Fractional Controller for Thin Plate Surface Temperature Control

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Abstract: Surface temperature control of a thin aluminium plate were investigated using closed loop control approach implemented using inverse problem. The one-dimensional model with periodic boundary condition was solved using the Laplace transform and both direct problem and inverse problem transfer functions were obtained. The resulting transfer functions were expanded using Zero-Pole expansion to obtain a finite order polynomial transfer function. Simulation results for closed loop control using fractional controllers ($FOPI^{\lambda}$, $FOPD^{\mu}$, and $FOPID^{\mu}$) were evaluated.

1 INTRODUCTION

Fractional order controllers start to be used more often recently with the purpose to obtain better performance of the system.

A heat conduction modelling for both steady state and unsteady state using periodic boundary conditions was presented in (Gebhart, 1971). Phase angle and magnitude of transfer functions of different order were provided (Ogata, 2010).

Inverse heat transfer problems of a metal plate have several solution methods, were presented in (Maillet, 2000). A detailed method was formulated for the design of FOPI, FOPD, and FOPID controllers (Monje, 2010). The heat flux and the temperature control on front surface using the measurement on the back surface of a finite slab, which is a standard problem, was estimated. The Laplace transform was used to get a solution of the resulting heat conduction equation to obtain the transfer functions, and then was expanded using Zero-Pole expansion (Feng, 2010).

A controller was designed with respect to gain and phase margin criteria to satisfy the robustness property for PID controller for the case of a ceramic infrared heater (Shekher, 2016). A detailed design of fractional order PID (FOPID) controller was proposed and the parameters of the controller were obtained according to the model characteristics and design specifications (Zheng, 2018). A stability regions study based on specified gain and phase margin of the fractional order PI controller to control integrating process was presented in (Cokmez, 2018). For FOPD controller design, a new tuning method of typical class of second order system was proposed and can ensure given gain crossover frequency and phase margin (Li, 2010 and Li, 2008). Smith predictor combined a fractional order controller is proposed to control the temperature of a steel slab reheating furnace, they introduce a simulation results for a fractional order proportional integrator controller (Batlle, 2013).

Laplace transform was used to get a solution for the one dimensional heat conduction equation, this done to obtain the transfer functions representing both problems resulted, direct problem and inverse problem of the system. Both Zero-Pole expansion and Taylor expansion were investigated using root locus plots. The number of terms used in the inverse transfer function was investigated to see the effect on the ill posedness of the problem. Zero-Pole expansion was adopted and simulations were done for a thin aluminium plate to control surface temperature of the plate on one side using inverse problem in closed loop control approach (Necsulescu, 2017). An approach was introduced to design a fractional order PI controller for controlling a DC motor speed and experimental results proved the efficiency of using such controller (Muresan, 2013). A fractional order controller that is able to deal with non-modelled dynamics was proposed for the cooperative cruise control (Flores, 2016). Interactive tools like Matlab and Labiew are used to teach fractional order control methods and how they can introduced in classical control course (Tan, 2016). A hybrid fractional order

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controller were optimized for a proportional derivative controller (Maurya, 2016).

In the current paper we compare several fractional order controller types (FOPI$^{\lambda}$, FOPD$^{\mu}$, and FOPID$^{\mu}$) to control the surface temperature of a thin plate on one side using the inverse problem in closed loop control approach; this was achieved by using the Laplace transform to solve the 1D heat conduction equation with periodic boundary conditions to get the transfer functions for both direct and inverse problem. We use periodic boundary conditions because of the possibility to represent temperature changing with time using Fourier series.

2 THEORY

2.1 Transfer Functions

The 1D heat conduction equation is given by:

$$\frac{\partial^2\theta}{\partial z^2} = \frac{1}{\alpha}\frac{\partial\theta}{\partial t} \tag{1}$$

where z is the 1D position variable 0 < z < L for a plate of thickness L.

Boundary conditions are the following for this study were:

$$\theta_1(0,t)=A\sin\omega t, \qquad \theta_2(L,t)=\text{free} \tag{2}$$

$$\emptyset_1(0,t)=\text{free}, \qquad \emptyset_2(L,t)=0 \tag{3}$$

where θ stand for the temperature, $\emptyset$ stand for the heat flux, α stand for thermal diffusivity, and subscript 1 and 2 indicate faces 1 and 2 of the plate.

Equation (1) can be written in complex domain as:

$$\frac{d^2\theta(z,s)}{dz^2} = \frac{s}{\alpha}\theta(z,s) \tag{4}$$

Boundary conditions in s-domain become:

$$\theta_1(0,s)=A\frac{\omega}{S^2+\omega^2}, \qquad \theta_2(L,s)=\text{free} \tag{5}$$

$$\emptyset_1(0,s)=\text{free}, \qquad \emptyset_2(L,s)=0 \tag{6}$$

Equations (5) and (6) define the thermal quadrupole ends, θ_1 and $\emptyset_1$ for input and θ_2 and $\emptyset_2$ for output.

The solution of (4), is:

$$\theta(z,s)=A_1\cosh(Kz)+A_2\sinh(Kz) \tag{7}$$

The heat flux is given by

$$\emptyset(z,s)=-Ks\frac{d\theta}{ds} \tag{7.1}$$

where

$$K=\sqrt{\frac{s}{\alpha}} \tag{8}$$

Applying boundary conditions, (5) and (6), to (7), gives the following results for A_1 and A_2 (Gebhart, 1971 and Maillet, 2000).

$$A_1=A\frac{\omega}{S^2+\omega^2}, \qquad A_2=-A\frac{\omega}{S^2+\omega^2}\tanh(KL) \tag{9}$$

For the above A_1 and A_2, the solutions become:

$$\theta(z,s) = A\frac{\omega}{S^2+\omega^2}[\cosh(Kz) - \tanh(KL)\sinh(Kz)] \tag{10}$$

$$\emptyset(z,s)=-KsA\frac{\omega}{S^2+\omega^2} * [\cosh(Kz) - \tanh(KL)\sinh(Kz)] \tag{11}$$

The boundary temperatures θ_1 and θ_2 are:

$$\theta_1=\theta(0,s)=A\frac{\omega}{S^2+\omega^2} \tag{12}$$

$$\theta_2 = \theta(L,s) = A\frac{w}{S^2+\omega^2}[\cosh(KL) - \tanh(KL)\sinh(KL)] = A\frac{\omega}{S^2+\omega^2}[1/\cosh(KL)] \tag{13}$$

The transfer function of the direct problem linking θ_2 to θ_1 is

$$G_1=\frac{\theta_2}{\theta_1}=\left[\frac{1}{\cosh(KL)}\right]=\text{sech}(KL) \tag{14}$$

The transfer function for the inverse problem is

$$G_2=\frac{1}{G_1}=\cosh(KL) \tag{15}$$

Given (8) for K in this formulation, the hyperbolic functions G_1 and G_2 depend on square root of s:

$$x=KL=\sqrt{\frac{s}{\alpha}}L \tag{16}$$

To overcome the computation problem in case of square root of s, Zero-Pole expansion is used to obtain equations in integer powers of s.

Zero-Pole expansion Gives the following equations:

$$G_1(s)=\frac{p_1p_2p_3p_4p_5p_6\cdots}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)(s-p_5)(s-p_6)\ldots} \tag{17}$$

where

$$p_n=-[\frac{(2k-1)\pi}{2}*\frac{\sqrt{\alpha}}{L}]^2, \quad n=1,2,3,\ldots p_n \tag{18}$$

and

$$G_2(s)=\frac{(s-z_1)(s-z_2)(s-z_3)(s-z_4)\ldots}{z_1 z_2 z_3 z_4\ldots} \tag{19}$$

where

$$z_n=-\left[\frac{(2k-1)\pi}{2}*\frac{\sqrt{\alpha}}{L}\right]^2, \quad n=1,2,3,\ldots\ z_n \tag{20}$$

The above Zero-Pole expansions (17) and (19) of $G_1=1/\cosh(x)$ and $G_2=\cosh(x)$ use integer number powers polynomials in s for simulation. These approximations proved appropriate for real-time surface temperature control of a plate.

Simulations were carried out for a thin 6061T6 Aluminium plate of thickness L = 12.7 [mm] and thermal diffusivity α= 6.9031e-5 [m^2/sec].

Simulations were done with M=4 terms for an inverse problem transfer function and N= 6 terms for the direct problem transfer function.

where

N & M stand for the number of terms for direct problem and inverse problem.

After we get the Transfer function to our plant:

$$G_1=\frac{5.169e8}{s^6+428.7s^5+6.337e4s^4+3.785e6s^3+8.636e7s^2+5.764e8s+5.169e8}$$

$$G_2=\frac{s^4+122.5s^3+3450s^2+2.497e4s+2.267e4}{2.267e4}$$

The resulting transfer function for the system is:

$$G=G_2*G_1=\frac{22805}{(S+178.5)(s+127.8)} \tag{21}$$

2.2 Control Approach

The block diagram for closed loop scheme is shown in Fig. 1, where

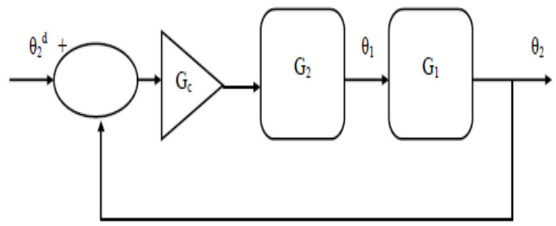

Figure 1: Block diagram for closed loop scheme.

θ_2^d is the desired input temperature.

θ_1 is the temperature output of the inverse problem.

θ_2 is the temperature output of the direct problem.

G_c is the transfer function of the controller.

G_1 is the direct transfer function.

G_2 is the inverse transfer function.

2.3 Controller Equations

The first controller is the fractional order proportional integral controller ($FOPI^{\lambda}$).

The fractional order PI controller formula is (Cokmez, 2018):

$$C(j\omega)=k_p[1+k_i(j\omega)^{-\lambda}] \tag{22}$$

$$C(j\omega)= k_p[1+k_i(\omega)^{-\lambda}\cos\left(\frac{\mu\pi}{2}\right)+jk_i(\omega)^{-\lambda}\sin\left(\frac{\mu\pi}{2}\right) \tag{23}$$

$$Arg[C(j\omega)]=\tan^{-1}\left[\frac{k_I(\omega)^{-\lambda}\sin\left(\frac{\mu\pi}{2}\right)}{1+k_I(\omega)^{-\lambda}\cos\left(\frac{\mu\pi}{2}\right)}\right] \tag{24}$$

$$|C(j\omega)|=K_p* \sqrt{[1+k_I(\omega)^{-\lambda}\cos\left(\frac{\mu\pi}{2}\right)]^2+[k_I(\omega)^{-\lambda}\sin\left(\frac{\mu\pi}{2}\right)]^2} \tag{25}$$

The open loop transfer function is:

$L(j\omega) = C(j\omega)G(j\omega)$

We want to satisfy three conditions to solve for variables:

1 - Robustness:

$$\left.\frac{d(Arg[L(j\omega)]}{d\omega}\right|_{\omega=\omega_{cg}}=0$$

2 - Gain crossover frequency:

$$|L(j\omega)|_{dB}=0$$

3 - Phase Margin:

$$Arg[L(j\omega)]|_{\omega=\omega_{cg}}=-\pi+\varphi_m$$

From criteria (3) we get:

$$\tan^{-1}\left[\frac{k_I(\omega_{cg})^{-\lambda}\sin\left(\frac{\mu\pi}{2}\right)}{1+k_I(\omega_{cg})^{-\lambda}\cos\left(\frac{\mu\pi}{2}\right)}\right]- \tan^{-1}\left[\frac{2\zeta\omega_{cg}\omega_n}{\omega_n^2-\omega_{cg}^2}\right]=-\pi+\varphi_m \tag{26}$$

From criteria (2) we get:

$$\frac{K_p\sqrt{[1+k_I(\omega_{cg})^{-\lambda}\cos\left(\frac{\mu\pi}{2}\right)]^2+[k_I(\omega_{cg})^{-\lambda}\sin\left(\frac{\mu\pi}{2}\right)]^2}}{\sqrt{(1-\frac{\omega_{cg}^2}{\omega_n^2})^2+4\zeta^2\omega_{cg}^2/\omega_n^2}}=1 \tag{27}$$

From criteria (1) we get:

$$K_I=\frac{-B\pm\sqrt{B^2-4A[A\omega_{cg}^{-2\lambda}+\lambda(\omega_{cg})^{-2\lambda-1}]}}{2[A\omega_{cg}^{-2\lambda}+\lambda(\omega_{cg})^{-2\lambda-1}]} \tag{28}$$

where

$$A=\frac{2\zeta\omega_n\left(\omega_n^2-\omega_{cg}{}^2\right)+4\zeta\omega_n\omega_{cg}{}^2}{(\omega_n^2-\omega_{cg}{}^2)^2+(2\zeta\omega_n\omega_{cg})^2}$$

and

$$B=2A\omega_{cg}^{-\lambda}\cos\left[\frac{\lambda\pi}{2}\right]+\lambda\omega_{cg}^{-\lambda-1}\cos\left[\frac{\lambda\pi}{2}\right]$$

The second controller is the fractional order derivative controller ($FOPD^{\mu}$).

The system transfer function formula is (Ogata, 2010):

$$G=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \tag{29}$$

we get:

$$|G(j\omega)|=\frac{1}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2+4\zeta^2\omega^2/\omega_n^2}} \tag{30}$$

$$Arg[G(j\omega)]=-\tan^{-1}\left[\frac{2\zeta\omega\omega_n}{\omega_n^2-\omega^2}\right] \tag{31}$$

The fractional order PD controller formula is (Monje, 2010 and Li, 2010):

$$C(j\omega)=k_p[1+k_d(j\omega)^{\mu}] \tag{32}$$

$$C(j\omega)=k_p\left[1+k_d(\omega)^{\mu}\cos\left(\frac{\mu\pi}{2}\right)+jk_d(\omega)^{\mu}\sin\left(\frac{\mu\pi}{2}\right)\right] \tag{33}$$

$$Arg[C(j\omega)]=\tan^{-1}\left[\frac{\sin\left[\frac{(1-\mu)\pi}{2}\right]+k_d(\omega)^{\mu}}{\cos\left[\frac{(1-\mu)\pi}{2}\right]}\right]-\frac{(1-\mu)\pi}{2} \tag{34}$$

$$|c(j\omega)|=K_p*\sqrt{(1+K_d\omega^{\mu}\cos\left(\frac{\mu\pi}{2}\right))^2+(1+K_d\omega^{\mu}\sin\left(\frac{\mu\pi}{2}\right))^2} \tag{35}$$

were:

K_p is the proportional gain.

K_d is the derivative gain.

The open loop transfer function is:

$L(j\omega)=C(j\omega)G(j\omega)$

We want to satisfy three conditions to solve for variables:

1 - Robustness:

$$\left.\frac{d(Arg[L(j\omega)])}{d\omega}\right|_{\omega=\omega_{cg}}=0$$

2 - Gain crossover frequency:

$$|L(j\omega)|_{dB}=0$$

3 - Phase Margin:

$$Arg[L(j\omega)]|_{\omega=\omega_{cg}}=-\pi+\varphi_m$$

From criteria (3) we get:

$$\tan^{-1}\left[\frac{\sin\left[\frac{(1-\mu)\pi}{2}\right]+k_d(\omega_{cg})^{\mu}}{\cos\left[\frac{(1-\mu)\pi}{2}\right]}\right]-\frac{(1-\mu)\pi}{2}-\tan^{-1}\left[\frac{2\zeta\omega_{cg}\omega_n}{\omega_n^2-\omega_{cg}^2}\right]=-\pi+\varphi_m \tag{36}$$

From criteria (2) we get:

$$\frac{K_p\sqrt{[1+K_d\omega_{cg}^{\mu}\cos(\frac{\mu\pi}{2})]^2+[K_d\omega_{cg}{}^{\mu}\sin(\frac{\mu\pi}{2})]^2}}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2+4\zeta^2\frac{{\omega^2}^2}{\omega_n}}}=1 \tag{37}$$

From criteria (1) we get:

$$\frac{\mu K_d\omega_{cg}^{\mu-1}\cos\left[\frac{(1-\mu)\pi}{2}\right]}{\cos^2\frac{(1-\mu)\pi}{2}+[\sin\frac{(1-\mu)\pi}{2}+K_d\omega_{cg}^{\mu}]^2}-\frac{2\zeta\omega_n\left(\omega_n^2-\omega_{cg}{}^2\right)+4\zeta\omega_n\omega_{cg}{}^2}{(\omega_n^2-\omega_{cg}{}^2)^2+(2\zeta\omega_n\omega_{cg})^2}=0 \tag{38}$$

From criteria (3) we can get a relation between K_d and μ as follows:

$$K_d=\frac{-B\pm\sqrt{B^2-4A^2\omega_{cg}^{2\mu}}}{2A\omega_{cg}^{2\mu}} \tag{39}$$

where:

$$A=\frac{2\zeta\omega_n\left(\omega_n^2-\omega_{cg}{}^2\right)+4\zeta\omega_n\omega_{cg}{}^2}{(\omega_n^2-\omega_{cg}{}^2)^2+(2\zeta\omega_n\omega_{cg})^2}$$

$$B=2A\omega_{cg}^{\mu}\sin\left[\frac{(1-\mu)\pi}{2}\right]-\mu\omega_{cg}^{\mu-1}\cos\left[\frac{(1-\mu)\pi}{2}\right]$$

The third controller is the fractional order proportional derivative controller ($FOPID^{\mu}$).

The fractional order PID controller formula is (Shekher, 2016 and Zheng, 2018):

$$C(s)=k_p[1+\frac{k_i}{(s)^{\lambda}}+k_d(s)^{\mu}] \tag{40}$$

For the current controller we have $\lambda=1$, we get:

$$C(j\omega)=k_p\{1+k_d(\omega)^{\mu}\cos\left(\frac{\mu\pi}{2}\right)+j\left[-k_i\omega^{-1}+k_d(\omega)^{\mu}\sin\left(\frac{\mu\pi}{2}\right)\right]\} \tag{41}$$

Let:

$$P(\omega)=1 \ +k_d(\omega)^{\mu} \cos\left(\frac{\mu\pi}{2}\right)$$

and

$$Q(\omega)=-\ k_i\omega^{-1}+k_d(\omega)^{\mu}\sin\left(\frac{\mu\pi}{2}\right)$$

Then

$$\text{Arg}[C(j\omega)]=\tan^{-1}[\frac{Q(\omega)}{P(\omega)}] \tag{42}$$

$$|C(j\omega)| = k_p\sqrt{P^2(\omega)+Q^2(\omega)} \tag{43}$$

We want to satisfy four conditions to solve for variables:

1 - Robustness:

$$\left.\frac{d(\text{Arg}[L(j\omega)]}{d\omega}\right|_{\omega=\omega_{cg}} = 0$$

2 - Gain crossover frequency:

$$|L(j\omega)|_{dB}=0$$

3 - Phase Margin:

$$\text{Arg}[L(j\omega)]|_{\omega=\omega_{cg}}= -\pi+ \varphi_m$$

4 - Noise rejection:

$$\left|T(j\omega)=\frac{C(j\omega)G(j\omega)}{1+C(j\omega)G(j\omega)}\right|_{dB} \leq A \text{ dB}$$

were A is a designed value.

According to specification (2) we get:

$$\frac{k_p\sqrt{P^2(\omega)+Q^2(\omega)}}{\sqrt{(1-\frac{\omega_{cg}^2}{\omega_n^2})^2+ 4\zeta^2\omega_{cg}^2/\omega_n^2}}=1 \tag{44}$$

From specification (3) we get:

$$\tan^{-1}\left[\frac{Q(\omega)}{P(\omega)}\right] - \tan^{-1}\left[\frac{2\zeta\omega\omega_n}{\omega_n^2-\omega^2}\right] = -\pi+ \varphi_m \tag{45}$$

From specification (1) we get:

$$\frac{P(\omega)*aa-Q(\omega)*pp}{P(\omega)^2+Q(\omega)^2} - \frac{2\zeta\omega_n\left(\omega_n^2-\omega_{cg}^2\right)+4\zeta\omega_n\omega_{cg}^2}{(\omega_n^2-\omega_{cg}^2)^2+(2\zeta\omega_n\omega_{cg})^2}=0 \tag{46}$$

From criteria (4) we get:

$$\frac{|C(j\omega)G(j\omega)|}{|1+C(j\omega)G(j\omega)|}=\frac{\sqrt{P^2(\omega)+Q^2(\omega)}}{\sqrt{[\frac{(1-\frac{\omega_{cg}^2}{\omega_n^2})^2}{K_p}+P(w)]^2+[Q(w)+2\zeta\frac{\omega_{cg}}{\omega_n K_p}]^2}}\leq A \tag{47}$$

3 RESULTS AND DISCUSSION

For the design purpose, the crossover frequency was set to be 20 (rad/sec) and the phase margin is set to be 65 degrees, all results had a one second of step time to see a clear step response away from Y-axis.

After we solve (26, 27, and 28) for the fractional order PI controller parameters, we get controller formula as follows:

$$\text{FOPI}= \ 1.3862 + 25.993(s)^{-1.486}$$

From Fig. 2, we see that the system reaches the desired response after 1.5 seconds, but it has an overshoot of 20%.

After we solve (37, 38, and 38) for the fractional order PD controller parameters, we get controller formula as follows:

$$\text{FOPD}= \ 1.5622 + 0.0086842(s)^{1.832}$$

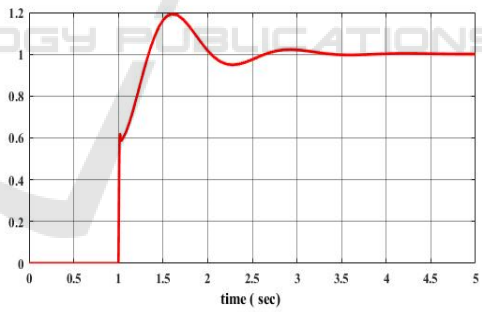

Figure 2: Step response using fractional order proportional controller (FOPI$^{\lambda}$).

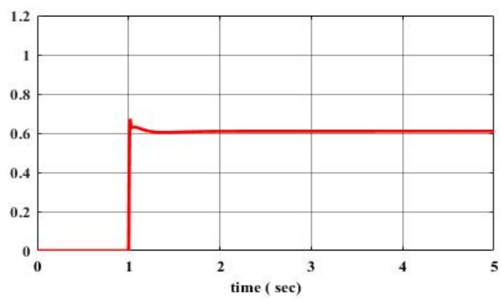

Figure 3: Step response using fractional order proportional derivative controller (FOPD$^{\mu}$).

From Fig. 3, we can see that the controller can't reach the desired steady state value no matter how long time we give the system.

After we solve (44, 45, 46, and 47) for the fractional order PID controller parameter we get the controller formula as follows:

$$\text{FOPID}= \ 0.9639 + \frac{2.9379}{s} - 0.0862(s)^{0.713}$$

From Fig. 4, we can see that the system reaches the desired response after 2.5 seconds, also with no overshoot.

From Fig. (2, 3, and 4), we see that the best controller is the fractional order proportional derivative controller (FOPID$^{\mu}$), since it achieves the desired response without overshoot.

Now we compare the results with the integer order controller (IOPID). From Fig. 5, we can see that the response is slower with an overshoot of about 8%, and this favours the fractional order proportional derivative controller (FOPID$^{\mu}$) over all other controllers, this is due to the fact that for this controller we have four parameters to change which gives a better design over all other controllers were only three parameters are available to change for design. The results for integer order controller were obtained by using Automatic Tuning Criteria in Matlab™.

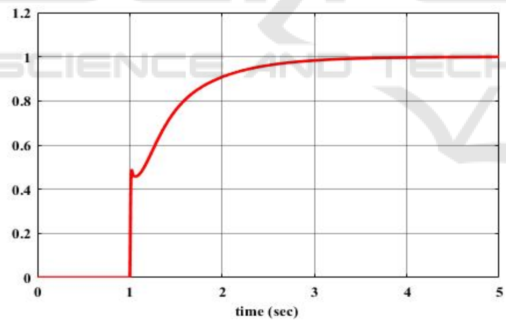

Figure 4: Step response using fractional order proportional derivative controller (FOPID$^{\mu}$).

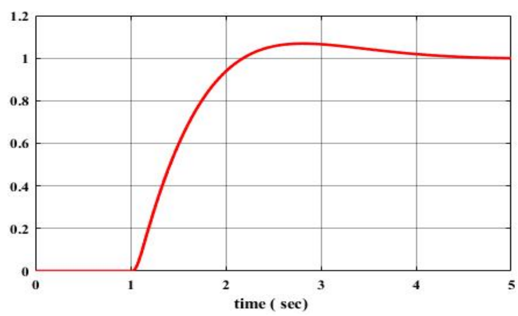

Figure 5: Step response using integer order proportional derivative controller (IOPID).

4 CONCLUSIONS

The fractional order controller has an advantages over integer order controller with respect to overshoot time, the fractional controller results show that we can get a response without overshoot. Fractional order controller design gives us more flexibility to choose five controller parameters compare to three controller parameters for integer order, which helps in control response time, overshoot and system stability. Surface temperature control for a metal thin plate has still to be further investigated and verified experimentally.

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