joseph, 1989; Aceves and Wabnitz, 1989; Barashenkov et al., 1998).

GSs in Kerr media have been the subject of intensive theoretical and experimental research due to their potential applications in optical signal processing such as optical delay lines, buffers and logic gates (Sipe, 1992; Sipe and Winful, 1988; de Sterke and Sipe, 1994; Sterke et al., 1997; Eggleton et al., 1997; Krauss, 2008). They have also been investigated theoretically in other nonlinear systems such as quadratic nonlinearity (Mak et al., 1998b), cubic-quintic nonlinearity (Atai and Malomed, 2001), coupled FBGs (Mak et al., 1998a; Baratali and Atai, 2012), nonuniform gratings (Atai and Malomed, 2005), waveguide arrays (Mandelik et al., 2004) and photonic crystals (Xie and Zhang, 2003; Neill and Atai, 2007; Monat et al., 2010).

It has previously been shown that propagation of light in dual-core and birefringent fibers with Kerr nonlinearity exhibits very rich dynamics (Atai and Chen, 1992; Atai and Chen, 1993; Nistazakis et al., 2002; Bertolotti et al., 1995; Chen and Atai, 1998; Chen and Atai, 1995). Therefore, in this paper, we investigate the existence and stability of GSs in a dual-core system with identical cores where each core contains a Bragg grating with dispersive reflectivity written in a cubic-quintic medium.

## 2 THE MODEL

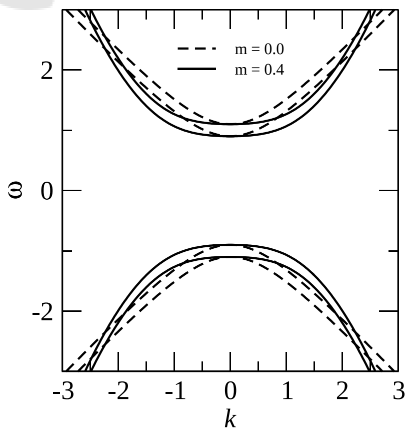


Figure 1: Examples of dispersion diagrams for $\lambda = 0.1$.

Propagation of light in two linearly coupled Bragg gratings (FBGs) with dispersive reflectivity in a



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cubic-quintic nonlinear medium is described by following equations:

$$\begin{aligned}
&iu_{1t}+iu_{1x}+\left(\frac{1}{2}|u_1|^2+|v_1|^2\right)u_1-\\
&\eta\left(\frac{1}{4}|u_1|^4+\frac{3}{2}|u_1|^2|v_1|^2+\frac{3}{4}|v_1|^4\right)u_1+\\
&\qquad v_1+\lambda u_2+mv_{1xx}=0,\\
&iv_{1t}-iv_{1x}+\left(\frac{1}{2}|v_1|^2+|u_1|^2\right)v_1-\\
&\eta\left(\frac{1}{4}|v_1|^4+\frac{3}{2}|v_1|^2|u_1|^2+\frac{3}{4}|u_1|^4\right)v_1+\\
&\qquad u_1+\lambda v_2+mu_{1xx}=0,\\
&iu_{2t}+iu_{2x}+\left(\frac{1}{2}|u_2|^2+|v_2|^2\right)u_2-\\
&\eta\left(\frac{1}{4}|u_2|^4+\frac{3}{2}|u_2|^2|v_2|^2+\frac{3}{4}|v_2|^4\right)u_2+\\
&\qquad v_2+\lambda u_1+mv_{2xx}=0,\\
&iv_{2t}-iv_{2x}+\left(\frac{1}{2}|v_2|^2+|u_2|^2\right)v_2-\\
&\eta\left(\frac{1}{4}|v_2|^4+\frac{3}{2}|v_2|^2|u_2|^2+\frac{3}{4}|u_2|^4\right)v_2+\\
&\qquad u_2+\lambda v_1+mu_{2xx}=0.
\end{aligned}\tag{1}$$

In Eqs. (1), $u_{1,2}(x,t)$ and $v_{1,2}(x,t)$ are the amplitudes of the forward- and backward-propagating waves in cores 1 and 2, respectively. $\eta > 0$ is a real parameter and denotes the strength of the quintic nonlinearity. $\lambda$ is coefficient of linear coupling between two cores and is real and positive. $m > 0$ is the strength of the dispersive reflectivity. It should be noted that equation (1) is normalized so that the group velocity coefficient 1. It should be noted that in the absence of dispersive reflectivity (i.e. $m = 0$), Eqs. (1) reduce to the model of Ref. (Islam and Atai, 2015).

To obtain the spectrum bandgap within which the GSs may exist, plane wave solutions $u_{1,2}, v_{1,2} \sim \exp(ikx - iwt)$ are substituted into linearized version of Eqs. (1) and after some straightforward algebraic manipulations the following dispersion relation is obtained:

$$\omega^2=\left(1-mk^2\right)^2+\lambda^2+k^2\pm 2\lambda\sqrt{\left(1-mk^2\right)^2+k^2}\tag{2}$$

Eq. (2) leads to the bandgap $\omega^2 < \omega_0^2 = (1-|\lambda|)^2$ for $m \leq 0.5$ and $\omega^2 < \omega_0^2 = \left(\frac{\sqrt{4m-1}}{2m}-|\lambda|\right)^2$ for $m > 0.5$. Examples of dispersion diagrams for various values of $m$ are shown in Fig. 1. It should be noted that since $m > 0.5$ may not be physically achievable, we will limit our analysis to $m \leq 0.5$.

# 3 SOLITON SOLUTIONS

To obtain the quiescent soliton solutions, we substitute $u(x,t) = U(x)e^{-i\omega t}$ and $v(x,t) = V(x)e^{-i\omega t}$ into Eqs. (1) which results in the following systems of coupled equations:

$$\begin{aligned}
-mU_{1xx}&=\omega V_1-iV_{1x}+\left(\frac{1}{2}|V_1|^2+|U_1|^2\right)V_1-\\
&\eta\left(\frac{1}{4}|V_1|^4+\frac{3}{2}|V_1|^2|U_1|^2+\frac{3}{4}|U_1|^4\right)V_1+\\
&U_1+\lambda V_2,\\
-mV_{1xx}&=\omega U_1+iU_{1x}+\left(\frac{1}{2}|U_1|^2+|V_1|^2\right)U_1-\\
&\eta\left(\frac{1}{4}|U_1|^4+\frac{3}{2}|U_1|^2|V_1|^2+\frac{3}{4}|V_1|^4\right)U_1+\\
&V_1+\lambda U_2,\\
-mU_{2xx}&=\omega V_2-iV_{2x}+\left(\frac{1}{2}|V_2|^2+|U_2|^2\right)V_2-\\
&\eta\left(\frac{1}{4}|V_2|^4+\frac{3}{2}|V_2|^2|U_2|^2+\frac{3}{4}|U_2|^4\right)V_2+\\
&U_2+\lambda V_1,\\
-mV_{2xx}&=\omega U_2+iU_{2x}+\left(\frac{1}{2}|U_2|^2+|V_2|^2\right)U_2-\\
&\eta\left(\frac{1}{4}|U_2|^4+\frac{3}{2}|U_2|^2|V_2|^2+\frac{3}{4}|V_2|^4\right)U_2+\\
&V_2+\lambda U_1.
\end{aligned}\tag{3}$$

There is no analytical solution for Eqs. (3). These equations can be solved numerically using the relaxation algorithm. We found that similar to the case of a single core Bragg grating with cubic-quintic nonlinearity (i.e. model of Ref. (Atai and Malomed, 2001)), there exist two distinct and disjoint families of solitons in the model of Eqs. (1) that are separated by a border. We refer to these soliton families as Type 1 and Type 2. In each of these families, there exist symmetric $(u_1 = u_2, v_1 = v_2)$ and asymmetric $(u_1 \neq u_2, v_1 \neq v_2)$ solitons. Examples of symmetric Type 1 and Type 2 soliton profiles are shown in Fig. 2. Soliton families differ in terms of their amplitude, phase, and parities. More specifically, as is shown in Fig. 3, $Re(u(x))$ and $Re(v(x))$ of Type 1 solitons are even and $Im(u(x))$ and $Im(v(x))$ are odd functions of $x$. The opposite occurs in the case of Type 2 solitons.

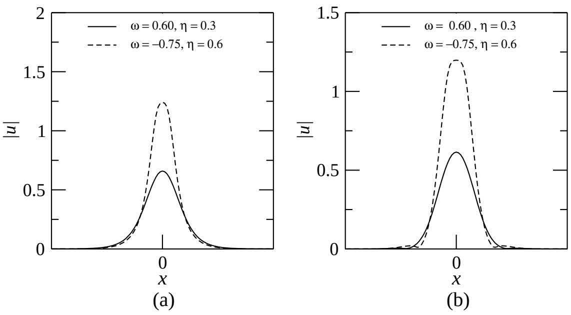


Figure 2: Symmetric Type 1 (solid line) and Type 2 (dashed line) stationary solitons profiles for (a) $\lambda = 0.1$, $m = 0.2$ (b) $\lambda = 0.1$, $m = 0.4$.

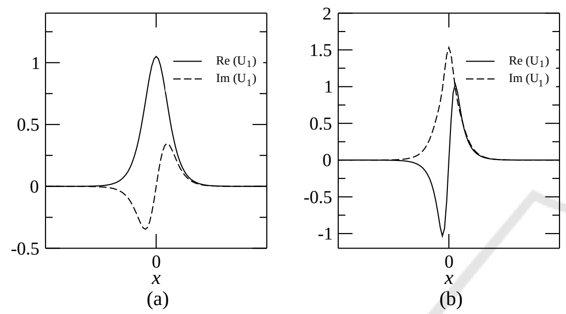


Figure 3: Real and Imaginary part of asymmetric gap solitons for $\lambda = 0.1$ and $m = 0.2$. (a) Type 1 at $\omega = 0.3$, $\eta = 0.2$,(b) Type 2 at $\omega = -0.2$, $\eta = 0.4$.

# 4 STABILITY OF QUIESCENT SOLITONS

To analyze the stability of the quiescent gap solitons, we have employed symmetrized split-step Fourier method to solve Eqs. (1) numerically. The numerical stability analysis shows that stable and unstable Type 1 and Type 2 solitons exist in the system. Examples of the evolution of asymmetric Type 1 and Type 2 solitons are shown in Fig. 4 and Fig. 5 respectively. Examples of propagation of symmetric Type 1 and Type 2 solitons are shown in Fig. 6. As is shown in Figs. 4 to 6 unstable solitons are either completly destroyed or sheds some energy in the form of radiation and evolves to another quiescent soliton in the system. Another interesting finding is that symmetric Type 2 solitons are always unstable.

The stability diagram for asymmetric and symmetric quiescent solitons corresponding to $m = 0.2$ at $\lambda = 0.1$ is displayed in Fig. 7. The dashed curve in Fig. 7 separates the Type 1 and Type 2 families of asymmetric solitons. As is shown in Fig. 7, there exist vast regions of stable solitons within the bandgap. Compared with the case of $m = 0$ (i.e. the model of Ref. (Islam and Atai, 2015)), it is found

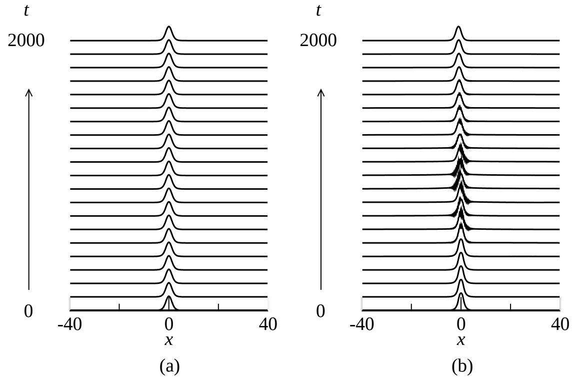


Figure 4: Examples of propagation of asymmetric Type 1 solitons for $\lambda = 0.1$, $m = 0.2$. (a) Stable asymmetric soliton for $\omega = 0.3$, $\eta = 0.20$ and (b) unstable asymmetric soliton for $\omega = -0.55$, $\eta = 0.01$. In this figure and following figures only $u_1$ component is shown.

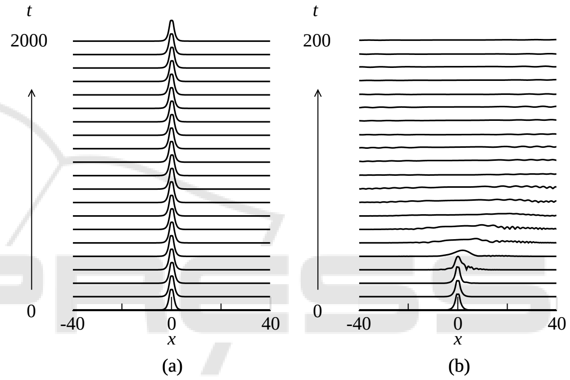


Figure 5: Examples of propagation of asymmetric Type 2 solitons for $\lambda = 0.1$, $m = 0.2$. (a) Stable asymmetric Type 2 for $\omega = -0.20$, $\eta = 0.40$, (b) unstable asymmetric Type 2 soliton for $\omega = 0.50$, $\eta = 1.0$.

that the presence of dispersive reflectivity leads to the expansion of stable regions. This finding is consistent with that for the single core Bragg grating with cubic-quintic nonlinearity and dispersive reflectivity (i.e. Ref. (Dasanayaka and Atai, 2010)).

# 5 CONCLUSIONS

We have numerically investigated the effect of dispersive reflectivity on the stability of quiescent gap solitons in coupled Bragg gratings with cubic-quintic nonlinearity. There exists a genuine bandgap within the linear spectrum of the system. Using the numerical techniques, it is found that stationary quiescent gap solitons exist throughout the bandgap. Furthermore, the model supports two disjoint families of quiescent solitons, namely Type 1 and Type 2 solitons. Additionally, both symmetric and asymmetric soli-

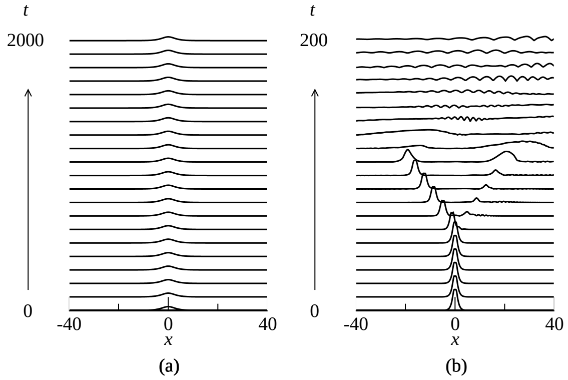


Figure 6: Example of evolution of symmetric solitons for $\lambda = 0.1$, $m = 0.2$ (a) stable Type 1 symmetric soliton for $\omega = 0.84$, $\eta = 0.01$ and (b) unstable Type 2 symmetric soliton for $\omega = -0.30$, $\eta = 0.39$.

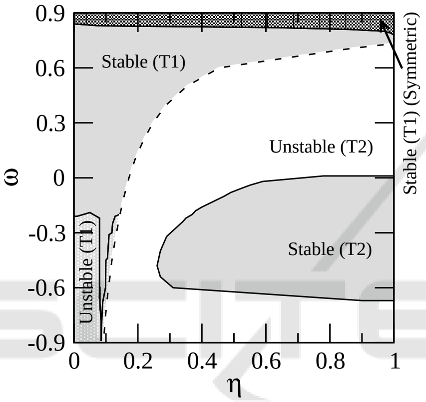


Figure 7: Stability diagram of asymmetric quiescent solitons for $m = 0.2$, $\lambda = 0.1$. The dashed curve is the border between Type 1 (T1) and Type 2 (T2) soliton families.

tons exist in each family.

We have conducted a numerical stability analysis for symmetric and asymmetric Type 1 and Type 2 solitons. It is found that symmetric Type 2 solitons are always unstable. For asymmetric solitons, the results of the stability analysis show that the presence of dispersive reflectivity has a stabilizing effect.

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