Investigating the Affordances and Constraints of SimReal for Mathematical Learning: A Case Study in Teacher Education

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Abstract: Visualizations tools are one of the most innovative technologies that emerged the last few years in educational settings. They provide new potentialities for mathematical learning by means of dynamic animations and representations, interactive simulations, and live streaming of lessons. Moreover, visualization tools have the potential to foster a visual, dynamic, distributed, and embodied mathematics rather than individual achievements and static representations. This paper uses the visualization tool SimReal in teacher education to explore the affordances of the tool for learning mathematics. It proposes a framework that captures the affordances of SimReal at a technological, pedagogical, and socio-cultural level. The aim of the article is to investigate the extent to which SimReal afford students' mathematical learning in teacher education. Based on the results, recommendations for future work are proposed.

1 INTRODUCTION

SimReal is a new visualization tool that is used to teach a wide range of mathematical topics both at the university and school level. SimReal uses a graphic calculator, video lessons, video live streaming, video simulations, and interactive simulations to teach mathematics (SimReal, 2018). In contrast to other digital tools such as GeoGebra, SimReal has more than 5000 applications and tasks in various areas of mathematics (Brekke and Hogstad, 2010). The tool can be divided in small subsets, while keeping the same structure and basic user interface. A subset of SimReal called Sim2Bil provides 4 windows for visualizations: simulation, graph, formula, and menu window (Hogstad et al., 2016),

There is a huge interest in visualization tools, but there are few research studies that address learning issues in authentic educational settings (Presmeg, 2014). Some research studies on SimReal focus on teaching mathematics at the undergraduate mathematical level (Brekke and Hogstad, 2010; Gautestad, 2015; Hogstad, 2012). The aim of these studies is to report on students' attitudes using SimReal as a supplement tool to ordinary teaching, and its usefulness in difficult and abstract mathematical areas. Hogstad et al. (2016) studied a subset of SimReal called Sim2bil that aims at exploring how engineering students use visualizations in their mathematical communication. Furthermore, Hadjerrouit and Gautestad (2018) used the theory of instrumental orchestration to analyze teachers' use of SimReal in an engineering class. Other studies were carried out in teacher education. Firstly, Hadjerrouit (2015) evaluated the suitability of the tool in teacher education using usability criteria. Secondly, Hadjerrouit (2017) addressed the affordances of SimReal and students' perceptions of the tool in teacher education. The present study is a continuation of these two studies. Based on the results achieved so far, the purpose of this work is to explore the affordances of SimReal and their impact on students' mathematical learning in teacher education.

The article is structured as follows. First, the theoretical framework is described, followed by the methodology. Then, the results are presented. Finally, a summary of the results, future work and recommendations conclude the article.

2 THEORETICAL FRAMEWORK

2.1 The Concept of Affordances

Among a wide range of theoretical approaches that can be applied to explore the impact of digital tools

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on mathematics learning (Geiger et al., 2012), the theory of affordances provides the most appropriate framework to address the impact of SimReal on learning mathematics in teacher education.

The term “affordance”, originally proposed by the perceptual psychologist James J. Gibson in his book “The Ecological Approach to Visual Perception” (Gibson, 1977), refers to the relationship between an object’s physical properties and the characteristics of a user that enables particular interactions between user and object. More specifically, Gibson used the term “affordance” to describe the action possibilities offered to an animal by the environment with reference to the animal’s action capabilities (Osiurak, et al., 2017)

The concept of affordances was introduced to the Human-Computer-Interaction community by Donald Norman in his book “The Psychology of Everyday Things” (Norman, 1988). Accordingly, the term “affordance” refers to the perceived and actual properties of the thing, primarily those fundamental properties that determine just how the thing could possibly be used.

A number of research studies used Norman’s ideas to implement the concept of affordances in various educational settings. For example, Turner and Turner (2002) specified a three-layer articulation of affordances: Perceived affordances, ergonomic affordances, and cultural affordances. Likewise, Kirchner et al. (2004) described a three-layer definition of affordance: Technological affordances that cover usability issues, educational affordances to facilitate teaching and learning, and, social affordances to foster social interactions. In mathematics education, Chiappini (2012) applied the notions of perceived, ergonomic, and cultural affordances to Alnuset, a digital tool for high school algebra.

De Landa (2002) emphasized that affordances are not intrinsic properties of the object. Rather affordances become actualized in specific context, e.g. the socio-cultural context of the classroom. In other words, affordances emerge from the relationship between the object and the particular environment with which it is interacting. From this perspective, the specific context of the mathematics classroom may include several artifacts or tools that interact with the user. Accordingly, the artifacts being used in a mathematics classroom have affordances and constraints. These may include paper-pencil techniques, the blackboard, Interactive White Board (IWB), Power Point slides, and diverse digital tools, such as Smart phones, IPad, GeoGebra, SimReal, and mathematics itself by means of symbols, notations, representations, etc. Artifacts with their affordances and constraints interact with the user.

2.2 SimReal Affordances

Based on the research literature described above and the specificities of mathematics education, this study proposes three categories of affordances and constraints at six different levels (Figure 1):

a) Technological affordances that describe the functionalities of the tool

b) Pedagogical affordances at four levels:

- Pedagogical affordances at the student level or mathematical task level
- Pedagogical affordances at the classroom level or student-teacher interaction level
- Pedagogical affordances at the subject level, that is the area of mathematics being taught
- Pedagogical affordances at the assessment level

c) Socio-cultural affordances that cover curricular, cultural, and ethical issues

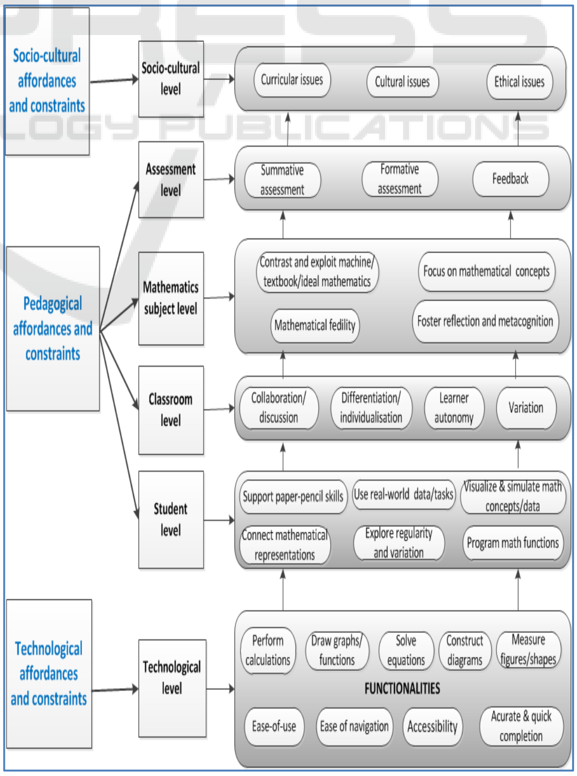

Figure 1: Three categories of SimReal Affordances at six different levels.

There are two types of technological affordances: Ergonomic and functional affordances. From the ergonomic point of view, these are ease-of-use, ease-of-navigation, accessibility at any time and place, accuracy and quick completion of mathematical activities. From the functional point of view, SimReal helps to perform calculations, draw graphs and functions, solve equations, construct diagrams, and measure figures and shapes. Technological affordances are a pre-requisite for any digital tool and provide support for pedagogical affordances.

There are several pedagogical affordances that can be provided at the student level, e.g., using the tool to freely build and transform mathematical expressions that support conceptual understanding of mathematics, such as collecting real data and create a mathematical model, using a slider to vary a parameter or drag a corner of a triangle in geometry software, moving between symbolic, numerical, and graphical representations, simulating mathematical concepts, or exploring regularity and change (Pierce and Stacey, 2010). At this level, the motivational factor is important, especially when using visualizations to engage students in mathematical problem solving. Furthermore, feedback in various forms to students' actions may foster mathematical thinking. Programming mathematical tasks may also be a way of using SimReal for learning and understanding.

Likewise, several pedagogical affordances can be provided at the classroom level (Pierce and Stacey, 2010). Firstly, affordances that result in changes of interpersonal dimensions, such as change of teachers' and students' role, less teacher-directed and more student-oriented instruction. Secondly, affordances that create more learner autonomy, resulting in students taking greater control over their own learning, and using SimReal as a "new" authority in assessing learning. Other affordances at this level are change of social dynamics and more focus on collaborative learning and group work, as well as change of the didactical contract (Brousseau, 1997). Variation in teaching and differentiation are other affordances offered by digital tools at this level (Hadjerrouit and Bronner, 2014). This may result in flipping the classroom, which is another way of using SimReal at this level.

Furthermore, three types of pedagogical affordances can be provided at the mathematical subject level (Pierce and Stacey, 2010). The first one is fostering mathematical fidelity, looking at congruence between machine mathematics and ideal or paper-pencil mathematics, and promoting faithfulness of machine mathematics (Zbiek et al. 2007). The second affordance is amplifying and reorganizing the mathematical subject. The former is accepting the goals to achieve those goals better. Reorganizing the mathematical subject means changing the goals by replacing some things, adding and reordering others. For example, in calculus there might be less focus on skills and more on mathematical concepts (Pierce and Stacey, 2010). In geometry, there might be emphasis on more abstract geometry, and away from facts, more argumentation and conjecturing (Pierce and Stacey, 2010). Likewise, it may be useful to support tasks that encourage metacognition, e.g., starting with real-world applications, and using SimReal to generate results.

Affordances at the assessment level consist of summative and formative assessment. Summative assessment is important for testing, scoring and grading, and it can be provided in form of statistics that the tool generates. Formative assessment is equally important for the learning process. Feedback is an essential condition for formative assessment. It can take many forms, e.g., immediate feedback to students' actions, a combination of conceptual, procedural, and corrective information to the students, or asking question types, etc.

Finally, several socio-cultural affordances can emerge at this level. Firstly, an important affordance is that SimReal should provide opportunities to concretize the mathematics subject curriculum in teacher education. Secondly, SimReal should be tied to teaching mathematics in schools, and support the learning of mathematics at the primary, secondary, and upper secondary level. In other words, SimReal should take the requirement of adapted education into account. Finally, other socio-cultural affordances can also emerge at this level, in particular those related to ethical, gender, and multi-cultural issues.

3 THE STUDY

3.1 Participants

Fifteen teacher students (*N=15*) from a technology and mathematics-based course in teacher education participated in this work. The students were categorized on the basis of their knowledge level in mathematics associated with their study programmes: Primary teacher education level 1-7, primary teacher education level 5-10, advanced teacher education level 8-13, and mathematics master's programme.

The recommended pre-requisites were basic knowledge of ICT (information and communications technology) and experience with standard digital tools like text processing, spreadsheets, calculators and Internet. No prior experience with SimReal was required.

3.2 Activities

A digital learning environment centered around SimReal was created over two weeks, starting from 25 August to 8 September 2016. An example of SimReal utilization is given in figure 2.

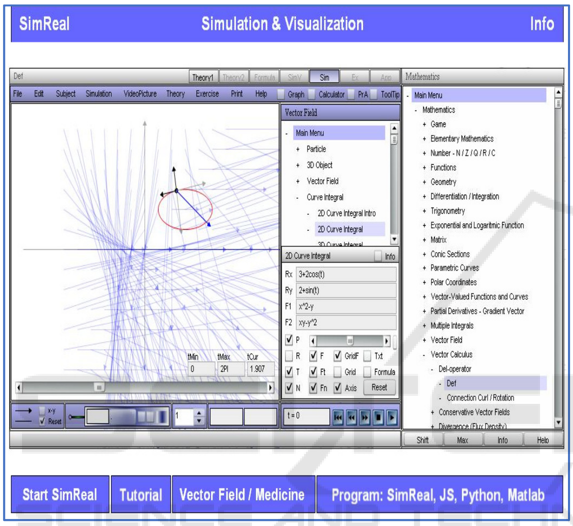

Figure 2: Example of SimReal utilization in mathematics education.

The environment included video lectures, visualizations, and simulations of basic, elementary, and advanced mathematics, and diverse online teaching material. Basic mathematics focused on games, dices, tower of Hanoi, and prison. Elementary mathematics consisted of multiplication, algebra, Pythagoras and Square theorems, and reflection. The topics of advanced mathematics were measurement, trigonometry, conic section, parameter, differentiation, and Fourier.

To assess experiences on specific mathematical topics that are of considerable interest for students, two specific mathematical tasks were chosen. The first one was Pythagoras theorem (Pythagoras theorem, 2018). There are many ways of representing Pythagoras. The theorem has also been given numerous proofs. These are very diverse, including both geometric and algebraic proofs, e.g., proofs by dissection and rearrangement, Euclid's proof, and algebraic proofs. Thus, Pythagoras is more than just a way of calculating the lengths of a triangle. An example of representing the theorem is given in the following figure (Figure 3).

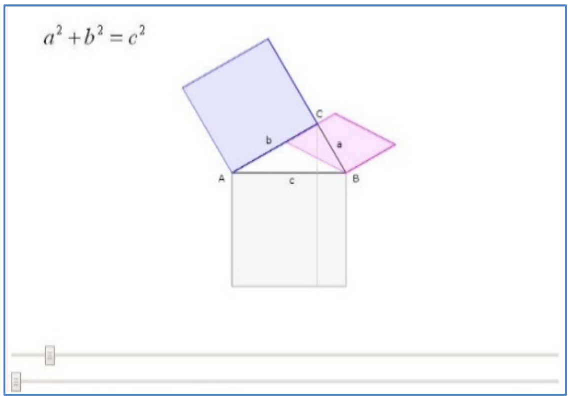

Figure 3: An example of representation of Pythagoras theorem.

The second task was the Square theorem (Square theorem, 2018). Like Pythagoras, there are many ways of using and representing the theorem (Figure 4).

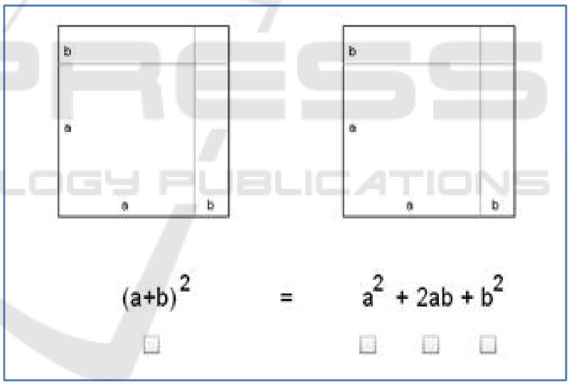

Figure 4: An example of representation of the Square theorem.

3.3 Methods

This work is a single case study in teacher education. It aims at exploring the affordances of SimReal for mathematical learning in teacher education. The study is exploratory in nature. Both quantitative and qualitative methods were used to collect and analyze students' experiences with SimReal. The following methods were used:

a) A survey questionnaire with a five-point Likert scale from 1 to 5, and quantitative analysis of the results

b) Students' comments in their own words to each of the statements of the survey questionnaire

c) Students' written answers to open-ended questions

d) Qualitative analysis of students' comments on point b and answers to open-ended questions to point c

e) Task-based questions on Pythagoras and Square theorem, and programming issues

The design of the survey was guided by the theoretical framework and the research goal. To measure the students' perceptions of SimReal, a survey questionnaire with a five-point Likert scale from 1 to 5 was used, where 1 was coded as the highest and 5 as the lowest (1 = "Strongly Agree"; 2 = "Agree"; 3 = "Neither Agree nor Disagree"; 4 = "Disagree"; 5= "Strongly Disagree"). The average score (MEAN) was calculated, and the responses to open-ended questions were analyzed qualitatively. The survey included 72 statements that were distributed as follows: Technological affordances (12), pedagogical affordances at the student level (11), classroom level (19), mathematical subject level (9), assessment level (10), and finally socio-cultural level (11). The students were asked to respond to the survey using the five-point Likert scale and to comment each of the statements in their own words. In addition, the students were asked to provide written answers in their own words to open-ended questions. The responses to students' comments to the survey and open-ended questions were analyzed qualitatively.

Of particular importance are task-based questions on Pythagoras and Square theorems to collect data on affordances when students engage with these mathematical tasks. An additional question on programming issues was given to the students to assess the affordances of programming languages for the learning of mathematics. Asking task-based questions provides supplementary information on the affordances of SimReal. This method also provides more nuanced information about the students' experiences with SimReal. The analysis of the data was guided by the specified affordances of the theoretical framework, and open-coding to bring to the fore information that was not covered by the theoretical framework.

4 RESULTS

4.1 SimReal Affordances

The results achieved by means of the survey questionnaires and open-ended questions show that the affordances of SimReal emerge at different educational levels. The affordances are less evident for students from the study programme 1-7. They become more visible at the middle level associated with the study programme 5-10. Affordances come to the fore at the advanced level, and for mathematics education students.

Globally, the vast majority of the students pointed out that SimReal still lacks an easy-friendly interface and that it is not easy to use, to start and to exit. For many students, the tool was accessible anywhere and anyplace, but the navigation through the tool is still not straightforward. On the positive side, SimReal has a ready-made mathematical content, and that the video lessons, simulations, animations, and live streaming are of good quality. This is reflected in many students' responses.

In terms of *pedagogical affordances at the student level*, many students think that SimReal provide real-world tasks, which engage them in mathematical problem solving, particularly when using visualizations to simulate mathematical concepts. Most students think that visualizations are useful to gain mathematical knowledge that is otherwise difficult to acquire, and they liked very much the combination of live streaming of lessons, video lectures, simulations, and animations. Most of the advanced mathematical exercises (trigonometry, differentiation, and conic section) were not difficult for them to understand. Likewise, SimReal provided affordances to explore variation and regularities in the way mathematics is taught, e.g., vary a parameter to see the effect of a graph. The students also think that SimReal is congruent with paper and pencil techniques. On the negative side, most students think that SimReal is not helpful to refresh students' mathematical knowledge.

In terms of *pedagogical affordances at the classroom level*, the majority agreed that they can use SimReal on their own, and that the use of the tool is not completely controlled by the teacher, and, as a result, they do not need much help from the teacher or textbooks to solve exercises. Likewise, most students think that the tool can be used as an alternative or supplement to textbooks and lectures. The tool also facilitates various activities (problem solving, video lectures, live streaming), and several ways of representing mathematical knowledge (texts, graphs, symbols, animations, visualizations). In terms of differentiation and individualization, many students believe that the level of difficulty of the mathematical tasks is acceptable, but it is relatively difficult to adjust the tool to the students' knowledge level. Even though the degree of autonomy is not very high, it is sufficient to allow students work at their own pace. On the negative side, the vast majority of the students think that SimReal does not support much cooperation or group work, and it does not have collaborative tools integrated into it.

In terms of *pedagogical affordances at the mathematical subject level*, most students agreed that SimReal provides a high quality of mathematical content. Many students also think that SimReal provides real-world applications and tasks that foster reflection, metacognition, and high-level thinking. Likewise, the vast majority found that SimReal is mathematically sound, and that the tool can display correctly mathematical formulas, functions, graphs, numbers, and geometrical figures. On the negative side, the overwhelming majority think that the software tool GeoGegra has a better interface, and it is better to express mathematical concepts. Finally, the combination of mathematics and practical applications in physics is evaluated as useful to gain mathematical understanding.

In terms of *affordances at the assessment level*, most students think that SimReal provides several assessment modes and give directly feedback in form of dynamic animations. This is a clear improvement compared to previous experiments. Likewise, SimReal provides satisfying solutions step-by-step, but not for all tasks. Still, SimReal does not provide several types of feedback, differentiated knowledge on student profiles, several question types, and statistics. Finally, the degree of interaction is evaluated as satisfying.

In terms of *affordances at the socio-cultural level*, most students think that SimReal is an appropriate tool to use in teacher education, but it does not take sufficiently into account the requirement for adapted education. Furthermore, most students believed that SimReal is appropriate to use in secondary schools, and in a lesser degree in middle and primary schools. On the negative side, the vast majority will not continue using video lessons and live streaming to learn mathematics, but some will still be using video simulations in the future. Nevertheless, the vast majority of the students think that the tool enables the teacher to concretize the mathematical subject curriculum.

Summarizing, it worth noting that affordances do not emerge in the same degree for all students. Rather they become actualized in relationship to the participants' knowledge level from the 4 categories of study programmes: Primary teacher education level 1-7 and level 5-10, master programme, and advanced teacher education level 8-13.

4.2 Affordances of Pythagoras Theorem

Students were engaged in 16 different approaches to exploring Pythagoras theorem. These were divided in paper-based (1-9) and SimReal-based approaches (10-16). The students were asked to report on SimReal affordances and critically reflect on their impact on learning Pythagoras by responding to 4 specific questions.

a) *If you should choose only one of the 16 different approaches of explaining Pythagoras, which of them would you prefer?*

The students provided a variety of solutions in order of priority according to the perceived affordances of the approaches. One suggestion was 2/5/7/12/15, and 16. Some students chose a sequence of approaches such as 2/3/5, or 2/3, or 7/14. One student provided another set of preferences that are worthwhile to study in details. Firstly, the student decided to use approach 15 as a brief introduction, and then Pythagoras 1, both as a simple presentation of the equation of the theorem and as a first visual proof of the theorem. Then, he suggested to use approach 2 as a general formula and 3 as a more specific or realistic one. The student also suggested a combination of approach 10 (paper-based) and 14 (SimReal-based), but without the written explanation or mathematical formula. Instead, one can start with a given problem such as "find the area of the pool or the area of the baseball field". After having discussed suggestions to approaching the solution, the student can then check the explanation provided by SimReal in terms of written text or mathematical formula. Finally, the student would demonstrate approach 4 using a rigorous proof through the usage of algebraic and geometrical properties. Summarizing, several affordances emerged in this situation: realistic task, pen-paper formulas, SimReal visualizations with written explanations of the theorem, rigorous mathematical proofs of the theorem, and a combination of paper-based and SimReal visualizations.

b) *If you should combine one of the pen/paper approaches and one of the digital simulations which of them would you prefer?*

The students provided a variety of approaches such as 3/12 and 9/12. As described above, one student combined approach 10 (pen/paper-based) with approach 14 (SimReal-based). After the use of the pen-and-paper solution and an attempt to calculate the blue area of figure 14, the students could try to calculate several areas of the figure. Example 10 and 14 have different approaches and content, but they complement each other in terms of their affordances.

c) *If you should choose the combination of the two approaches 9 and 12, how would you in detail explain Pythagoras?*

A variety of explanations were provided to explain Pythagoras using various elements such as the layout and colors of the figure, the dynamic of the simulation, and mathematical explanations. A good combination of 9 and 12 is as follows. The student starts with 9 (angle A=90°), and notes that the pink and the blue area of the bottom square is equal to the other two corresponding square areas (blue and pink). Furthermore, the sum of the blue and pink square areas is equal to the bottom square area, consisting of these two rectangles. Moving on to 12, it is worth mentioning that their area remains unchanged. Before using the SimReal-based simulation, some figures of rectangles on the blackboard would be useful, revising or presenting the formula for the area of the parallelogram. The figures should be of various parallelograms, including interaction with the student by considering different bases each time and different heights. The student concluded that a combination of moving the scroll bar and considering cases on the blackboard depending on the position of the parallelogram and its height. This could be a good reasoning step to explain why the area remains the same. The digital simulation could be used to clarify the question.

d) *Do you think that teaching different ways of Pythagoras by combining pen/paper and SimReal-based simulations would help in the understanding of this topic, or do you think it would be confusing for the students?*

The students think that the combination of pen/paper and SimReal simulations is helpful to understand Pythagoras depending on time and pedagogical constraints, and students' knowledge level as well. They think that it would be positive to use several approaches to teach Pythagoras considering various students' knowledge levels and learning styles when solving problems. It is therefore important to present mathematical tasks using different ways. By showing a figure describing Pythagoras theorem, the teacher has a good opportunity to explain the mathematical formula in his/her own words, before showing a SimReal simulation of the theorem. This may motivate the students, and stimulate their curiosity. Approach 9 or 3 combined with simulation 12 would give a good effect. In most cases, a good combination of pen/paper and SimReal simulations is preferable, but there may be some confusing cases that make the understanding of the topic more difficult. In those cases, the task should be either pen/paper solution or SimReal simulation, but not both approaches, even though the teaching may be less efficient. As a result, a good way of teaching Pythagoras is a combination of SimReal affordances with paper-pencil solutions.

4.3 Affordances of Square Theorem

Students were engaged in 6 different approaches to exploring the Square theorem. These are divided in paper-based (1-3) and SimReal-based approaches (4-6). The students were asked to study them and report on their affordances by responding to 4 specific questions.

a) *Pen/paper proofs (1, 2, 3) versus SimReal-based proofs (4,5,6) of the Square theorem*

Most students preferred a combination of pen/paper with SimReal proofs, but those participating in the task should not just passively read the proofs. They should rather take advantage of the dynamic visualizations provided by SimReal. Regarding the Square theorem, the pen-paper approaches 1-2-3 do not necessarily promote students' understanding, because these are based on a more mechanical calculation method. SimReal simulation 4 is good approach for visualizing the theorem. However, the second and third approaches are somewhat tricky to understand geometrically, but still better than just formulas. Therefore, approaches 4-6 should be used to create dynamic images of the Square theorem. Another student preferred pen/paper proofs (1-3) and think that these methods are mostly used to describe algebraic operations and expressions. However, these approaches are important only if the teacher takes a more practical approach to the theorem, and the geometrical SimReal-based approaches could be used to enhance the understanding of the theorem.

b) *In what way do you think the use of SimReal can provide a better understanding of the Square theorem?*

The students think that SimReal provides affordances to improve the understanding of the Square theorem by visualizing mathematical concepts. More specifically, one student suggested a quiz, and another a "fill the blanks" exercise, where a student could get a direct result or feedback if the answer is correct or not. Globally, the students think that the digital visualizations are beneficial for visually strong students, considering the fact that upper secondary mathematics becomes more theoretical the higher up the grade, and, as a result, there is less focus on conceptual understanding, and why and how to carry out calculations. Digital simulations can have therefore a positive effect on student learning and help them to see how mathematical formulas work.

c) *Give some comments about how you could think to improve either by pen/paper or SimReal the understanding of the Square theorem*

Students provided many ways of improving the understanding of the Square theorem. One solution is giving exercises both with symbols and numbers, but also allowing the use of expansion like (a + b) ^2 = (a + b) (a + b), until the student becomes familiar with the theorem. The paper-and-pen exercises 1-2-3 show specific procedures on how a student can change and calculate the Square theorem tasks, but the procedures would have been clearer if there was a headline for each example to show how the theorem works. SimReal solutions 4-5-6 have digital simulations with explanations, color coding, and reference to formulas. These cover the Square theorem quite well, and there is no need for improvement. Likewise, SimReal simulations can make it easier for students to see how the formulas work, and this is especially true for the 1st and 2nd approach to the Square theorem.

d) *Do you prefer learning the Square theorem in one way or do you feel a better understanding learning it in different ways?*

As already stated above, most students think that a combination of different approaches is the most appropriate way to provide a better understanding of mathematics, while also being careful not to use several approaches at the same time as this might be counterproductive. They also argued that it is important to see mathematics from different angles. Using new methods to explain the solution to a single problem will give new perspectives about the problem and the corresponding solution, and how these are interrelated. A good example is the figure-based and algebraic proofs of the Square theorem. Showing different point of views of the theorem (like a geometrical one) and applications of the theorem could indeed be very efficient.

Summarizing, a comparison of the results in terms of affordances achieved by means of task-based questions reveal that these are globally in line those achieved by the survey questionnaire and open-ended questions in terms of pedagogical affordances at the student level. The issues that correspond very well are the usefulness of visualizations for understanding the Square and Pythagoras theorem, the congruence of SimReal-based visualizations with paper-pencil techniques, and a combination of different representations and approaches to the theorems.

4.4 Programming Affordances

Programming has rapidly grown as an innovative approach to learning mathematics at different levels. The topic will become compulsory in schools from the study year 2020. As a result, it is expected to improve SimReal by including programming tasks using Python and other programming languages. Given this consideration, it was worthwhile to ask the students about the affordances of programming.

a) *Would it be of interest for you to program your own simulations in teaching mathematics?*

The study reveals that SimReal can provide more affordances in terms of programming mathematical concepts. Basically, most students think that teachers with experience in programming mathematical simulations and visualizations will open a new way of teaching mathematics. For example, a teacher could focus on subjects and tasks that are difficult for the students to comprehend. Another possibility is to program tasks that are not already covered by SimReal, but that are already available online. Most importantly for teachers is the use of different methods to promote understanding and making new connections. Hence, it may be worthwhile to take advantage of simulations and explanations combined with some programming examples so that the knowledge to be learned is presented with various methods.

b) *Do you think it would be of interest and help that students program their own simulations?*

The participants think that students would be interested in programming visualizations if they have acquired sufficient skills in this matter. This would contribute to enhanced motivation and increased understanding of mathematics, because the students will be forced to fully comprehend mathematics before they could program visualizations. Likewise, it could be of help for the students if they could program their simulations by themselves. However, it is crucial that they focus on the mathematical part of the task rather than programming issues alone. Programming their own simulations could be motivating for those students who are both interested and knowledgeable in programming. This presupposes, however, that the students have understood the mathematics before getting started with programming. Students having difficulties in mathematics should rather spend their time on it. Hence, programming would be helpful if it contributes to the learning of mathematics. Likewise, advanced mathematics requires a higher level of programming knowledge, and it may therefore be necessary to evaluate whether students have sufficient understanding of mathematics to be able to program themselves. Finally, only one student pointed out that he would not spend time on programming, even though he sees an advantage in it. Summarizing, programming mathematical tasks can contribute to the understanding of mathematics, but it is demanding in terms of efforts and time for novice students.

5 DISCUSSION

The purpose of this work is to assess the impact of SimReal affordances on students' mathematical learning in teacher education. The study provided an important amount of empirical data on what students perceived as affordances of SimReal and their impact on learning mathematics at different levels. Although this study does not aim to capture all potential affordances, it is possible to make reasonable interpretations of the results and draw some recommendations for using SimReal in teacher education.

In terms of technological affordances, there is a need for a better and intuitive user interface and navigation for different types of users. From a pedagogical affordance point of view, SimReal affords many students to do mathematics both at the student and classroom level. It provides variation in teaching mathematics, and visualizations are considered as useful to gain mathematical knowledge that is otherwise difficult to acquire. The combination of live streaming of lessons, video lectures, simulations, and animations is highly valued. Students can work at their own pace, without much interference from the teacher. SimReal also facilitates various activities and several ways of representing knowledge. In terms of affordances, SimReal needs to be better adjusted to the student knowledge levels, and it should provide a better support for group work. Furthermore, there is a need for feedback and review modes, more differentiation and individualization, including the possibility of programming their own videos and visualizations. At the mathematics subject level, the tool has a high quality of mathematical content. Moreover, the mathematical notations are correct and sound.

The study shows that the affordances of SimReal make mathematics easier to understand, because these provide a concrete way of making mathematical concepts more dynamic. In addition, SimReal provides a huge variation of visualization examples for the teacher to use in classroom, e.g., SimReal can support the understanding of Square and Pythagoras theorems by visualizing the dynamic behavior of the theorems. Nevertheless, a combination of pen and paper, digital visualizations, and chalk-blackboard could be more efficient to teach mathematics than just SimReal alone. Moreover, the students think that videos can speed up the interest and motivation for doing mathematics. Videos could be used as a supplement to mathematics on the blackboard and paper-pencil, and as an alternative way of sharing knowledge and explaining mathematics. Videos are especially important because these are one of the main sources of information for young students. Most students also think that programming mathematical tasks can provide more affordances for the learning of mathematics.

Moreover, many students think that the tool is appropriate to use in teacher education and upper secondary school level, and it enables to concretize the curriculum. At the assessment level, works need to be done to improve the feedback function.

Summarizing, the theoretical framework has proven to be useful to address the affordances of SimReal and their impact on mathematical learning in teacher education. Nevertheless, the research literature reveals that the concept of affordances can be reconceptualised and extended by considering ontological issues (Burlamaqui and Dong, 2015). As already stated, affordances are not properties that exist objectively. Rather affordances emerge in the socio-cultural context of the classroom, where a number of other artifacts and their affordances interact with SimReal, e.g., paper-pencil, black board, textbooks, Smart Phones, Power Point slides, mathematical tasks and their representations, etc. A reconceptualization of the concept of affordances needs to take in consideration new and more powerful theories such as Actor-Network Theory (ANT), which does not consider technology simply as a tool, but rather as an actor with agency that serves to reorganize human thinking (Latour, 2005). In this regard, Wright and Parchoma (2011) criticized the value of affordances, and proposed Actor-Network-Theory as an alternative framework that may contribute to greater critical consideration of the use of the concept "affordances". The theory of assemblage may also contribute to the understanding of affordances and its relationship to mathematical learning, which is understood as "an indeterminate act of assembling various kinds of agencies rather than a trajectory that ends in the acquiring of fixed objects of knowledge" (De Freitas and Sinclair, 2014, p. 52). Moreover, Withagen, et al. (2017) argued that affordances are not mere possibilities for action, but can also have the potential to solicit actions. Hence, the concept of agency can contribute to a better understanding of affordances.

6 CONCLUSIONS

The purpose of this article is to assess the affordances of SimReal for mathematical learning in teacher education by asking students to respond to a survey questionnaire and open-ended questions. In addition, the students had the opportunity to comment the items of the survey in their own words. Task-based questions were also used to provide more nuanced information about the students' engagement with the Pythagoras and Square theorem, and their views on programming affordances as well. The data collected by means of these methods provided an important amount of information that gave a better sense and interpretation of the results achieved in this study. Even though, the results are promising, it is still difficult to generalize the findings because of the small sample size (*N=15*). In fact, amongst this sample size there is already variance with regard to the different primary teacher education levels. However, it would have better for the research study to have less variance with such a small sample size and ensure that one or two of those groups have a larger representation.

In future studies, students' recommendations will be considered to improve the teaching of mathematics with SimReal. In terms of technological affordances, there is a need for a better and intuitive user interface and navigation for different type of users. In terms of pedagogical affordances, there is a need for better feedback and review modes, more differentiation and individualization, and the possibility of programming their own videos and visualizations. The concept of affordances will be refined by considering other theories, such as Action-Network Theory, agency, and assemblage theory. It is also planned to look at students' learning styles, for example between visual and verbal students. Finally, the data collection and analysis methods will be improved to ensure more validity and reliability.

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