# Automatic Algorithmic Complexity Determination Using Dynamic Program Analysis

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Abstract: Algorithm complexity is an important concept in computer science concerned with the efficiency of algorithms. Understanding and improving the performance of a software system is a major concern through the lifetime of the system especially in the maintenance and evolution phase of any software. Identifying certain performance related issues before they actually affect the deployed system is desirable and possible if developers know the algorithmic complexity of the methods from the software system. In many software projects, information related to algorithmic complexity is missing, thus it is hard for a developer to reason about the performance of the system for different input data sizes. The goal of this paper is to propose a novel method for automatically determining algorithmic complexity based on runtime measurements. We evaluate the proposed approach on synthetic data and actual runtime measurements of several algorithms in order to assess its potential and weaknesses.

# **1** INTRODUCTION

The performance of a software application is one of the most important aspect for any real life software. After the functional requirements are satisfied, software developers try to predict and improve the performance of the software in order to meet user expectations. Performance related activities include modification of the software in order to reduce the amount of internal storage used by the application, increase the execution speed by replacing algorithms or components and improve the system reliability and robustness (Chapin et al., 2001).

Simulation, profiling and measurements are performed in order to assess the performance of the system during maintenance (McCall et al., 1985), but using just measurements performed on a developer machine can be misleading and may not provide sufficient insight into the performance of the deployed system on possible different real life data load. Profiling is a valuable tool but, as argued in (W. Kernighan and J. Van Wyk, 1998), no benchmark result should ever be taken at face value.

Analysis of an algorithm, introduced by (Knuth, 1998) is concerned with the study of the efficiency of algorithms. Using algorithm analysis one can compare several algorithms for the same problem, based on the efficiency profile of each algorithm or can reason about the performance characteristics of a given algorithm for increasing size of the input data. In essence, studying efficiency means to predict the resources needed for executing a given algorithm for various inputs.

## 1.1 Motivation

While in case of library functions, especially standard library functions, complexity guarantees for the exposed methods exist, such information is generally omitted from the developer code and documentation. The main reason for this is the difficulty of deducing said information by the software developer. Analyzing even a simple algorithm may require a good understanding of combinatorics, probability theory and algebraic dexterity (Cormen et al., 2001). Automated tools, created based on the theoretical model presented in this paper can overcome this difficulty.

Knowledge about algorithmic complexity can complement existing software engineering practices for evaluating and improving the efficiency of a software system. The main advantage of knowing the complexity of a method is that it gives an insight into the performance of an operation for large input data sizes. Profiling and other measurement based techniques can not predict the performance characteristics of a method for other than the data load under which

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Czibula, I., Oneţ-Marian, Z. and Vida, R. Automatic Algorithmic Complexity Determination Using Dynamic Program Analysis. DOI: 10.5220/0007831801860193 In Proceedings of the 14th International Conference on Software Technologies (ICSOFT 2019), pages 186-193 ISBN: 978-989-758-379-7 Copyright © 2019 by SCITEPRESS – Science and Technology Publications, Lda. All rights reserved the measurements were performed.

Knowledge about algorithmic complexity is also beneficial for mitigating security issues in software systems. There is a well documented class of lowbandwidth denial of service (DoS) attacks in the literature that exploit algorithmic deficiencies in software systems (Crosby and Wallach, 2003). The first line of defence against such attacks would be to properly identify algorithmic complexity for the operations within the software system.

The main contributions of this paper is to propose and experimentally evaluate a deterministic approach to find the asymptotic algorithmic complexity for a method. To the best of our knowledge there is no other approach in the literature that automatically determines algorithmic method complexity based on runtime data analysis.

## 2 RELATED WORK

In this section we will present a short overview of some existing approaches from the literature related to algorithmic complexity.

The first approaches, for example (Le Métayer, 1988), (Wegbreit, 1975) and (Rosendahl, 1989), were based on source code analysis and the computation of mathematical expressions describing the exact number of steps performed by the algorithm. While such methods can compute the exact complexity bound of an algorithm, they were defined for functional programming languages and recursive functions.

More recent approaches can be applied for other programming languages as well, but many of them focus only on determining the complexity of specific parts of the source code (usually loops). Such approach is the hybrid (composed of static and dynamic analysis) method presented in (Demontiê et al., 2015) and the modular approach presented in (Brockschmidt et al., 2014).

Goldsmith et al. introduced in (Goldsmith et al., 2007) a model that explains the performance of a program as a feature of the input data. The focus is on presenting the performance profile (*empirical computational complexity*) of the entire program and not the identification of algorithmic complexity at the method level.

Benchmark is a library, written in C++, that supports benchmarking C++ code (Benchmark, 2016). Though it is not its main functionality, it also supports the empirical computation of asymptotic complexities. A comparison of our approach to the results provided by Benchmark are presented in Section 5. While they do not focus on determining the complexity of the source code directly, there are several approaches that try to find performance bugs (pieces of code that function correctly, but where functionality preserving changes can lead to substantial performance improvement) in the source code, for example: (Luo et al., 2017), (Olivo et al., 2015) and (Chen et al., 2018).

An approach using evolutionary search techniques for generating input data that trigger worst case complexity is presented in (Crosby and Wallach, 2003). Such approaches are complementary to our approach which assumes that testing data already exists.

## **3** METHODOLOGY

The most used representation for algorithm ana-lysis is the one proposed in (Knuth, 1998), the asymptotic representation, based on the *Big O* notation, a convenient way to deal with approximations introduced by Paul Bachmann in (Bachmann, 1894).

**Definition 1.** O(f(n)) denotes the set of g(n) such that there exist positive constants C and  $n_O$  with  $|g(n)| \le C * f(n)$  for all  $n \ge n_0$ .

If we denote by T(n) the number of steps performed by a given algorithm (*n* is the size of the input data), then the problem of identifying the algorithmic complexity becomes finding a function f(n) such that  $T(n) \in O(f(n))$ . The basic idea is to find a function f(n) that provides an asymptotic upper bound for the number of steps that is performed by the algorithm.

When analyzing algorithmic complexity, we are not differentiating between functions like  $f(n) = 2n^2 + 7$  and  $f(n) = 8n^2 + 2n + 1$  the complexity in both cases will be  $T(n) \in O(n^2)$ . The result of algorithm analysis is an approximation indicating the *order of growth* of the running time with respect to the input data size (Cormen et al., 2001). While the complexity function can be any function, there is a set of well known and widely used functions in the literature used to communicate complexity guarantees. In this paper we use the functions from Table 1, but the set can be extended without loosing the applicability of the proposed method. The considered functions represent complexity classes that appear frequently for real life software systems (Weiss, 2012).

In conclusion, the problem of identifying algorithmic complexity becomes selecting the most appropriate function f(n) from a predefined set, such that f(n)best describes the order of growth of the running time of the analyzed method for increasing input data sizes.

In this paper we introduce an automated approach for identifying algorithmic complexity for a method in a software system based on runtime measurements. The basic idea is to measure the execution time of the analyzed method for different input sizes and use these measurements to identify the best fitting complexity function. The proposed approach consists of three steps described in the following sections.

## **3.1** Step 1 - Data Collection

Our approach requires multiple executions of the analyzed method with different input data and the recording of the elapsed time for each execution. For this we instrument the analyzed methods and implement code that executes the target methods for different inputs. Using Java profilers we were able to slightly modify the entry and exit points of the targeted methods in order to extract the information we needed, such as execution time and input parameters. It is important to note, that although these changes did impact the execution a bit, they did not alter the original flow of instructions in any way. This approach was chosen since the profiler we used was not only lightweight, adding little overhead to the original application, but it was also highly reusable, being compatible with any method we needed to analyze.

The result of the measurement step for a given method *mtd* in a software system is a list of pairs  $M_{mtd} = \{(t_i, n_i)\}_{i=1}^m$  where *m* is the number of measurements performed for the method *mtd*,  $n_i$  is the size of the input data for the *i*-th measurement and  $t_i$  measures the elapsed time for running the analyzed method with an input data of size  $n_i$ .

## 3.2 Step 2 - Data Fitting

Given a set of measurements  $M_{mtd} = \{(t_i, n_i)\}_{i=1}^m$  and a function f(n) from Table 1, we want to identify the best coefficients  $c_1, c_2$ , such that the function fits the actual measured data.

For example, for  $f(n) = c_1 n \cdot log_2(n) + c_2$  we try to find  $c_1, c_2$  such that  $t_i \approx c_1 n_i \cdot log_2(n_i) + c_2$  for every measurement pair. For this purpose we use the nonlinear least square data fitting method (Hansen et al., 2012), where we need to solve:

$$\underset{c_1,c_2}{\text{minimize}} f(c_1,c_2) = \sum_{i=1}^{m} (t_i - (c_1 n_i \cdot \log_2(n_i) + c_2))^2$$

For determining the best values for the coefficients  $c_1$  and  $c_2$ , we used the *scipy.optimize* library from Python, more exactly the *curve\_fit* function from this library (Scipy, 2019). Implementations for the curve fitting algorithm are available in Matlab and other programming languages and libraries as well.

In the definition of the *Big O* notation, the only requirement regarding the constant *C* is that it should be positive, but no upper limit is set, since the inequality from the definition should be true for all values of  $n \ge n_0$ . However, in our approach the value *n* (i.e., the size of the input data) is finite, it has a maximum value, which means that the values of the constants  $c_1, c_2$  should also be restricted. If we consider the function  $f(n) = c_1 log_2(n) + c_2$ , and the maximum value of n (the maximum input size in the set  $M_{mtd}$ ) is 100000,  $log_2(100000)$  is approximately 17. By allowing the value of  $c_1$  to be 17, we actually transform the function into  $log_2^2(n)$ .

In order to avoid the problem presented above, we introduce an algorithm to automatically identify the upper bounds for the coefficients. We restrict the values of  $c_1$  and  $c_2$  to be in an interval  $[b_l, b_u]$ . The value of the lower bound,  $b_l$ , is set to 0.1 for every function. The value of the upper bound,  $b_{\mu}$  is computed separately for every function f(n) from Table 1 and for every set of measurements,  $M_{mtd}$ . For computing the bounds, we searched for the maximum value of n in  $M_{mtd}$ , denoted by  $n_{max}$ . We also considered the functions from Table 1 with  $c_1 = 1$  and  $c_2 = 0$ , ordered increasingly by their value for  $n_{max}$ . For every function f(n), we considered the previous function,  $f_{prev}(n)$  (we added the constant function, f(n) = 1 as well, to have a previous for every function). The value of the upper bound is computed as  $b_u = p * (f(n_{max}) - f_{prev}(n_{max}))$ , where p is a value between 0 and 1, denoting the percent of the difference we want to consider. In all our experiments presented below, the value of p = 0.05 was used. If the value  $b_u$  is less than 0.1 (the lower bound) we set  $b_u$  to be 0.2. Table 2 contains the bounds computed for the functions from Table 1 for  $n_{max}$  equal to 10<sup>4</sup>. Since we did not present actual data sets yet, this value was taken just to provide an example for the bounds. In Table 2 only the upper bounds for the functions are given.

# 3.3 Step 3 - Select the Best Matching Complexity Function

For a set of measurements  $M_{mtd} = \{(t_i, n_i)\}_{i=1}^m$ , and for every function from Table 1 we computed at Step 2 the parameters  $c_1$ ,  $c_2$  such that they represent the best fit for the given function.

The aim of this step is to choose one single function that is the most accurate description of the measurement data and consequently identify the complexity class that the analyzed method belongs to. In order to determine the best matching function, we compute the RMSE (root mean square error) for every function

Name	Function	Complexity
		class
F1	$c_1 \cdot log_2(log_2(n)) + c_2$	$log_2(log_2(n))$
F2	$c_1 \cdot \sqrt{\log_2(n)} + c_2$	$\sqrt{log_2(n)}$
F3	$c_1 \cdot log_2(n) + c_2$	$log_2(n)$
F4	$c_1 \cdot log_2^2(n) + c_2$	$log_2^2(n)$
F5	$c_1 \cdot n + c_2$	п
F6	$c_1 \cdot nlog_2(n) + c_2$	$nlog_2(n)$
F7	$c_1 \cdot n^2 + c_2$	$n^2$
F8	$c_1 \cdot n^2 log_2(n) + c_2$	$n^2 log_2(n)$
F9	$c_1 \cdot n^3 + c_2$	$n^3$
F10	$c_1 \cdot n^4 + c_2$	$n^4$

Table 1: Complexity classes considered for the experimental evaluation.

Table 2: Examples of bounds for a data set with the maximum input data size  $10^4$ .

Func.	Upper Bound	Func.	Upper bound
F1	0.2	F6	6143.856
F2	0.132	F7	$4.99 \cdot 10^{6}$
F3	0.478	F8	$6.14 \cdot 10^{7}$
F4	8.164	F9	$4.99 \cdot 10^{10}$
F5	491.172	F10	$5 \cdot 10^{14}$

*F* from Table 1 considering the values for  $c_1$  and  $c_2$  from Step 2. We pick as the complexity class best describing our data, the function *F* with the minimum RMSE.

# 4 COMPUTATIONAL EXPERIMENTS

In order to assess the effectiveness of the proposed approach, we will perform a series of experiments on synthetic data and actual measurements for different algorithms. The experiments were chosen based on the literature review, existing approaches in the literature use similar test systems. In the first experiment we exemplify the potential of our approach using synthetic measurement data. The second experiment is performed on a small code base with various sorting methods. Similar experiments where performed in the literature in (Wegbreit, 1975) in order to evaluate the effectiveness of their proposed approaches. The scope of the last experiment is to illustrate the potential of the proposed approach beyond identifying the runtime complexity of a method.

Table 5. Theden functions used to generate data sets.			
Name	Function $f(n)$	Complexity	
		class	
HF1	$5n^2 + 20$	$n^2$	
HF2	$3n^2 + 7n + 20$	$n^2$	
HF3	$log_2(n) + 7$	$log_2(n)$	
HF4	$log_2(n) + \sqrt{log_2(n)} +$	$log_2(n)$	
	$log_2(log_2(n))$		
HF5	$4log_{2}^{2}(n)+$	$log_2^2(n)$	
	$11log_2(n) + 25$		
HF6	$n^2 log_2(n) + 12n^2$	$n^2 log_2(n)$	
HF7	$9n + 15\sqrt{n} + 7$	п	
HF8	$25n^3 + 2n^2 + 500n + 4$	$n^3$	
HF9	$n^2 + 500nlog_2(n)$	$n^2$	
HF10	$20nlog_2(n) + 100n + 3$	$nlog_2(n)$	

#### Table 3: Hidden functions used to generate data sets.

## 4.1 Synthetic Data

The aim of the first experiment is to illustrate the proposed method, to verify its potential and evaluate possible limitations. We generate multiple sets of synthetic measurement data based on different mathematical functions. The assumption is that the number of steps performed by a given method can be described with a function. The set of functions used for these experiments and the complexity class they belong to is presented in Table 3. We used 10 different functions in order to simulate multiple possible running scenarios. From now on we will call these functions *hidden functions*, based on the idea that, while they describe the number of steps performed by an algorithm, in general, they are not known.

We generate a separate data set for every hidden function. Each data set contains 50 pairs of values [n, f(n)] where n is ranging from 100 to 10 million, evenly distributed in the mentioned interval. Intuitively every function corresponds to a method in a software system and every generated pair corresponds to a measurement (execute the method and collect input data and elapsed time).

Using the generated data, we try to predict the runtime complexity of every method, the input data for every experiment is one data set and the output is a function representing the complexity of the associated method.

The first step is to compute the coefficients for every considered complexity class from Table 1 based on the points generated for the hidden functions from Table 3. For this we used the method described in Section 3.2.

The next step is to compute the root mean squared error (*RMSE*) between the data and every considered complexity function.

The last step is to select the complexity function that best describes the analyzed data set, basically we pick the one with the smallest RMSE.

We performed the experiment for every data set and our approach is able to correctly identify the complexity class for every considered hidden function from Table 3.

#### 4.2 Sorting Methods Case Studies

The aim of this case study is to classify algorithms into different complexity classes based on measurement data. We measured (using the methodology from Section 3.1) the running time for various sorting methods in order to automatically determine their complexity. We choose this experiment as other approaches in the literature related to the topic of algorithmic complexity use as a case study similar examples.

The analyzed project includes various sorting methods: insertion sort, selection sort, merge sort, quicksort, bubble sort, heapsort and code that executes those methods for different lists of numbers.

#### 4.2.1 Collecting Measurements

The code will invoke each sorting method for arrays sorted in ascending order, descending order and with elements in random order, varying the length of the array for each invocation. We have chosen these types of input array orderings, because, for many sorting algorithms, they are connected to the best, worst and average case runtime complexity of the algorithm.

We generated the input arrays of numbers in ascending, descending and random order. As presented above we have considered 6 different sorting algorithms, but for quicksort we considered two different implementations: one in which the first element of the arrays is always chosen as the pivot (this version will be called quicksort first), and one where the middle element was always chosen as the pivot (quicksort *middle*). The expected correct results for all sorting algorithms and the 3 input array orderings are presented in Table 4, where asc. means ascending, desc. means descending and rnd. means random.

We run the experiments on two regular laptops with different hardware specifications. A first group of data sets were created on one laptop, denoted by  $L_1$ , where for each of the above mentioned 7 sorting algorithms (considering both versions of quicksort) and for each of the three input array orderings (ascending, descending and random) we have created four data sets. This gives us a total of 84  $L_1$  data sets.

A second group of data sets were created on a second laptop, denoted by  $L_2$ , where for each of the sort-

Table 4: Correct complexity classes for the considered algorithms and input array orderings.

Algorithm	Asc.	Desc.	Rnd.
insertion sort	п	$n^2$	$n^2$
selection sort	$n^2$	$n^2$	$n^2$
merge sort	$nlog_2(n)$	$nlog_2(n)$	$nlog_2(n)$
quicksort first	$n^2$	$n^2$	$nlog_2(n)$
quicksort mid.	$nlog_2(n)$	$nlog_2(n)$	$nlog_2(n)$
bubble sort	п	$n^2$	$n^2$
heapsort	$nlog_2(n)$	$nlog_2(n)$	$nlog_2(n)$

ing algorithms (for quicksort one single version was considered, quicksort first) and for each of the input array orderings, we have created one data set. This gives us a total of  $18 L_2$  data sets.

In order to test the generality of our approach, we have created data sets containing measurements for C++ code as well. For these data sets, the measurements were performed using the Benchmark library (Benchmark, 2016). For each of the 7 sorting algorithms, and for each of the three input array orderings we have created two data sets, a short one and a long one (the exact number of points is given below). For this group, we have a total of 42 data sets and we call it the BM data sets.

Each of the 144 data sets contains between 31 and 37 pairs of measurements representing the size of the array and the run-time in nanoseconds. The set of 37 different array sizes is  $S = \{s \in \mathbb{N} | s = i \cdot 10^j, s < =$  $2 \cdot 10^6, i \in \mathbb{N} \cap [1,9], j \in \mathbb{N} \cap [2,6]$ . The exact number of measurements for every sorting algorithm and group is presented in Table 5. The data sets with less than 37 points do not contain measurements for the last points of the above list because running the corresponding algorithms would have taken too much time (for example, in case of *bubble sort* in group  $L_1$ , the last data point is for an array of length 500000).

Table 5: Number of measurements for data sets.

Group	Sorting method	Number of points
	insertion sort	37
	selection sort	37
	merge sort	37
1.	quicksort first	32
	quicksort middle	32
	bubble sort	32
	heapsort	37
$L_2$	all data sets	36
BM	short data sets	31
	long data sets	37

# 4.2.2 Determining the Complexity for Sorting Data

We have performed an experiment similar to the one presented in Section 4.1: for every data set we determined the complexity class which is the best match for the points from the data set. We have considered the 10 complexity classes from Table 1.

From the 144 data sets, our algorithm returned the correct complexity class (i.e., the ones from Table 4) for 137 data set, having an accuracy of 95%.

Considering these results we can conclude that our approach can identify the complexity class based on runtime measurement data with a good accuracy.

### 4.2.3 Data Set Size Reduction

The next step of this experiment is to verify if the obtained accuracy is maintained even if we reduce the number of recorded measurement samples. From every data set we have randomly removed points: first we removed one point randomly and run our approach to determine the complexity class to which the remaining points belong. Then, starting from the original data set, we removed two points and determined the complexity class for the remaining points. And so on, until only 5 points remained. We repeated each experiment 100 times and counted how many times the correct result was obtained, which is the accuracy for the given data set.

For every sorting algorithm and input array ordering (ascending, descending, random), we computed the average accuracy over the 7 data sets (6 in case of quicksort middle) and two of these are presented in Figure 1. The accuracy is computed for data sets with the same number of points left.

Due to lack of space, Figure 1 does not contain all sorting algorithms. We did not include the results for quicksort middle, heap sort, selection sort and merge sort because for these algorithms the accuracy is almost 100% for each case. We did not include bubble sort either, which is similar to insertion sort.

The two sorting algorithms from Figure 1 contain lower accuracy values for some array orderings. In case of insertion sort, descending and random array have almost perfect accuracy (above 98% for every case), but for ascending arrays the accuracy is between 83% and 53%. The reason for these decreased values is that for the two ascending *BM* data sets our approach did not find the correct complexity class for the initial data sets and it has an accuracy of 0 (for these two data sets) when we start removing points.

Quicksort first is another sorting algorithm for which our approach could not find the correct complexity class in every case and this is visible on Figure 1 as well. For quicksort first, an incorrect result was returned for the ascending and descending array from the  $L_2$  group, and for these two data sets the accuracy is constantly 0 when we remove elements.

As expected, the quality and quantity of the measurements influence the accuracy of our approach, but the results are still promising and further work will be done in order to identify bottlenecks.

## 4.3 Map Case Study

The aim of this case study is to show the potential of the proposed approach to identify non-obvious performance related problems in real life software systems.

The Map (Dictionary) is a widely used data type described as an unsorted set of key-value elements where the keys are unique. For this discussion we will refer to the *HashMap* implementation available in the *Java* programming language.

Given an implementation similar with the one from Figure 2 an average software developer would assume an algorithmic runtime complexity n for *tar-getMethod* where n is the number of keys in the Map. For most use cases this assumption is true. However, we managed to write specially crafted code for which, based on runtime measurements, our approach returned the complexity classes  $n^2$  and  $nlog_2n$ , which are worse than developers' intuition.

While the results may look surprising or erroneous, they are in fact correct. In order to get a complexity class of  $n^2$ , we need to make sure that every key has the same hash code, so they will all be added into the same bucket. In this way finding one key will have a complexity of n.

The  $nlog_2n$  complexity is obtained when the keys are comparable, because in this situation in the Java implementation of *HashMap* the individual buckets will switch to a balanced tree representation whenever the amount of elements within exceeds a threshold. Assuming that all elements have the same hash code, searching for a key will have a complexity of  $log_2n$ . After a closer look into the *HashMap* documentation, specifically this change (OpenJDK, 2017), we find out about this lesser known behaviour.

Our approach can be used to develop automated tools that identify performance related problems and to indicate possible security issues that can be exploited by malicious users or systems that try to exploit algorithmic deficiencies. While in this experiment we created special code to reproduce the performance issue, measurement data can be collected by other means (Section 3.1).



(a) Insertion sort

(b) Quicksort first

Figure 1: Average accuracy over the sorting data sets after the removal of points.

```
public void targetMethod(HashMap<K, Integer> map) {
   for(K key : map.keySet()) {
      Integer value = map.get(key);
      // ...
}
```

```
}
```

Figure 2: HashMap data collection function.

# 5 DISCUSSION AND COMPARISON TO RELATED WORK

In Section 4.1 we illustrated experimentally that our approach is able to correctly identify the runtime complexity even from a small number of measurement samples.

Input and output blocking instructions, network communication as well as multithreaded operations are not specifically handled with our approach, so at first glance one might see this as a problem. However, this is fine considering that our analysis would be performed in a test, where one could mock blocking instructions. Furthermore, even if it was not possible to eliminate these aspects from our analysis, we would still want to be able to measure the performance of the rest of the code.

The running time may not reflect the number of steps performed by the algorithm. For practical reasons the actual runtime is more important, and with the aid of other tools such as profilers, we can identify the factors that influenced said execution durations.

The most similar approach to the one proposed in this paper is Benchmark (Benchmark, 2016), but it can only be used for computing the complexity of C++ code. However, since it is an open source library, we implemented their method in Python to compare it with our proposed approach. We have used two settings for the experiments: one in which all the complexity classes considered for our approach (the ones from Table 1) were considered and one experiment in which only those six complexity classes were considered, which are used in the Benchmark library as well (this included the constant complexity class as well, which is not used in our approach).

For the first setting, the Benchmark implementation found the correct complexity class for 55 out of 144 data set, an accuracy of 38%. For the second setting, the results were better, 83 correctly classified data sets, resulting in an accuracy of 58%. The accuracy for our approach on these data sets, was 95%, which is a lot better. Moreover, for the data set for which our approach did not find the correct complexity class, neither did the benchmark implementation.

However, Benchmark was developed for C++ code, so an explanation for the poor performance can be the large number of data sets (102 out of 144) containing measurements for Java code. In order to investigate this theory, we measured the accuracy separately for the C++ data sets (generated using Benchmark) and the Java data sets. The accuracies are presented in Table 6.

Table 6: Accuracies for C++ and Java data sets for our approach and the Benckmark implementations.

Method	C++ data set	Java data set
Benchmark 10	76%	23 %
complexity classes		
Benchmark 6	81 %	48%
complexity classes		
Our approach	90%	97 %

From Table 6 we can see that considering only the data sets generated for C++ code, the accuracy of the Benchmark implementation is approximately twice as high as for the Java data sets. This suggests that determining the complexity class for Java code is more complicated than determining it for C++ code and more complex methods are needed for it.

Other approaches in the literature focus on a specific type of algorithms (such as (Zimmermann and Zimmermann, 1989) dealing with divide and conquer algorithms) or specific parts of the code (such as loops in (Demontiê et al., 2015)). Our approach is more general, it is not constrained by the type of algorithm used in the analyzed method. We focus on an entire method that has the additional benefit that the instrumented code used to collect runtime data is less costly since our only concern is the execution time of the method.

# 6 CONCLUSIONS AND FUTURE WORK

In this paper we have introduced an approach for automatically determining the algorithmic complexity of a method from a software system, using runtime measurements. The results of the experimental evaluation show the potential of our approach. Although the approach has a good accuracy, further improvement is possible by analyzing the particularities of the misclassified examples.

The next step would be automatizing this whole process and making it readily available to developers. The research and experiments described in this paper serve as solid groundwork for creating such a tool that allows for real time algorithm complexity verification. The need for this functionality becomes clear when one considers the benefits gained, such as the ability to easily identify potential performance or security misconceptions developers might have when writing the code. The exact manner in which said tool might function still needs analysis and experimentation, but a possible form might be akin to unit tests that are executed within a continuous integration environment.

## REFERENCES

Bachmann, P. (1894). Die Analytische Zahlentheorie.

- Benchmark (2016). Benchmark library. https://github.com/google/benchmark.
- Brockschmidt, M., Emmes, F., Falke, S., Fuhs, C., and Giesl, J. (2014). Alternating runtime and size complexity analysis of integer programs. In *International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, pages 140–155. Springer.
- Chapin, N., Hale, J. E., Kham, K. M., Ramil, J. F., and Tan, W.-G. (2001). Types of software evolution and software maintenance. *Journal of Software Maintenance*, 13(1):3–30.
- Chen, Z., Chen, B., Xiao, L., Wang, X., Chen, L., Liu, Y., and Xu, B. (2018). Speedoo: prioritizing performance optimization opportunities. In 2018 IEEE/ACM 40th

International Conference on Software Engineering (ICSE), pages 811–821. IEEE.

- Cormen, T. H., Stein, C., Rivest, R. L., and Leiserson, C. E. (2001). *Introduction to Algorithms*. McGraw-Hill Higher Education, 2nd edition.
- Crosby, S. A. and Wallach, D. S. (2003). Denial of service via algorithmic complexity attacks. In *Proceedings of the 12th Conference on USENIX Security Symposium Volume 12*, SSYM'03, pages 3–3, Berkeley, CA, USA. USENIX Association.
- Demontiê, F., Cezar, J., Bigonha, M., Campos, F., and Magno Quintão Pereira, F. (2015). Automatic inference of loop complexity through polynomial interpolation. In Pardo, A. and Swierstra, S. D., editors, *Programming Languages*, pages 1–15, Cham. Springer International Publishing.
- Goldsmith, S. F., Aiken, A. S., and Wilkerson, D. S. (2007). Measuring empirical computational complexity. In Proceedings of the the 6th Joint Meeting of the European Software Engineering Conference and the ACM SIGSOFT Symposium on The Foundations of Software Engineering, ESEC-FSE '07, pages 395–404, New York, NY, USA. ACM.
- Hansen, P., Pereyra, V., and Scherer, G. (2012). *Least Squares Data Fitting with Applications*. Least Squares Data Fitting with Applications. Johns Hopkins University Press.
- Knuth, D. E. (1998). The Art of Computer Programming, Volume 3: (2Nd Ed.) Sorting and Searching. Addison Wesley Longman Publishing Co., Inc., Redwood City, CA, USA.
- Le Métayer, D. (1988). Ace: An automatic complexity evaluator. ACM Trans. Program. Lang. Syst., 10:248–266.
- Luo, Q., Nair, A., Grechanik, M., and Poshyvanyk, D. (2017). Forepost: Finding performance problems automatically with feedback-directed learning software testing. *Empirical Software Engineering*, 22(1):6–56.
- McCall, J. A., Herndon, M. A., Osborne, W. M., and States., U. (1985). Software maintenance management [microform] / James A. McCall and Mary A. Herndon, Wilma M. Osborne. U.S. Dept. of Commerce, National Bureau of Standards.
- Olivo, O., Dillig, I., and Lin, C. (2015). Static detection of asymptotic performance bugs in collection traversals. In ACM SIGPLAN Notices, volume 50, pages 369– 378. ACM.
- OpenJDK (2017). Hashmap implementation change. https://openjdk.java.net/jeps/180.
- Rosendahl, M. (1989). Automatic complexity analysis. In *Fpca*, volume 89, pages 144–156. Citeseer.
- Scipy (2019). Python scipy.optimize documentation. https://docs.scipy.org/doc/scipy/reference/optimize.html.
- W. Kernighan, B. and J. Van Wyk, C. (1998). Timing trials, or, the trials of timing: Experiments with scripting and user-interface languages. *Software: Practice and Experience*, 28.
- Wegbreit, B. (1975). Mechanical program analysis. Communications of the ACM, 18(9):528–539.
- Weiss, M. A. (2012). Data Structures and Algorithm Analysis in Java. Pearson Education, Inc.
- Zimmermann, P. and Zimmermann, W. (1989). The automatic complexity analysis of divide-and-conquer algorithms. Research Report RR-1149, INRIA. Projet EURECA.