Uncertain Formal Concept Analysis for the Analyze of a Course Satisfaction Questionnaire

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Abstract: The Formal Concept Analysis (FCA) is a method of data analysis often used in data mining. This method proposes to build a collection of formal concepts from a set of objects and their properties. These formal concepts can be ordered to provide a concept lattice. Several researches have demonstrated a link between the possibility theory and the formal concept analysis. Thus, it is possible to take into account the uncertainties of the properties by using the possibility theory before propagating them during the computation of the formal concepts. We propose in this paper an experimentation of the uncertain formal concept analysis for the extraction of knowledge in a satisfaction questionnaire for a course of professionalization in bachelor. Some questions can be open questions where the answers provided by students are given freely. For this purpose, we perform a text mining processing in order to categorize and classify the answers. During this processing, uncertainties can appear. In this research, we will handle these uncertainties by using the uncertain formal concept analysis. Then, we will extract the uncertain formal concepts from the concept lattice by using queries and represent the reduced lattice concepts with a visualization tool.

1 INTRODUCTION

The realization of a satisfaction questionnaire at the end of a course at the university allows us to know better the students' expectations. It gives us also a quality evaluation of our pedagogy, our course resources and so on. Many universities have already proposed this kind of evaluation by using for example the learning management system such as Moodle. It contributes to the continuous improvement of the courses.

It is easy to build a quiz but the processing of answers is often complex. Closed questions can be defined as questions where the answer is in a set of possible answers, for example *yes* or *no*. This kind of question sometimes presents difficulties for analysis. The second category of questions is called open questions. These are questions where answers are provided freely and the student needs to give his opinion or express his feelings. These free answers increase even more the difficulty of processing. The processing of answers often uses data mining or natural language processing to analyse the syntax, semantics or to extract information by text mining. For open questions, the goal of processing is to categorize and classify answers in order to provide the synthesis of the answers which can help in decision making.

For our experimentation we would like to extract the answers given by the majority of the students. As presented in (Belohlavek et al., 2007), the Formal Concept Analysis (FCA) can be used to do this. A formal concept is described in (Bedek et al., 2015) as a set of objects which have the same properties. Several researches have been published for educational application (Bedek et al., 2015; Fernandez-Manjon and Fernandez-Valmayor, 1998; Kickmeier-Rust et al., 2016) or social network (Snášel et al., 2008). We can also find several studies which propose to use fuzzy logic for imprecise properties as in (Belohlavek, 2004; Zou and Deng, 2017; Cross et al., 2010). Nevertheless, we rarely found applications of the formal concept analysis taking into account uncertainty. It is however possible. Authors in (Dubois and Prade, 2009; Dubois and Prade, 2012; Dubois and Prade, 2015; Dubois et al., 2007; Navarro et al., 2012; Miclet et al., 2011) propose a generalization of the formal concept analysis by using the possibility theory.

We propose in this paper an experimentation of the uncertain formal concept analysis. Our application consists in analysing a satisfaction questionnaire for a course of PPP (Professional Personal Project) in bachelor.

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Our questionnaire consists of closed questions and open questions. For our open questions we must perform a processing to classify the answers of the students. Unfortunately, this processing generates uncertainties because an answer can belong, though in different measure, to several classes at the same time. The uncertainties must be propagated in the formal concept analysis to avoid losing information.

Our study will first propose the description of the formal concept analysis and show how it can be generalized by the possibility theory. Then we will present our application and the processing of the answers to open questions. Finally, we will show several results which illustrate our approach.

2 THE POSSIBILITY THEORY

The possibility theory was developed in 1978 by L.A. Zadeh after the fuzzy set theory in the paper (Zadeh, 1978). This theory proposes to take into account imprecision and uncertainty often attached to knowledge. Knowledge can be modeled by a possibility distribution. For example, if Ω is the universe and π_x a possibility distribution of the variable x defined from Ω in $[0,1]$, then if $\pi_x(u) = 0$ then $x = u$ is impossible. If $\pi_x(u) = 1$ then $x = u$ is possible. In the possibility theory there are two fundamental measures, the possibility measure noted Π and the necessity measure noted N (Dubois and Prade, 1988) defined from the set of parts of Ω, $P(\Omega)$ in $[0,1]$. These measures can be defined as below:

$$\forall A \in P(\Omega), \Pi(A) = \sup_{x \in A} \pi(x). \tag{1}$$

$$\forall A \in P(\Omega), N(A) = 1 - \Pi(\neg A) = \inf_{x \notin A} 1 - \pi(x). \tag{2}$$

The possibility theory is not additive but maxitive. This very important characteristic explains why the properties of the probability theory cannot be applied to the possibility theory. The possibility theory is a non-additive theory of the uncertain:

$$\forall A, B \in P(\Omega), \Pi(A \cup B) = \max(\Pi(A), \Pi(B)). \tag{3}$$

We can present two other operators of the possibility theory. The first one is the measure of guaranteed possibility:

$$\forall A \in P(\Omega), \Delta(A) = \inf_{x \in A} \pi(x). \tag{4}$$

And the second one is the measure of potential necessity:

$$\forall A \in P(\Omega), \nabla(A) = 1 - \inf_{x \notin A} \pi(x). \tag{5}$$

As we will see in the next section, all these operators can be used to generalize the formal concept analysis.

3 FORMAL CONCEPT ANALYSIS

The formal concept analysis is a data analysis approach introduced by R. Wille (Wille, 1982). It consists in analysing concepts by using a concept lattice. The concept can be viewed as a basic component of human thinking.

There are two notions to define the concept present in philosophy: the intent and the extent. The intent is the definition of the concept and the extent denotes the elements to which it applies. Mathematically, a formal context is a triple $(O, P, \Re)$ where $O = \{o_1, ..., o_n\}$ is the set of objects, $P = \{p_1, ..., p_m\}$ is the set of properties, and $\Re$ is a relation such as $\Re \subseteq O \times P$.

In fact if $(o, p) \in \Re$, then the object o has the property p. It can also be noted $o\Re p$. Generally, we represent this in a table where the lines are the objects and the columns are the properties. The relation is represented by a 0 if $(o, p) \notin \Re$ or by a 1 if $(o, p) \in \Re$. These values correspond to the values of the table.

We can define a valuation $\vartheta(o, p)$ which returns the value of the table for object o and property p. A formal concept of $(O, P, \Re)$ is a pair (X, Y) such that $X \in O$ and $Y \in P$ where Y is the set of properties shared by all objects of X. We can use as in (Belohlavek et al., 2007) the notation $X^{\uparrow} = Y$ or $Y^{\downarrow} = X$. For example, in the formal context below the sets $(\{o_2, o_3, o_5\}, \{p_2, p_3\})$ and $(\{o_4, o_5\}, \{p_1, p_2\})$ are two formal concepts.

Table 1: Example of a formal context.

Object	p_1	p_2	p_3
o_1	0	1	0
o_2	0	1	1
o_3	0	1	1
o_4	1	1	0
o_5	1	1	1

The set of all formal concepts of $(O, P, \Re)$ is noted $\beta(U, V, \Re) = \{(X, Y) | X^{\uparrow} = Y, Y^{\downarrow} = X\}$. If $\leq$ is a partial order such that for $(X_1, Y_1), (X_2, Y_2) \in \beta(X, Y, \Re)$, then $(X_1, Y_1) \leq (X_2, Y_2)$ si $X_1 \subseteq X_2$ ou $Y_2 \subseteq Y_1$. Then we can build a concept lattice based on the partial order $\leq$. A concept lattice can be visualized by using a Hasse diagram. For example, we can show the concept lattice of the previous example performed by using the tool ConExp (*http://sourceforge.net/projects/conexp*):

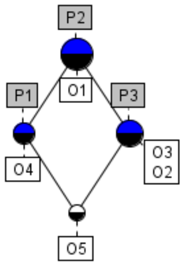

Figure 1: Concept lattice of the example.

We can see that the notation is reduced because we use the legacy of properties and objects. When the properties are many-valued, it is necessary to perform a transformation of the context into a binary formal context. We can take as an example the following many-valued context:

Table 2: Example of a many-valued context.

Object	Score	Quality
o_1	5	low
o_2	15	high
o_3	15	medium
o_4	12	low
o_5	18	medium

We can see that the score is numerical with a range in $[0,20]$, so we must propose a categorization of the values by defining for example three classes. The first one is low for the values in $[0,7]$, the second is medium for the values in $[8,14]$, and the last class is high for the values in $[15,20]$. It can be transformed into the following binary formal context:

Table 3: The transformation of the many-valued context into a binary formal context.

Object	S_{low}	S_{medium}	S_{high}	Q_{low}	Q_{medium}	Q_{high}
o_1	1	0	0	1	0	0
o_2	0	0	1	0	0	1
o_3	0	0	1	0	1	0
o_4	0	1	0	1	0	0
o_5	0	0	1	0	1	0

So far, the properties were perfectly known but if the properties are uncertain, it is interesting to propagate the uncertainty to the lattice concept. We propose to use the possibility theory (Zadeh, 1978) to take into account uncertainty as proposed by the authors in (Dubois et al., 2007). Thus, we obtain the possibility distribution $\pi_{o_p}(u)$ defined for $u \in \Omega$, which is the possibility that the property p of the object o is u. This possibility distribution must be normalized. The authors propose also to extend the formal concept analysis by defining four operators inspired from the possibility theory. If $\Re$ is a relation, then we can define $R(o) = \{p \in P|(o,p) \in \Re\}$ and $R^t(p) = \{o \in O|(o,p) \in \Re\}$. If S is a subset of O, we obtain the following operators:

- $(S)^{\Pi} = \{p \in P|R^t(p) \cap S \neq \oslash\}$
- $(S)^{N} = \{p \in P|R^t(p) \subseteq S\}$
- $(S)^{\Delta} = \{p \in P|R^t(p) \supseteq S\}$
- $(S)^{\nabla} = \{p \in P|R^t(p) \cup S \neq O\}$

As an example, we propose to show the results of these operators for Table 1 and for different sets of objects:

Table 4: Example of the application of the four operators.

S	$(S)^{\Pi}$	$(S)^{N}$	$(S)^{\Delta}$	$(S)^{\nabla}$
(o_1,o_2,o_3)	(p_2,p_3)	$(\oslash)$	(p_2)	$(\oslash)$
(o_2)	(p_2,p_3)	$(\oslash)$	(p_2,p_3)	(p_1,p_3)
(o_4,o_5)	(p_1,p_2,p_3)	(p_1)	(p_1,p_2)	(p_1,p_3)
(o_1,o_2,o_3,o_4)	(p_1,p_2,p_3)	$(\oslash)$	(p_2)	$(\oslash)$

$(S)^{\Pi}$ is the set of properties possessed by at least one object of S. $(S)^{N}$ is the set of properties possessed only by the objects of S. In other words, if the property is in $(S)^{N}$, it is not possible that another object not present in S should have this property. $(S)^{\Delta}$ is a set of properties shared by all objects of S. $(S)^{\nabla}$ is a set of properties that are not satisfied by at least one object in $\bar{S}$. The behaviour of the operator $(.)^{\Delta}$ corresponds to the formal concept analysis. In our research, we will use this operator. The authors in (Dubois et al., 2007) remark that the four operators have already been proposed in scientific works without reference to the possibility theory. These operators can be presented in the cube of oppositions for formal concept analysis (*AffIrmo nEgO*):

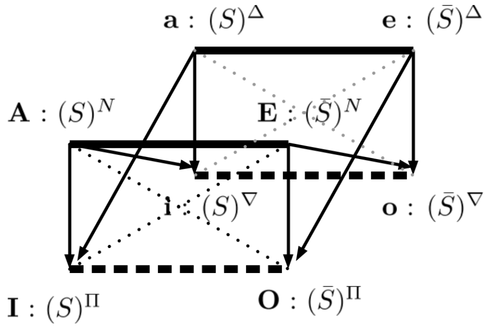

Figure 2: Cube of oppositions.

These operators are defined for a set of objects S but we can easily propose equivalent operators for the sets of properties noted $(.)^{-1\Pi}$, $(.)^{-1N}$, $(.)^{-1\Delta}$, $(.)^{-1\nabla}$ as in (Dubois et al., 2007). It is possible to use fuzzy sets to model imprecise properties as in (Belohlavek,

2004) but there is less research taking into account uncertainty and ignorance which can be partial or full. Certainty is associated with the necessity measure in the possibility theory. This measure can be used in the table of the formal context. As a result, the formal context can be viewed as a weighted family of formal contexts obtained by the threshold of uncertain values of the table. The authors in (Dubois and Prade, 2015) propose to use a pair of necessity measures $(\alpha(o,p),\beta(o,p))$ with $\alpha(o,p) = N((o,p) \in \Re)$ and $\beta(o,p) = N((o,p) \notin \Re)$ which represents the certainty that the object has the property (resp. does not have the property). Moreover, we must satisfy the property $min(N((o,p) \in \Re), N((o,p) \notin \Re)) = 0$ of the possibility theory. The pairs $(1,0)$ and $(0,1)$ represent the fact that the object has the property or not. On the other hand, if $1 > max(\alpha(o,p),\beta(o,p)) > 0$, the ignorance is partial, and if we have $(0,0)$, the ignorance is total. We can define the uncertain formal context in the following formula:

$$\Re' = \{(\alpha(o,p),\beta(o,p)) | o \in O, p \in P\} \tag{6}$$

For this first experimentation, we propose to consider only the simple case where the uncertain values $(\alpha(o,p),0)$ and $(0,\beta(o,p))$ are changed into sure values. So $(\alpha(o,p),0)$ is changed in $(1,0)$ and $(0,\beta(o,p))$ is changed in $(0,1)$. For example, $(0.5,0)$ can be transformed into (1,0) in the table and $(0,0.7)$ can be transformed into (0,1). As a result, we obtain a formal context noted $\Re'_+$ and we can now extract all formal concepts. We can provide below an example of an uncertain context:

Table 5: Example of an uncertain context.

Object	p_1	p_2	p_3
o_1	(0,1)	(1,0)	(0.2,0)
o_2	(0,0.5)	(1,0)	(1,0)
o_3	(0.5,0)	(1,0)	(1,0)
o_4	(1,0)	(1,0)	(0.8,0)
o_5	(1,0)	(1,0)	(1,0)

In this example, we can see that $(\{o_1,o_2\},\{p_2,p_3\})$ and $(\{o_3,o_4,o_5\},\{p_1,p_2,p_3\})$ are formal concepts. We have come up with the following formula in order to compute the certainty degree of a formal concept $C = (X,Y)$:

$$N(C) = \min_{o \in X, p \in Y} N((o,p) \in \Re) \tag{7}$$

In the previous example, the certainty of $(\{o_1,o_2\},\{p_2,p_3\})$ is 0.2 and the certainty of $(\{o_3,o_4,o_5\},\{p_1,p_2,p_3\})$ is 0.5. This indicates that there is one or more properties which are uncertain in these formal concepts. The interest of this approach is to take into account uncertain properties and to provide a concept lattice where the formal concept can be weighted by the degree of certainty. If we perform a purely binary reasoning, in which an object has the property or not, we can fail to discover the core knowledge. Among the existing algorithms described in (Kuznetsov and Obiedkov, 2003) for computing formal concepts, we have chosen Ganter Algorithm *Next Closure* (Ganter, 1987) for our experimentation. This algorithm finds all intents or extents. We have adapted this algorithm to compute uncertain formal concepts. To do this, we propose to modify the previous function R for a certainty threshold s: $R_s(o) = \{p \in P | N((o,p) \in \Re) > s\}$ and $R^t_s(p) = \{o \in O | N((o,p) \in \Re) > s\}$, and to define two new operators based on these functions $(.)^{\Delta_s}$ and $(.)^{-1\Delta_s}$:

$$(S)^{\Delta_s} = \{p \in P | R^t_s(p) \supseteq S\} \tag{8}$$

$$(S)^{-1\Delta_s} = \{o \in O | R_s(o) \supseteq S\} \tag{9}$$

We also have to define the closure operator $\oplus_s$:

$$X \oplus_s i = ((X \cap \{p_1,...,p_{i-1}\}) \cup \{p_i\})^{-1\Delta_s\Delta_s} \tag{10}$$

And finally we have to present the comparison operator $<_i$ (lexicographic order). If $X \in P$ and $Y \in P$ then $X <_i Y$ if:

$$\begin{cases} p_i \in Y - X \text{ and} \\ X \cap \{p_1,...,p_{i-1}\} = Y \cap \{p_1,...,p_{i-1}\} \end{cases} \tag{11}$$

Moreover, we have $X < Y$ if an i exists such that $X <_i Y$ is verified. If $s = 0$ then we perform the computation of the formal concepts for the formal context $\Re'_+$. The algorithm for the computation of uncertain formal concepts is the following:

Algorithm 1: Uncertain NextClosure.

```
Input  : R is an uncertain context
Output: The set of intents noted I
begin
    V = ∅^{-1Δ0Δ0}
    Save(V)
    while V ≠ P do
        for i ← |P| to 1 do
            V+ = V ⊕0 i
            if V <i V+ Exit of the loop.
        Save(V+)
        V ⟵ V+
```

If we use a comparison operator between all concepts then we build the lattice concept.

4 NATURAL LANGUAGE PROCESSING

In the satisfaction questionnaire of our experimentation we have an open question which requires a free answer. However, it is possible to provide a set of classes for the possible answers. So, if we have a set of samples for each class, then it is possible to do a supervised classification of the students' answers. To do this, we must perform the processing of the samples to extract the key words and eliminate useless characters and words. The training set can be the provided samples of the classes. Then, we have to perform the supervised learning before the classification of all answers. A neural network can be used for this classification. The processing of the samples of the classes and of the students' answers consists at first in constructing the corpus. Then we change the case, we eliminate undesired characters, punctuation, numbers and useless words. We resume below the processing:

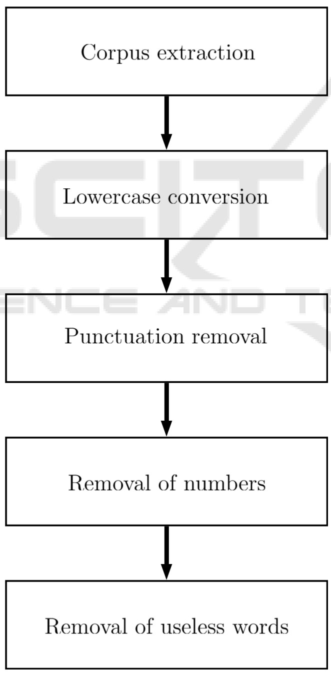

Figure 3: Processing of the corpus.

After this processing, we perform the computation of the Document Term Matrix for the samples of the classes and the students' answers. The classification will use for the training set the DTM of the samples of the classes. Then the students' answers are classified. A significant problem in natural language processing is the problem of a spelling mistake. Our idea is to use a measure of resemblance between the words during the computation of the DTM and during the classification. We propose to use a fuzzy measure in $[0,1]$. Several string metrics exist to measure the resemblance of two strings (Christen, 2006; Jaro, 1989). The most famous are the distance of Levenshtein, Jaccard, Damerau-Levenshtein, hamming, the longest common subsequence, Smith-Waterman and Jaro-Winkler (Winkler, 1999).

All these metrics didn't return measures in $[0,1]$ except the distance of Jaro-Winkler. This measure has also been positively evaluated by the author of (Christen, 2006). So we will use this measure for our study. If we consider the following example which concerns several spellings of the word *intelligence* and if we calculate for each word the Jaro-Winkler distance, we obtain:

Table 6: Comparison of the Jaro-Winkler distance for the word *intelligence*.

Word	Jaro-Winkler distance
Intelligence	1.0
Inteligence	0.95
Inelligence	0.89
Intelijence	0.92
Hazard	0

We can notice that if the words are very close to the word *intelligence*, then the Jaro-Winkler distance returns a value close to 1. If the measure is not close enough to 1, then we can consider that the words are different. So if $d_{JW}(word\ 1, word\ 2) < \eta$, then the word 1 is different from the word 2.

In the previous example, if $\eta = 0.8$ then the word *hazard* is different from *intelligence*. The next step is the construction of the DTM. For example for the students' answers the result is the following:

Table 7: Example of DTM matrix for the students' answers.

student	intelligences	gardner	quiz	proust	cv	...
student 1	1.0	0.96	0.0	0.0	0.0	...
student 2	0.0	0.0	0.0	0.82	0.0	...
student 3	1.0	1.0	0.0	0.0	0.0	...
...	...	...	...	...	...	...
student N	0.0	0.0	0.0	0.0	0.0	...

When the DTM are computed for the students' answers and the samples of the classes, we can perform the classification of the students' answers by a neural network. The learning of the coefficient of the neural network is performed by using a backpropagation of the gradient. The algorithm is presented below:

Algorithm 2: Backpropagation of the gradient.

Input : E: Sets of samples, N: Number of neurons, NM: Number of values in input, NC: Number of classes, μ: a constant, E_{max}: maximum error, I_{max}: maximum number of iterations.

Output: ω_1, ω_2

begin
- $\sum_{Error} \longleftarrow +\infty$
- $count \longleftarrow 0$
- **while** $\sum_{Error} > E_{max} \wedge count < I_{max}$ **do**
  - $\sum_{Error} \longleftarrow 0$
  - $count \longleftarrow count + 1$
  - **forall the** $x \in E$ **do**
    - **for** $i \leftarrow 0$ **to** NC **do**
      - $RC1 \longleftarrow NeuronsLayer(\omega_1, x_i, N, NM)$
      - $RF \longleftarrow NeuronsLayer(\omega_2, RC1, NC, N)$
      - **for** $j \leftarrow 0$ **to** NC **do**
        - $\eta \longleftarrow ErrorComputation(RF, i)$
        - $e_j = RF_j \times (1 - RF_j) \times \eta$
        - **for** $k \leftarrow 0$ **to** N **do**
          - $\omega_2^{j,k} \longleftarrow \omega_2^{j,k} + \mu \times e_j \times RC1_k$
      - $\sum_{Error} = \sum_{Error} + |\eta|$
      - **for** $j \leftarrow 0$ **to** N **do**
        - **for** $k \leftarrow 0$ **to** NM **do**
          - $s \longleftarrow 0$
          - **for** $l \leftarrow 0$ **to** NC **do**
            - $s \longleftarrow s + \omega_2^{l,j} \times e_l$
          - $\omega_1^{j,k} \longleftarrow \omega_1^{j,k} + \mu \times RC1_j(1 - RC1_j) \times s \times x_{i,k}$

The result of the classification can be interpreted as a possibility measure after normalization for each class. We can also compute a necessity measure which is in fact certainty. This necessity measure can be taken into account in the formal concept analysis.

5 EXPERIMENTATION

The experimentation concerns the analysis of a satisfaction questionnaire for a course of bachelor. 144 students have answered the questionnaire which consisted of closed questions and open questions. To simplify the processing, we take into account only one open question, the one which is the most important for us. There are 32 other closed questions where the answers are many-valued. We have transformed these many-valued data accordingly to the proposed solution of part 3 (as in the example of table 3) and performed a processing of the open question by using the method presented in the previous section. The result is an uncertain formal context where the columns are the possible answers (the properties of the FCA) and the lines are the students (the object of the FCA). Concerning the open question, we have as many columns as possible classes. The values of the table are the necessity measures. When an answer is certain, the value is 1. The uncertain values are between 0 and 1. Finally, we have a table of 158 columns and 144 lines. To simplify the computation of the lattice, we can apply a filter on the formal concepts by using the criteria such as the number of students, the certainty of the formal concept, the score, the properties, and the presence of one student or a group of students in the formal concepts. We present below several examples of results in order to illustrate some of these criteria. The first one is an example of filtering depending on the number of students:

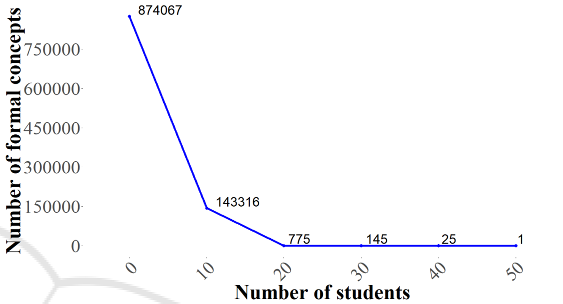

Figure 4: Filtering of the formal concepts according to the number of students.

We can see as expected that more there are students, fewer there are formal concepts. The second criterion is to filter the formal concept by making vary the threshold of certainty between 0 and 1. The obtained result is as follows:

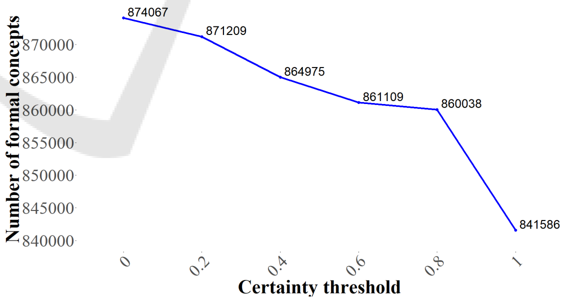

Figure 5: Filtering of the formal concepts according to the threshold of certainty.

We can see that the number of formal concepts is still high for the certainty equal to 1. These previous filters are not sufficient to extract the concepts representing the students' most frequent answers. So we have imagined to use a score representing the students' most common answers. If $\beta(U, V, \Re)$ is the set of formal concepts and $C = (X, Y)$ a formal concept with X the extent and Y the intent, then the score can be the following:

$$Score(C) = \frac{|X| + |Y|}{\max\limits_{(u,v)\in\beta(U,V,\mathfrak{R})} |u| + |v|} \quad (12)$$

We have filtered the formal concepts by using the score and we obtain the following result:

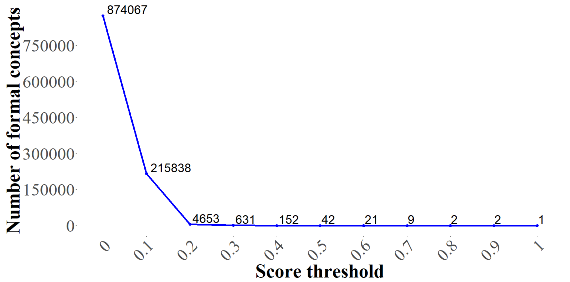

Figure 6: Filtering of the formal concepts by using the score.

We can also define filters on the properties or objects. All these criteria can be used in a query on formal concepts. The aim of the query is to perform complex filtering of the formal concepts to extract only those which are interesting. The query is a multi-criteria filtering of the formal concepts. Most of the papers concerning the formal concept analysis are focused on two topics: performance computation or visualization of information. After the computation, the results of the queries can be visualized to highlight the best results. We might have build a Hasse diagram but we have preferred to present a personalized visualisation of the information. For example, if a formal concept represents a lot of students' answers, then it can be displayed as a circle with a radius proportional to its significance. If it represents few answers then the radius is smaller. The uncertainty of the formal concept can be represented by a colour gradation (red, yellow and green) from red for less certain formal concepts to green for certain formal concepts. Several visualization tools exist. We have chosen Gephi because it is free and has a lot of functionalities. The results of the queries are transformed in a format compatible with Gephi and can be easily imported. For example, we can extract the concepts for a given score, with a set of properties and at least 20 students. The query can be seen as a set of criteria $B_\phi = (\phi_1 \wedge \ldots \wedge \phi_n)$ which must be all satisfied. Then we have $B_\phi \models B_C$ with B_C the set of formal concepts which verify all criteria of B_ϕ. We present below an example of a query:

$$B_\phi = \begin{cases} P_1 \vee P_2 \vee P_3 \vee P_4 \vee P_5 \vee P_6 \vee P_7 \vee P_8 \\ Score(C) \geq 0.1 \\ Card(X) \geq 20 \end{cases} \quad (13)$$

With $C = (X,Y)$ a formal concept, *Card* the number of properties or objects of the concept and P_i the classes of the open question. In our experimentation we obtain for this query the result below:

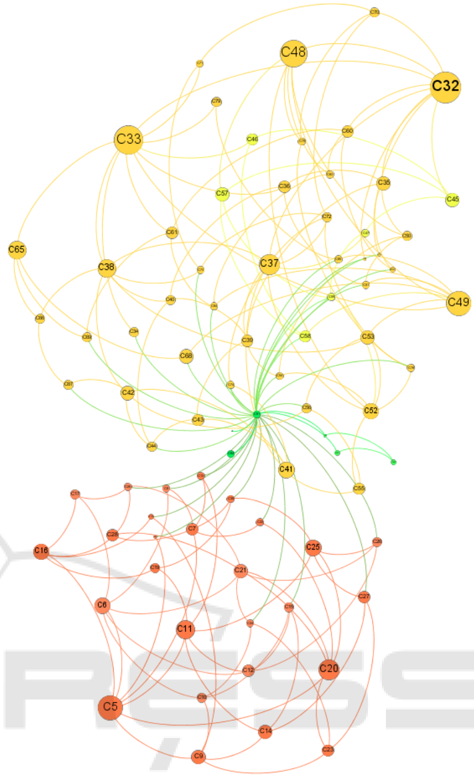

Figure 7: Example of a query result.

In the above figure, we can see the uncertainty of the formal concepts and the formal concepts which represent a lot of students' answers. The formal concept C5 seems to be a good solution but it is too uncertain because it is marked in red. There are several formal concepts with a certainty medium which can be a good compromise because we have no good results with a big green circle. For example, the formal concept C33 is interesting. We can deduce from this concept that the majority of the students have appreciated the co-facilitating of the course and found useful the topic concerning the theory of multiple intelligences. We can also define a score of relevance for a query in order to present the ranking of the results from the best solution to the less pertinent one.

6 CONCLUSIONS

The formal concept analysis can be extended by using the possibility theory leading to the uncertain formal concept analysis. We have applied the uncertain formal concept analysis to the analysis of a satisfaction questionnaire for a course of bachelor. To do this, we have performed a natural language processing for open questions before extracting formal concepts. The goal was to extract the lattice concepts by taking into account uncertainty generated by the processing of open questions. We have proposed to use queries to extract interesting formal concepts. In order to improve the presentation of the results of the queries, we have proposed a visualization which highlights the uncertainty of the formal concept by using a colour gradation and a circle with a radius proportional to the number of answers. In future, we would like to improve our experimentation in order to obtain more experimental results and comparative evaluations. Particularly, we have to evaluate better the query on the lattice concept and the use of the score of relevance. In this study, we have limited our approach to a simple case of the certainty computation of a formal concept but we wish to propose a general frame for certainty computation. On the other hand, we also need to improve the performance of the lattice concept computation. We have explored only the use of the guaranteed possibility operator, so we would like to explore the interest of the use of the other operators.

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