Improving the Attitude towards Mathematics via an ICT Rearrangement of the 8th Grade Math Curriculum

Borislav Lazarov
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Blok 8 Georgi Bonchev Str., Sofia, Bulgaria

Keywords: Attitude towards Mathematics, Dynamic Geometry Software, Synthetic Competence, Individual Educational Trajectory.

Abstract: Under consideration is an experimental teaching in mathematics of an eight-grade high abilities student, whose attitude towards mathematics initially was negative. The educational goal includes developing student's competence of synthetic type, i.e. a package of knowledge, skills and attitudes, which are multifunctional and transferable. The European framework of key-competences is chosen to design an individual educational trajectory (IET). An original didactic model is adopted as theoretical base for this IET. The main issue in the IET was how to change the student's attitude towards mathematics to positive. For resolving this issue, a rearrangement of the syllabus is done by incorporating ICT. A large part of the routine paper-and-pencil drills was replaced with dynamic-geometry-software exercises. Parallel to the compulsory lessons in math, the IET included two project-oriented initiatives. The goal of these initiatives were to reinforce the synthesis of the student's analytical knowledge and skills, which were built separately in math, ICT and arts. On this stage of the IET, the informal learning served as an accelerator in turning attitude in positive direction. At the final stage of the experimental teaching, the student covers the general standard for positive attitude towards mathematics.

1 INTRODUCTION

The concept of competence in European Commission understanding consist of 3 components: knowledge, skills and attitude (EU, 2006). The development of knowledge and skills are parts of any math curriculum in Bulgaria. There is a long tradition in teaching any particular math topic in secondary school and valid didactic methods are approved for different age groups (Ganchev and Kuchinov, 1996). The modern textbooks follow these methods and the teachers are more or less prepared to apply them into school practice. However, the recent international assessments pointed on a general drop in performance of the Bulgarian secondary school students in math (PISA, 2016). It appears that Bulgarian eighth-graders are not competent to cover some international educational standards. Therefore, if the knowledge-skills part of the education is still in a framework that assures good results in the past, then maybe the problem is in the students' attitude.

We do not know examples of Bulgarian good practice on improving the attitude toward mathematics among eighth grade students. Outside Bulgaria (where the European framework was adopted) the attitude is related to high achievement, or motivation, or engagement, which is not the case we are interested in (Blackweir, 2016, p.16-17). Therefore, we constituted a teaching experiment to look for some possible stimuli for an eight-grader to study math having in mind the Bulgarian educational context.

The eighth grade mathematics curriculum in Bulgarian secondary school is crucial in forming students' deductive abilities. The main didactical instrument for this mission is the geometry part that upgrades the axiomatic approach introduced in seventh grade by 4 modules: vectors, rigid motions, circle and geometrical constructions. Every module is elaborated in a traditional mode lessons: introductory example, definitions, theorems, drills. The students' activities are supposed to be performed by hand drawing and calculating, as well as deductive reasoning based on a carcass of statements of Euclidean geometry. We advocate the need of renovation this traditional approach.

The experimental teaching described below presents an attempt to reform the teaching method in eighth grade math by introducing some computer

Lazarov, B.
Improving the Attitude towards Mathematics via an ICT Rearrangement of the 8th Grade Math Curriculum.
DOI: 10.5220/0007744203910397
In *Proceedings of the 11th International Conference on Computer Supported Education (CSEDU 2019)*, pages 391-397
ISBN: 978-989-758-367-4

applications, mainly dynamic geometry software (DGS), but also computer algebra system (CAS). As we will show, this reform went far beyond the simply technical issues and caused deep rethinking of the bases.

2 THE INITIAL CONDITIONS

The student V started the eighth grade mathematics classes declaring she hate math. V was studious and she was doing regularly the obligatory math assignments but without any enthusiasm. The most unpleasant for her were the routine work: calculations, algebraic transformations etc. During the first term, the V's test results were mediocre that contrasts her high intellectual abilities in general. Such status quo was unacceptable and the need for change was obvious. In this connection, the following questions were top of the agenda:
1) how the attitude towards mathematics of a student of high potential abilities could be changed to positive;
2) what part of the curriculum should by reorganized.

Therefor we designed a midterm individual educational trajectory (IET). By IET we will understand *the organizational framework and plan for realization of a medium term educational process that are coherent with the individual specifics of the learner and provides opportunities for the optimal development of his/hers creative potential* (Lazarov, 2012).

3 THE IET: A BIT OF THEORY

We summarized our previous experience in tutoring advanced students in (Lazarov 2013A), where we proposed a didactic model (called DMT) for designing an IET. The movement along the IET follows a kind of spiral. However, it is more convenient to decompose this spiral into two directions: vertical and horizontal. The vertical direction is an iterative procedure that includes the following components.

1) Formation of an individual informational environment.
2) Individualization of the didactical resources, including selection of the individual (re)searching instruments.
3) Individualization of setting the educational goal, including flexible approach to achieve it.
4) Individualization of the learning temps, investigation activities, layout style.
5) Taking into account the individual reflexive abilities and self-organization aptitude in searching a synergetic effect. [ibid]

The DMT allows visualizing the iterations of an IET as climbing floors. Here is the list of the steps to be done in the k-th floor of the IET (the program maximum).

…(k – 1) →

- A proximal educational goal (of learning, investigative or research type) is mapped out with respect to the actual knowledge, skills and attitude (KSA) of the student (actual development).
- A (very limited) informational resource is determined which is focused on the goal.
- All needed activities to extend the actual KSA to a level required for reaching the stated goal are performed.
- Student proceeds to the goal in his own manner.
- Analysis of the achievements (tutor responsibility).

→ (k+1) …

The horizontal direction connects the iterations in a specific way for which we will give details in the next section.

4 IMPLEMENTATION OF THE THEORY

The traditional style math teaching requires just a white-board and some posters visualizing basic geometry shapes and math formulae. The students' classroom activities corresponding to such teaching are related mainly to problem solving by paper-and-pencil techniques. Therefore, the first step in our plan was to expand the classroom equipment by computers with DGS. We designed for V an IET, based on implementation of DGS.

4.1 The Ground Floor of V's IET

Initially the informational environment was limited to the textbook (Paskaleva et al., 2013). Following the syllabus, the linear simultaneous equations stood on agenda. This was the right moment to start our plan for overcoming V's negative attitude to math. Among the didactical tools we selected was learning by syntax (Lazarov and Karakoleva, 2011). The correct usage of the DGS requires from V a kind of mathematics competence related to basic concepts of the analytical geometry: Cartesian coordinates, line-equation correspondence, graphic interpretation of simultaneous equations etc. Studying the DGS interface V realized the need to learn the math concepts deeply. The benefits came quickly: the paper-and-pencil solution of simultaneous equations is written on several rows; the DGS solution is limited to entering the equations and interpretation of the picture. Let us emphasize that we passed the obligatory content in full scale to cover knowledge-and-skills standards. V did just a part of the routine as a DGS activity.

One can see the V's uncertain first attempt in using DGS in Figure 1. The genuine problem was to solve graphically the simultaneous equations $x+2y=3$, $y+x/2=0$. There are expendable elements as vertical lines that appear in searching points of intersection that actually do not exist.

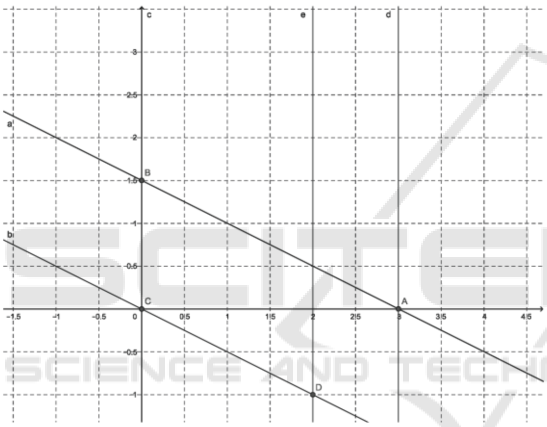

Figure 1: V's first attempts in applying DGS.

On this floor, V made successfully the first steps in overcoming the routine calculations by application of CAS.

About the attitude component of the competence, our proximal educational goal was related with the first two points of the *Mathematical literacy* key-competence, as well as points ii. and iii. of the *Learning to learn* key-competence (see the Appendix). So, our program maximum at this stage was fulfilled: V started dealing math with no bad feelings.

4.2 The Second Floor of V's IET

The next big step that V made was related to geometry lessons: the rigid motions. The chapter in the textbook started with a discussion about how to imagine a plane that can move onto itself. Here the advantages of the DGS versus paper-and-pencil style are indisputable. No imagination was needed in general but one can see the matter of a particular rigid motion just dealing with DGS instruments to perform translation, rotation, or symmetry.

V and a classmate of her started a project about congruent shapes in real life environment. They captured images of buildings, electric lighting poles, street tessellation etc. and classified the repeating shapes according to the type of the rigid motion that could be recognized. For instance, the front view of some buildings repeat the same form of the floors; hence, they could be obtained by translation about a vertical vector. Our educational goal in its attitude part referred to points i., iv., and vi. of the key-competence *Learning to learn* (see the Appendix).

We will skip more details of that stage of V's IET just mentioning that V's attitude towards math ceased being negative. Such change was probably caused by the decontextualization of the math theory connecting it with the everyday student's experience.

4.3 The Third Floor of V's IET

The project-oriented education was a part of the school strategy at that time (Lazarov, 2018). Moreover, students like working on educational projects and we catch this opportunity to apply integrated approach. V was no exception. The project about the rigid motion in the city environment lifted the V's enthusiasm.

'Rigid motion' in Bulgarian translation is 'ednakvost' (pl. ednakvosti). *Ednakvost* has an everyday meaning, which is slightly different from the mathematical concept. It is close to 'mapping a figure onto congruent figure' (figuratively, copy-pasting). Here the 'rigid' part is not obligatory that allows extending the math concept beyond the geometry. In the following, we will use the word *ednakvost* for expanding the meaning of the concept of rigid motion beyond geometry.

So, we started another project on ednakvosti, this time about figures in art. Our idea was to integrate with math as much art as possible. After consulting the music teacher, who suggested to consider the music of Ravel's bolero, we decided to go further and to analyze the Maya Plisetskaya's bolero dance (YouTube, 2016). In our opinion, this movie is a pure synthesis of music, dance, and filming. The project idea was to observe how the show is composed of figures in music and dance that are multiplied through different ednakvosti.

The rhythmic part of Ravel's bolero repeated 168 times the initial figure. V recognized here translation (Figure 2). Another translation V observed in the two

Figure 2: Translation in the rhythmic section (a slide from V's presentation).

melodies, which are on top of this rhythm (Figure 3). The second translation moves the musical figure from flute to clarinet, then to bassoon, and so on. This was the first coordinate of the translation vector. The second coordinate is in the loudness, constantly rising from pianissimo to fortissimo.

Figure 3: Translation in the melody (a slide from V's presentation).

Another ednakvoti V saw in the Maya Plisetskaya's dance. She recognized symmetry and rotation. V managed to cut the YouTube clip into short parts, which illustrated the corresponding ednakvost. For this purpose, V found the free software WEVIDEO in the www, learned by herself how it works and applied it to a presentation. Let us point that the filming of the dance emphasized the dance figures in a very suitable manner for the project goals. V caught this opportunity. E.g. she splitted one of the movements into symmetrical origin and image, which are shown together in the next movement that completes the symmetry (ibid., 0:22-1:27); similarly, V extracted rotation (ibid., 8:04-8:15).

V presented the project to her teachers and schoolmates. At the end of the presentation, after the final accords of bolero, she declared 'ednakvosti are everywhere'. This we accepted as the turning point in change V's attitude towards math to positive. However, we consider the attitude in a synthesis with some attitude components from other key-competences, e.g. *Cultural awareness* and *ICT* ones (Appendix).

4.4 The Top Floor of V's IET

The time to harvest the fruits from the V's metamorphosis came with the curriculum unit about Geometrical constructions. However, the application of DGS in standard tasks requires deep revision of the method. The ruler-and-compass axiomatic is not coherent with DGS instruments. The conflict points are discussed in details by Lazarov (2011).

Another issue relates the reasoning. Here two points stand on agenda:
1) the traditional proof that the construction algorithm produces the proper figure can be replaced by a check for dynamical steadfastness of the DGS construction;
2) the conditions under which a figure exists can be replaced by examination of the initial data.

4.4.1 The First Point

We illustrate the reasoning instead the proof by the example in Figure 4. The theorem states that the symmetric point of the orthocenter of a triangle about any side lies on the circumcircle.

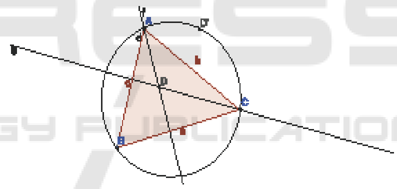

Figure 4: V's applet illustrates the property of the symmetric point of the orthocenter of a triangle.

The V's applet allows checking different type of triangles: acute, obtuse, right. The symmetric point D' of the orthocenter D of triangle ABC about the line BC lies on the circumcircle, every time when any of the vertices A, B, C changes its location. Let us point that the rigor proof is not easy even for some math teachers.

4.4.2 The Second Point

The analysis of existence of a figure we illustrate by Figure 5 where V constructed isosceles triangle having given base *a* and the circumradius *r*.

She introduces two sliders: one for the base a, another for the radius r. The dynamic construction allows examining the existence of a triangle with the given properties.

Let us explain that the change of the method does not change the educational goal of the topic Geomet-

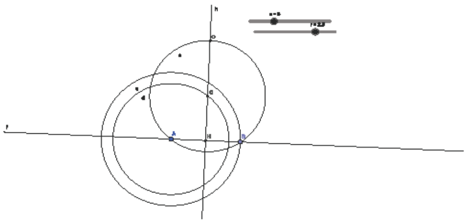

Figure 5: Construction of an isosceles triangle with given base and circumradius.

rical constructions. The replacement of the axiomatic of ruler-and-compass by the DGS operators develops the same kind of algorithmic thinking. Moreover, the computer-based application of the theory is a solid ground to cultivate some designer abilities. E.g., V made just for fun an applet for topological transformation of a dog's face into a cat's face and vice versa.

At this stage, we emphasize the last three points of math key-competence, again in synthesis with key-competence *Learning to learn* (Appendix).

5 DEVELOPING COMPETENCE ALONG THE IET

The DMT describes in general a hypothetic movement inside the floors of an IET (Lazarov, 2013). However, the details are rather specific for any particular IET. Bulgarian educational legislation operates with the category of key competences (MON, 2015), while PISA framework takes mathematical literacy as the basic construct (PISA, 2018). Our standing point differs from both cases: building up a kind of synthetic competence is what should be the educational goal in the secondary school (Lazarov, 2013B).

The synthetic competence is a construct for the methodology when designing IET, but it becomes a personal attribute for any individual as an outcome of the IET. Let us emphasize that the synthetic competence is context sensitive. This is why the educational context should be challenging and to include stimuli of synthetic type. In our case, some of the stimuli were the two project-oriented initiatives on ednakvosti (remember 4.2. and 4.3.). Other stimuli were set during the discussions on extracurricular topics as the nine-point circle of a triangle, Euler's line, etc. Any time the DGS illustrations were done by V. As result of her activity an archive of DGS applets were created.

The knowledge and skills built on any floor of V's IET were tested by implementation in DGS activities that assured a kind of transferability and multifunktionality of knowledge and skills. Parallel to this the V's attitude towards mathematics lifted on any floor (in our opinion). Therefore, we can speak for a development of V's knowledge-skills-attitude package in a context that is different from the one, in which the KSA package was elaborated. In other words, the V's synthetic competence was upgraded at any floor of her IET.

Formation of the V's individual informational environment included web-based resources, e.g. the paper about Ravel's bolero (WikipediA, 2018). V was not restricted in copy-pasting some paragraphs from internet papers, but the citation ethics was strictly under control. We agree with Petrovic (2018) about the benefits of informal learning even in such reproductive form. Moreover, we consider this type of informal learning as an opportunity to decontextualize the classroom knowledge. However, the key-competence *ICT* attitude points (Appendix) must be taken into account in any copy-paste activities.

6 CONCLUDING WORDS

The Bulgarian secondary school theoretical framework adopts explicitly the key-competences from the European one (EU, 2006). However, the responsibility for development of any key-competence is spread all over the subjects and vice versa: any subject covers several key-competences. E.g. any of the themes Plane figures and solid bodies, Probability and Functions covers 7 of 8 key-competences (MON, 2015) (in fact there are 9 key-competences in Bulgarian educational legislation). Implicitly this means that a kind of synthetic competence is in the scope of the compulsory education, which is coherent with our standing point about one competence of synthetic type.

Further, the attitude as a component of the synthetic competence inherits the synthetic nature of the competence. This was clearly seen during our experimental teaching.

The concept of attitude is operationalized more comprehensive in Bulgarian legislation (MON, 2015) than the points we cite in the Appendix. However, the existing instruments to measure the student's attitude in quantitative way, which we know, do not correspond to the individual approach we applied. E.g., take the attitude point 3.1.iii and think about what it means to register 30% higher *respect for truth*. The common instrument for this purpose is an inquiry of the type given by Zakariya (2017, p. 82).

On the other hand, a qualitative evaluation is possible and we observed (indirectly) rather positive lift of V's attitude towards math. Taking into account that the V's math test results are related mainly to the knowledge and skills side of her learning process, we would like to think that the higher score she achieved at the end of the school year is also influenced by the positive attitude she get during the experimental teaching presented here.

ACKNOWLEDGEMENTS

The study is supported by the Educational and Research Program *Cherorizec Hrabar* of the Institute of Mathematica and Informatics, Bulgarian Academy of Sciences. The author thanks the reviewers for the comments.

REFERENCES

Blackweir, J. (2016). *Attitudes towards mathematics: Development and validation of an Attitudes towards mathematics*. Master thesis, Graduate School of Education, Australia.

EU (2006). Recommendation of the European parliament and of the council of 18 December 2006 on key competences for lifelong learning. Official Journal of the European Union 30.12.2006, L 394/15.

Ganchev, I. and Kuchinov, Y. (1996). Organizatsia i metodika na uroka po matematika. Sofia. Modul. (Organization and methodic of mathematics lesson) In Bulgarian.

Lazarov, B. (2011). Paper and Pencil versus ICT – Battlefield Geometry. In E-learning, distance education or ... The education of 21st century international conference. Sofia, Bulgaria 6-8 April 2011. Conference proceedings, p. 123-130.

Lazarov, B. (2012). Individualna obrazovatelna treaktoria – izsledvane na chasten sluchay. Matematika i informatika. No 3, 2012. p. 238-248. (Individual educational trajectory – a case study) In Bulgarian.

Lazarov, B. (2013A). Application of some cybernetic models in building individual educational trajectory. Information Models and Analyses. Vol. 2, No1, 2013, p. 90-99.

Lazarov, B. (2013B). Developing Synthetic Competence along Individual Educational Trajectory. In Proceedings of the International Conf. on Math. Edu. on Creativity & Giftedness (August 9–10, 2013). The Korean Society of Mathematical Education. p. 251-262.

Lazarov, B. (2018). Topic-oriented Upgrade of Subject-oriented Educational System. In Tarasenkova, N. (Eds). Current Status and Prospects of Mathematical Education: Monograph. ISBN 978-615-00-2441-7 In L. Kyba (A. Ed.). Budapest, Hungary: SCASPEE. p. 37-53

Lazarov, B. and Karakoleva, S. (2011). Using CAS syntax for education in mathematics. In *Vissheto obrazovanie v Bulgaria i strategia „Evropa 2020”*. (Bulgarian Higher Education and the Europe 2020 Strategy.) MVBU Pblsh. ISBN 978-954-9432-43-5, p. 914-921.

MON (2015). NAREDBA № 5 ot 30.11.2015 g. za obshtoobrazovatelnata podgotovka. DV, br. 95 ot 08.12.2015. (Bulgarian Ministry of Education. Regulation No 5 of general education) In Bulgarian.

Paskaleva, Z., Paskalev, G., and Alashka, M. (2013). Matematika 8. klas. Arhimed 2. (Mathematics for 8th grade) In Bulgarian.

Petrovic O. (2018). Digital Media's Alteration Mechanism for Informal Learning. In *Proceedings of the 10th International Conference on Computer Supported Education - Volume 2*: CSEDU, ISBN 978-989-758-291-2, p. 321-330. DOI: 10.5220/0006772303210330.

PISA (2016). https://www.businessinsider.com/pisa-worldwide-ranking-of-math-science-reading-skills-2016-12

PISA (2018). Directorate for Education and Skills Programme for International Student Assessment. Edu/Pisa/Gb(2018)4, 6 April 2018.

WikipediA (2018). https://en.wikipedia.org/wiki/Boléro.

YouTube (2016). Music by Maurice Ravel, choreography by Maurice Béjart, dancing Maya Plisetskaya. Bolero. https://www.youtube.com/watch?v=6vAvpUyFvrU.

Zakariya, Y.F. (2017). Development of Attitudes towards Mathematics Scale (ATMS) using Nigerian Data – Factor Analysis as a Determinant of Attitude Subcategories. International Journal of Progressive Education, Volume 13 Number 2. p. 74-84.

APPENDIX

The attitude component of Mathematics key-competence in EUROPEAN COMMISSION, 2018. ANNEX to the Proposal for a Council Recommendation on Key Competences for Lifelong Learning (page C 189/9) is defined as *A positive attitude in mathematics is based on the respect for truth and a willingness to look for reasons and to assess their validity*. (https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32018H0604(01)&rid=7)

Below is an extraction from the communication Key Competences in the Knowledge Based Society – A framework of eight key competences. European Commission (Sept. 2004). The quotations are taken from the column ATTITUDES of the table on pages 5-13. Here the key-competence number corresponds the source numbering.

3.1. Key-competence *Mathematical literacy*

i. overcoming 'fear of numbers';

ii. willingness to use numerical computation in order to solve problems in the course of day-to-day work and domestic life;
iii. respect for truth;
iv. willingness to look for reasons to support one's assertions;
v. willingness to accept or reject the opinions of others on the basis of valid (or invalid) reasons or proofs.

4. Key-competence *ICT* (*digital competence*)
   i. desire critically to assess information available;
   ii. awareness that the lower threshold to access information may need to be balanced by higher standards of ethics and taste – ability to distinguish what is 'accessible' from what is 'acceptable'.
5. Key-competence *Learning to learn*
   i. adaptability and flexibility;
   ii. self-motivation and confidence in one's capability to succeed;
   iii. a self-concept that upholds one's willingness to change and further develop competences;
   iv. sense of initiative (to learn);
   v. positive appreciation of learning as a life-enriching activity.
8. Key-competence *Cultural awareness*

   The term 'cultural awareness' comprises an appreciation of popular culture and general social mores, as well as the ability to appreciate literature, art, music and other forms of creative expression.