

# Robust Finite-time Position and Attitude Tracking of a Quadrotor UAV using Super-Twisting Control Algorithm with Linear Correction Terms

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**Keywords:** Quadrotor, Unmanned Aerial Vehicle, Position and Attitude Tracking, Finite-time Convergence, Super-Twisting Algorithm.

**Abstract:** This work investigates the problem of finite-time position and attitude trajectory of quadrotor unmanned aerial vehicle systems based on a modified second order sliding mode algorithm. The selected algorithm is a modified super-twisting with both nonlinear and linear correction terms. It ensures robustness against unknown dynamics and perturbations and allows fast finite-time convergence even when the trajectories of the considered system are far from the user-chosen switching surface. In addition, this algorithm is very attractive since it solves the major problems of the first and second order sliding mode, namely, the chattering phenomenon and the required unavailable information for practical implementation. To show the effectiveness of the used modified structure of the super-twisting algorithm, simulation results are presented for the considered quadrotor system.

## 1 INTRODUCTION

Sliding Mode Control (SMC) is known to be one of the most effective and powerful robust nonlinear techniques that attracts the community of automation researcher (Utkin et al., 1999). Indeed, SMC is well known for its three good features: insensitivity to a class of uncertainties, simplicity and finite-time convergence. This controller uses switching input signals to force the trajectories of the system to reach in finite-time the so-called sliding surface and then to move throughout this latter towards the equilibrium point. SMC has been tested in simulation and in real time on different nonlinear systems such as power systems (Kali et al., 2018a; Kali et al., 2019), robotic manipulator systems (Feng et al., 2002; Kali et al., 2015) and underactuated systems as Unmanned Aerial Vehicles (UAVs) (Runcharoon and Srichatrapimuk, 2013). Nevertheless, its real-time implementation suffers from the chattering phenomenon (Boiko and Fridman, 2005) caused by high switching signals. This phenomenon is the major disadvantage of SMC since it can cause several problems as bad performances and the degradation or/and deterioration of the moving mechanical parts.

In literature, several published works tried to re-

duce or eliminate this problem (Tseng and Chen, 2010; Lee et al., 2009; Kali et al., 2018b; Besnard et al., 2012). The most effective method is the proposed Second Order Sliding Mode (SOSM) (Levant, 2003). Unlike the discontinuous classical SMC, the SOSM control signals that fed into the system are continuous (Kali et al., 2017a; Kali et al., 2017b) since the the switching signals act on the derivative of the control inputs. However, its real-time implementation still limited due to the lack of required informations (measurement of the derivative of the selected sliding surface). This problem has been solved by the proposition of the Super-Twisting Algorithm (STA) (Benallegue et al., 2008; González-Hernández et al., 2017b; González-Hernández et al., 2017a; Ibarra and Castillo, 2017; Kali et al., 2018).

In this paper, the modified STA (structure with nonlinear and linear terms) will be designed and tested by simulations on a quadrotor UAV system. The choice of UAVs belongs to the fact that to the authors' best knowledge, unlike the standard STA, this modified structure has never been studied, and consequently, used for UAV systems. Moreover, control of flight robot systems is an attractive field of research since the number of applications where these systems are used keeps growing. Among the most developed

applications, we cite exploration, construction, visual inspection, transportation (Segales et al., 2016; Singh and Frazier, 2018)... In addition, UAVs belong to the class of nonlinear underactuated systems that suffer from uncertainties due to the variation of the inertia and mass that can happens during a transportation task (Yang and Xian, 2017) and external perturbations due to environmental changes as wind (Ceccarelli et al., 2007).

The rest of this article is organized as follows. In Section 2, the mathematical model of the considered quadrotor UAV system is described. In Section 3, the used stucture of STA is studied for the position and attitude tracking problem in the presence of uncertainties. Simulation is conducted in Section 4 on the considered system are provided to exhibit the effectiveness of the used STA structure. The last section concludes this article.

## 2 QUADROTOR UAV MODEL AND PRELIMINARIES

The quadrotor UAVs are aerial robotic systems that consist of four independent motors mounted on a rigid cross structure as shown in Fig. 1.

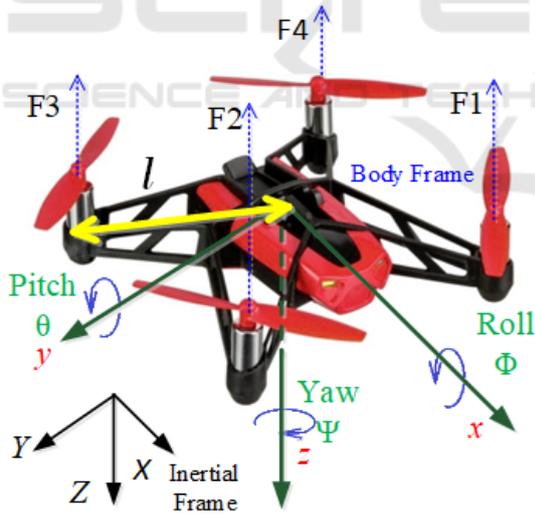


Figure 1: Quadrotor structure, forces, angles and frames.

### 2.1 Position and Attitude Dynamical Model

The mathematical model of most of the UAV systems is based on 6-Degrees Of Freedom (DOF)  $[x, y, z, \phi, \theta, \psi]^T \in R^6$ . This latter contains the position vector  $[x, y, z]^T \in R^3$  that includes the altitude  $z$

and the attitude or Euler angles vector  $[\phi, \theta, \psi]^T \in R^3$  with  $\phi$  represents the roll,  $\theta$  represents the pitch and  $\psi$  represents the yaw.

On the one hand, the position dynamic model is given as in (Wu et al., 2017) by:

$$\begin{aligned}\ddot{x} &= -\frac{k_{f_{tx}}}{m}\dot{x} + (\cos(\psi)\sin(\theta)\cos(\phi) + \sin(\psi)\sin(\phi))\frac{u_1}{m} + d_x \\ \ddot{y} &= -\frac{k_{f_{ty}}}{m}\dot{y} + (\sin(\psi)\sin(\theta)\cos(\phi) - \cos(\psi)\sin(\phi))\frac{u_1}{m} + d_y \\ \ddot{z} &= -\frac{k_{f_{tz}}}{m}\dot{z} - g + \cos(\theta)\cos(\phi)\frac{u_1}{m} + d_z\end{aligned}\quad (1)$$

where  $k_{f_{tx}}$ ,  $k_{f_{ty}}$  and  $k_{f_{tz}}$  are drag coefficients of translation,  $m$  denotes the quadrotor's mass,  $g$  denotes the constant of gravity,  $d_x$ ,  $d_y$  and  $d_z$  are uncertain functions and  $u_1$  is the vertical force.

On the other hand, the attitude dynamic model is given as in (Wu et al., 2017) by:

$$\begin{aligned}\ddot{\phi} &= \frac{1}{I_x}(-k_{f_{ax}}\dot{\phi}^2 + (I_y - I_z)\dot{\theta}\dot{\psi} - J_r w_r \dot{\theta} + u_2) + d_\phi \\ \ddot{\theta} &= \frac{1}{I_y}(-k_{f_{ay}}\dot{\theta}^2 + (I_z - I_x)\dot{\phi}\dot{\psi} + J_r w_r \dot{\phi} + u_3) + d_\theta \\ \ddot{\psi} &= \frac{1}{I_z}(-k_{f_{az}}\dot{\psi}^2 + (I_x - I_y)\dot{\phi}\dot{\theta} + u_4) + d_\psi\end{aligned}\quad (2)$$

where  $u_2$ ,  $u_3$  and  $u_4$  are respectively the roll, pitch and yaw torques,  $k_{f_{ax}}$ ,  $k_{f_{ay}}$  and  $k_{f_{az}}$  are the coefficients of the aerodynamic friction,  $I_x$ ,  $I_y$  and  $I_z$  denote the moments of inertia,  $J_r$  denotes the motor inertia,  $d_\phi$ ,  $d_\theta$  and  $d_\psi$  are the uncertain functions and  $w_r$  is the rotor speed that is related to the torques by the following equations:

$$\begin{aligned}u_1 &= b(w_1^2 + w_2^2 + w_3^2 + w_4^2) \\ u_2 &= bl(w_1^2 + w_4^2 - w_2^2 - w_3^2) \\ u_3 &= bl(w_1^2 + w_2^2 - w_3^2 - w_4^2) \\ u_4 &= c(w_1^2 + w_3^2 - w_2^2 - w_4^2) \\ w_r &= w_1 - w_2 + w_3 - w_4\end{aligned}\quad (3)$$

where  $c$ ,  $b$  and  $l$  represent respectively the drag coefficient, the thrust coefficient and the length of the moment arm.

### 2.2 Problem Formulation

As said before, the objective of this work is to design a robust nonlinear control technique that ensures a finite-time convergence of the 6-DOF vector  $[x, y, z, \phi, \theta, \psi]^T$  of the quadrotor system to the desired known trajectory vector  $[x_d, y_d, z_d, \phi_d, \theta_d, \psi_d]^T$  despite the presence of uncertainties and perturbations. In the subsequent section, the used controller will be derived based on the following assumptions:

- **Assumption 1:** The vectors  $[x, y, z, \phi, \theta, \psi]^T$  and  $[\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi}]^T$  are available for measurements.
- **Assumption 2:** The reference vector  $[x_d, y_d, z_d, \phi_d, \theta_d, \psi_d]^T$  and its time derivatives are known. Moreover, the desired Euler angles are limited to:

$$|\phi_d| < \pi/2,$$

$$|\theta_d| < \pi/2,$$

$$|\psi_d| < \pi.$$

- **Assumption 3:** The Euler angles are limited to:

$$|\phi| < \pi/2,$$

$$|\theta| < \pi/2,$$

$$|\psi| < \pi.$$

- **Assumption 4:** The first time derivative of uncertain functions  $d_j$  for  $j = x, y, z, \phi, \theta, \psi$  is bounded such as:

$$|\dot{d}_j| \leq \delta d_j$$

$$|\dot{d}_j| \leq \delta d_{1j} + \delta d_{2j} |S_j|$$

where  $\delta d_j$  is a positive constants,  $\delta d_{1j}$  is chosen to be equal to  $\delta d_j$ ,  $\delta d_{2j}$  is an arbitrary chosen positive constant and  $|S_j|$  is the selected switching surface for each trajectory that will be given in the design procedure.

### 3 MODIFIED SUPER-TWISTING ALGORITHM

In this section, the used STA structure for the finite-time both position and attitude trajectory tracking of uncertain quadrotor UAV systems will be designed in two steps. The first step consists on designing the controller for the outer position loop while the second step consists on generating the desired roll and pitch angles and on designing the inner attitude loop.

#### 3.1 Position Controller Design

The position dynamic model given in (1) can be rewritten as follows:

$$\ddot{\chi} = [\ddot{x}, \ddot{y}, \ddot{z}]^T = F(\dot{\chi}) + U_\chi + d_\chi \quad (4)$$

where  $F(\dot{\chi}) = \left[ -\frac{k_{f1x}}{m} \dot{x}, -\frac{k_{f1y}}{m} \dot{y}, -\frac{k_{f1z}}{m} \dot{z} \right]^T$  represents the known dynamics,  $d_\chi = [d_x, d_y, d_z]^T$  represents the vector of uncertainties and disturbances and  $U_\chi =$

$[u_x, u_y, u_z]^T$  with  $u_x$ ,  $u_y$  and  $u_z$  are virtual control inputs defined as follows:

$$\begin{aligned} u_x &= \frac{1}{m} (\cos(\psi) \sin(\theta) \cos(\phi) + \sin(\psi) \sin(\phi)) u_1 \\ u_y &= \frac{1}{m} (\sin(\psi) \sin(\theta) \cos(\phi) - \cos(\psi) \sin(\phi)) u_1 \\ u_z &= g - \frac{1}{m} \cos(\theta) \cos(\phi) u_1 \end{aligned} \quad (5)$$

Now, let us select the sliding surface as follows:

$$\begin{aligned} S_\chi &= \dot{e}_\chi + \lambda_\chi e_\chi \\ &= \dot{\chi} - \dot{\chi}_d + \lambda_\chi (\chi - \chi_d) \end{aligned} \quad (6)$$

where  $e_\chi \in R^3$  is the position tracking error vector with  $\chi_d \in R^3$  is the vector of desired positions such as  $\chi_{1d} = x_d$ ,  $\chi_{2d} = y_d$  and  $\chi_{3d} = z_d$  and  $\lambda_\chi$  is a diagonal matrix with strictly positive elements. The aim of the SOSM is to ensure robustness, high precision and lower chattering. To this end, the following STA structure is selected:

$$\begin{cases} \dot{S}_\chi = -M_1 |S_\chi|^{0.5} \text{sign}(S_\chi) - M_2 S_\chi + \dot{\eta} \\ \dot{\eta} = -M_3 \text{sign}(S_\chi) - M_4 S_\chi \end{cases} \quad (7)$$

where  $|S_\chi|^{0.5} = \text{diag}(|S_{1\chi}|^{0.5}, |S_{2\chi}|^{0.5}, |S_{3\chi}|^{0.5})$ ,  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$  are  $(3 \times 3)$  diagonal matrices where the elements will be chosen to satisfy the stability of the closed-loop system and  $\text{sign}(S_\chi) = [\text{sign}(S_{1\chi}), \text{sign}(S_{2\chi}), \text{sign}(S_{3\chi})]^T$  with:

$$\text{sign}(S_{i\chi}) = \begin{cases} 1, & \text{if } S_{i\chi} > 0 \\ 0, & \text{if } S_{i\chi} = 0 \\ -1, & \text{if } S_{i\chi} < 0 \end{cases} \quad \text{for } i = 1, 2, 3 \quad (8)$$

Now, let us calculate  $\dot{S}_\chi$  based on (6) and using the nominal position dynamic model (4) as follows:

$$\begin{aligned} \dot{S}_\chi &= \ddot{e}_\chi + \lambda_\chi \dot{e}_\chi \\ &= \ddot{\chi} - \ddot{\chi}_d + \lambda_\chi \dot{e}_\chi \\ &= F(\dot{\chi}) + U_\chi - \ddot{\chi}_d + \lambda_\chi \dot{e}_\chi \end{aligned} \quad (9)$$

Therefore, solving (7) using (9) gives the following proposed improved super-twisting control algorithm:

$$\begin{aligned} U_\chi &= -F(\dot{\chi}) + \ddot{\chi}_d - \lambda_\chi \dot{e}_\chi - M_1 |S_\chi|^{0.5} \text{sign}(S_\chi) \\ &\quad - M_2 S_\chi - M_3 \int_0^t \text{sign}(S_\chi) dt - M_4 \int_0^t S_\chi dt \end{aligned} \quad (10)$$

Finally, the total thrust  $u_1$  can be obtained using the following formula (Zhao et al., 2015):

$$u_1 = m \sqrt{u_x^2 + u_y^2 + (u_z - g)^2} \quad (11)$$

**Theorem 3.1.** (Wang et al., 2018) Consider the position model of the quadrotor system given in (4), the

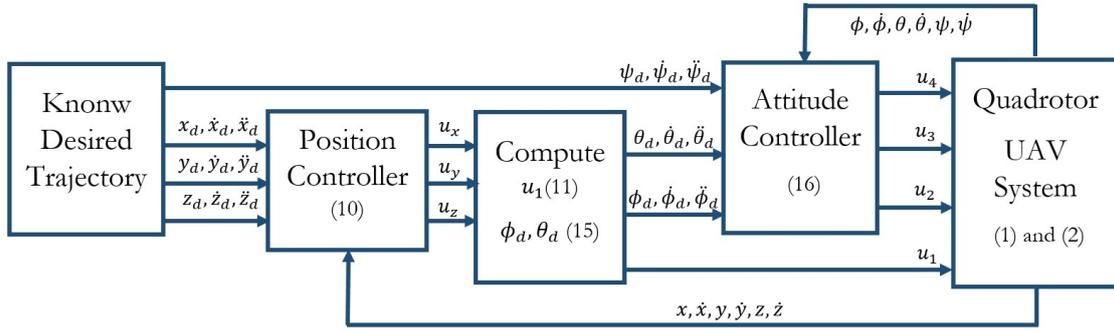


Figure 2: Block diagram of the closed-loop quadrotor UAV.

proposed controller in (11) ensures finite-time convergence if the diagonal elements for  $i = 1, 2, 3$  of the matrices  $M_1, M_2, M_3$  and  $M_4$  in (10) are chosen as follows:

$$\begin{aligned} M_{1i} &> 2\sqrt{\delta d_{1i}}, M_{2i} > \frac{1}{2}\sqrt{2\delta d_{2i}}, M_{3i} > \delta d_{1i}, \\ M_{4i} &> \frac{M_{1i}^2(2M_{2i} - \delta d_{2i}) + v_i(5M_{2i}^2 + 2\delta d_{2i})}{2v_i - M_{1i}^2}, \quad (12) \\ v_i &= \frac{1}{2}M_{1i}^2 + M_{3i} - \delta d_{1i} \end{aligned}$$

**Proof.** Refer to (Wang et al., 2018).

### 3.2 Attitude Controller Design

The same methodology used for the position tracking will be adopted in this part. First of all, let us rewrite the attitude dynamical model given in (2) as follows:

$$\dot{\Theta} = H(\dot{\Theta}) + GU_{\Theta} + d_{\Theta} \quad (13)$$

where  $\Theta = [\phi, \theta, \psi]^T$  represents the attitude state vector,  $G = \text{diag}\left(\frac{1}{I_x}, \frac{1}{I_y}, \frac{1}{I_z}\right)$  is the non-singular control matrix,  $d_{\Theta} = [d_{\phi}, d_{\theta}, d_{\psi}]^T$  is the vector of uncertainties and disturbances and  $U_{\Theta} = [u_2, u_3, u_4]^T$  and  $H(\dot{\Theta}) = [H_1(\dot{\Theta}), H_2(\dot{\Theta}), H_3(\dot{\Theta})]^T$  denotes the known nonlinear dynamics with:

$$\begin{aligned} H_1(\dot{\Theta}) &= \frac{1}{I_x} (-k_{f\alpha}\dot{\phi}^2 + (I_y - I_z)\dot{\theta}\dot{\psi} - J_r w_r \dot{\theta}) \\ H_2(\dot{\Theta}) &= \frac{1}{I_y} (-k_{f\alpha}\dot{\theta}^2 + (I_z - I_x)\dot{\phi}\dot{\psi} + J_r w_r \dot{\phi}) \quad (14) \\ H_3(\dot{\Theta}) &= \frac{1}{I_z} (-k_{f\alpha}\dot{\psi}^2 + (I_x - I_y)\dot{\phi}\dot{\theta}) \end{aligned}$$

The objective of this part is to ensure the fast converge to zero of the attitude tracking error  $e_{\Theta} = \Theta - \Theta_d$  where  $\Theta_d = [\phi_d, \theta_d, \psi_d]^T$  is the vector of the known desired trajectories. Here, the desired roll and pitch angles are generated from the virtual con-

trollers (Zhao et al., 2015) as follows:

$$\begin{aligned} \phi_d &= \arcsin\left(\frac{m}{u_1}(u_x \sin(\Theta_{3d}) - u_y \cos(\Theta_{3d}))\right) \\ \theta_d &= \arctan\left(\frac{1}{u_z + g}(u_x \cos(\Theta_{3d}) + u_y \sin(\Theta_{3d}))\right) \quad (15) \end{aligned}$$

**Theorem 3.2.** (Wang et al., 2018) Consider the attitude model of the quadrotor system given in (13), the proposed control law is given by:

$$\begin{aligned} U_{\Theta} &= -G^{-1}(H(\dot{\Theta}) - \ddot{\Theta}_d + \lambda_{\Theta}\dot{e}_{\Theta} + \zeta) \\ &\quad - G^{-1}(N_1|S_{\Theta}|^{0.5}\text{sign}(S_{\Theta}) + N_2 S_{\Theta}) \quad (16) \\ \zeta &= \int_0^t N_3 \text{sign}(S_{\Theta}) dt + N_4 \int_0^t S_{\Theta} dt \end{aligned}$$

where  $S_{\Theta} = \dot{e}_{\Theta} + \lambda_{\Theta}e_{\Theta}$  is the classical switching surface with  $\lambda_{\Theta} \in \mathbb{R}^{3 \times 3}$  is a diagonal positive matrix.

Moreover, The above controller ensures finite-time convergence if the gains of the diagonal positive matrices  $N_1, N_2, N_3$  and  $N_4$  are chosen for  $i = 1, 2, 3$  as follows:

$$\begin{aligned} N_{1i} &> 2\sqrt{\delta d_{1i}}, N_{2i} > \frac{1}{2}\sqrt{2\delta d_{2i}}, N_{3i} > \delta d_{1i}, \\ N_{4i} &> \frac{N_{1i}^2(2N_{2i} - \delta d_{2i}) + p_i(5N_{2i}^2 + 2\delta d_{2i})}{2p_i - N_{1i}^2}, \quad (17) \\ p_i &= \frac{1}{2}N_{1i}^2 + N_{3i} - \delta d_{1i} \end{aligned}$$

**Proof.** Refer to (Wang et al., 2018).

Finally, the architecture of the proposed control system is represented in Fig. 2.

## 4 NUMERICAL SIMULATION

In this work, numerical simulation is performed using MATLAB/Simulink software to validate the used improved super-twisting algorithm. The considered quadrotor is the parrot-rolling spider quadrotor (Mathworks, 2018) described by (1) and (2). The

used physical parameters given in Table 1 can be found in (Mathworks, 2018).

Table 1: Physical parameters of the quadrotor UAV.

Parameter	Value	Unit
Mass, $m$	0.068	Kg
Moment of inertia, $I_x$	$0.0686 \cdot 10^{-3}$	$\text{Kg.m}^2$
Moment of inertia, $I_y$	$0.0920 \cdot 10^{-3}$	$\text{Kg.m}^2$
Moment of inertia, $I_z$	$0.1366 \cdot 10^{-3}$	$\text{Kg.m}^2$
Motor inertia, $J_r$	$1.0209 \cdot 10^{-7}$	$\text{Kg.m}^2$
Gravity, $g$	9.81	$\text{m/s}^2$

The simulation is performed such as the initial 6-DOF vector is chosen to be  $[0, 0, 0, 0, 0, 0]$ . In addition, the desired scenario is given by choosing the following desired references:

$$\begin{aligned} x_d(t) &= 2 \sin(0.5t) \text{ m} \\ y_d(t) &= 2 \cos(0.5t) \text{ m} \\ z_d(t) &= 1 \text{ m} \\ \psi_d(t) &= 0 \text{ rad} \end{aligned}$$

In addition, the disturbances are introduced at time  $t = 8 \text{ s}$ . The chosen profile for the disturbances on the position is given for  $j = x, y, z$  by:

$$d_j = \begin{cases} 0, & \text{if } t < 8 \\ 0.5 \sin(2\pi t), & \text{if } t \geq 8 \end{cases}$$

while the disturbances on the Euler angles are given for  $j = \phi, \theta, \psi$  by:

$$d_j = \begin{cases} 0, & \text{if } t < 8 \\ 0.2 \sin(2\pi t), & \text{if } t \geq 8 \end{cases}$$

Moreover, for this scenario, the chosen sliding surface and improved super-twisting gains are given in Table 2.

Table 2: Proposed controller gains.

Gains	Value
$\lambda_\chi = \text{diag}(\lambda_{\chi 1}, \lambda_{\chi 2}, \lambda_{\chi 3})$	$\text{diag}(5, 5, 5)$
$M_1 = \text{diag}(M_{11}, M_{12}, M_{13})$	$\text{diag}(7.5, 7.5, 7.5)$
$M_2 = \text{diag}(M_{21}, M_{22}, M_{23})$	$\text{diag}(6, 10, 10)$
$M_3 = \text{diag}(M_{31}, M_{32}, M_{33})$	$\text{diag}(6, 6, 6)$
$M_4 = \text{diag}(M_{41}, M_{42}, M_{43})$	$\text{diag}(3, 3, 3)$
$\lambda_\Theta = \text{diag}(\lambda_{\Theta 1}, \lambda_{\Theta 2}, \lambda_{\Theta 3})$	$\text{diag}(20, 20, 20)$
$N_1 = \text{diag}(N_{11}, N_{12}, N_{13})$	$\text{diag}(5, 7, 10)$
$N_2 = \text{diag}(N_{21}, N_{22}, N_{23})$	$\text{diag}(15, 20, 2)$
$N_3 = \text{diag}(N_{31}, N_{32}, N_{33})$	$\text{diag}(4, 5.5, 5.5)$
$N_4 = \text{diag}(N_{41}, N_{42}, N_{43})$	$\text{diag}(10, 17, 1)$

The results obtained are given in figures 3-6. The used nonlinear method ensures the high accuracy convergence of the position and attitude trajectories to their desired known references in finite-time as depicted in figures 3 and 4. These good performances

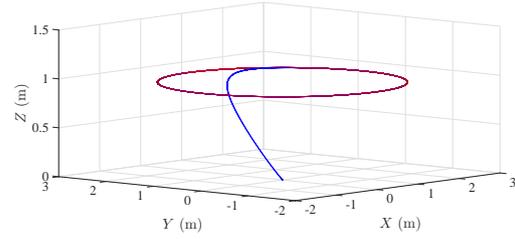


Figure 3: Finite-time 3D tracking.

are due to the ability of the proposed controller to reject the uncertain functions. Indeed, Fig. 5 confirms the good results since all the tracking error values are very small. Finally, Fig. 6 shows that the chattering is reduced in the control torque inputs. Moreover, the obtained values of the control torque inputs are in the range of acceptable torques for the considered parrot-rolling spider quadrotor system.

## 5 CONCLUSIONS

In this paper, the robust super-twisting control algorithm with nonlinear and linear correction terms has been designed and simulated on a quadrotor UAV for finite-time position and attitude tracking in the presence of unknown dynamics and perturbations. The chosen controller has never been used for underactuated systems such as the considered aerial robotic systems. Moreover, this algorithm allows fast finite-time convergence, reduces chattering and rejects the effects of the unmodelled and unknown dynamics and unexpected perturbations. The simulation has been carried out on parrot-rolling spider quadrotor. The results obtained showed good performances even in the presence of uncertainties. Future works will be conducted to implement in real-time the proposed controller and to make the convergence time faster during the sliding phase.

## ACKNOWLEDGEMENTS

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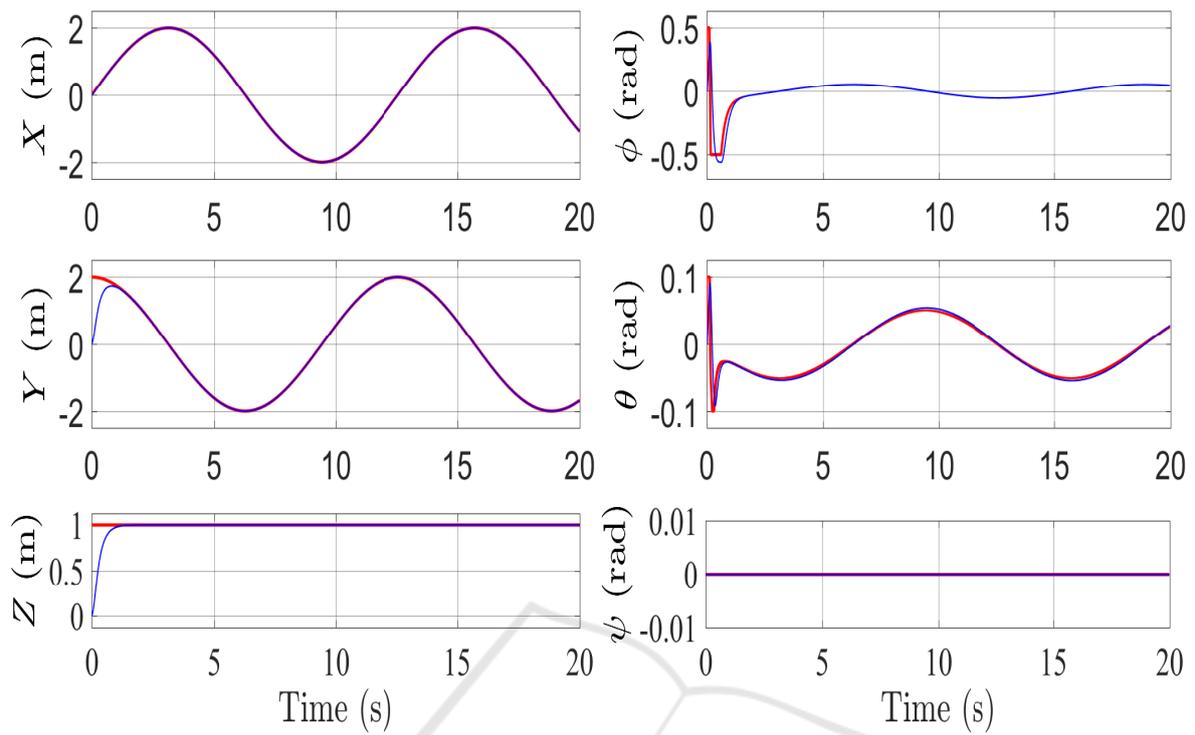


Figure 4: Finite-time position and attitude trajectory tracking.

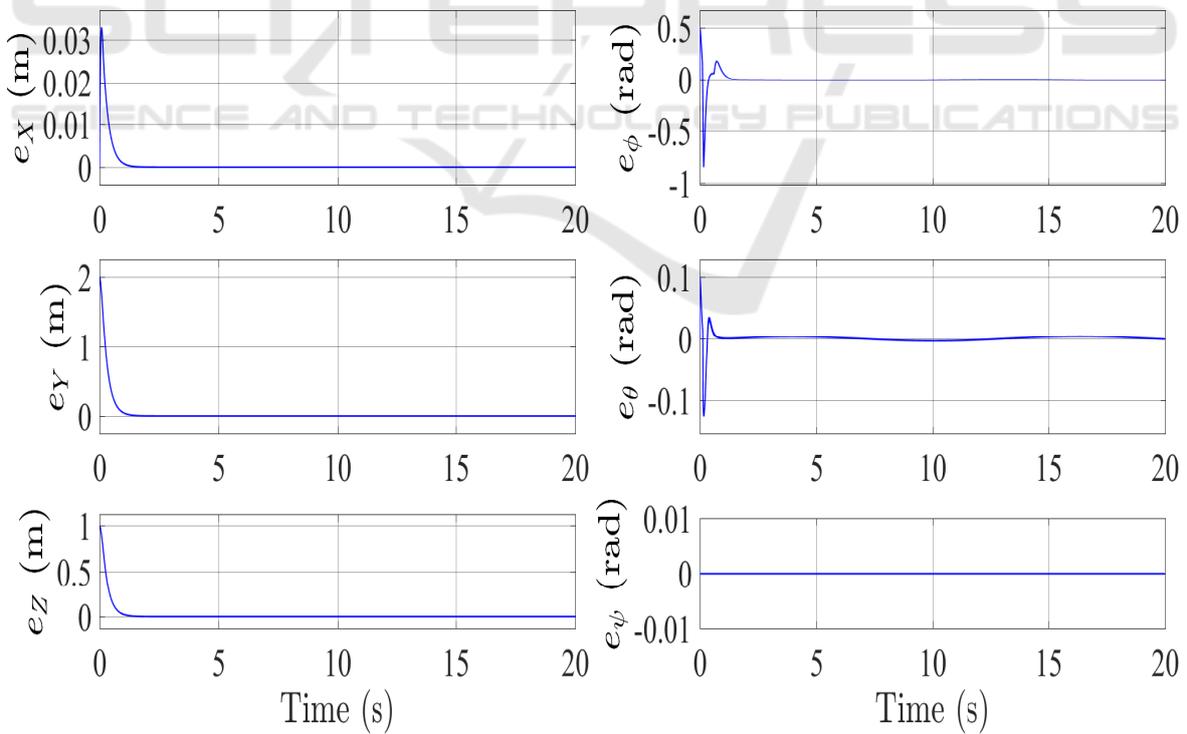


Figure 5: Finite-time position and attitude tracking error.

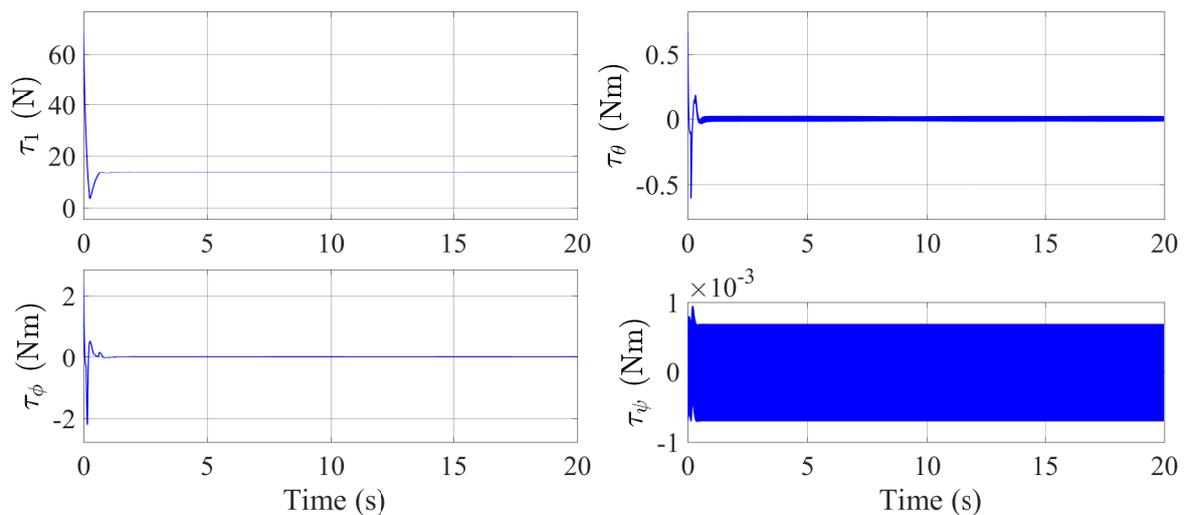


Figure 6: Computed control torque inputs.

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