Computation of Trajectory Sensitivities with Respect to Control and Implementation in PSAT

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Abstract: Trajectory sensitivity based analysis is widely regarded as an important tool for real time protection scheme of power systems. Model Predictive Control (MPC) for voltage instability is one such protection scheme which computes a sequence of control actions depending upon the trajectory behaviour of the dynamics of the power systems. Thus, computation of trajectory sensitivities with respect to control input can be an integral part for designing a real-time protection scheme. In this context, it is important to note that for the state-of-the-art Power System Analysis Tool (PSAT)(Milano, 2005), (Milano et al., 2008), while it is relatively easy to compute the trajectory sensitivities with respect to any system variables, the computation of trajectory sensitivities with respect to control inputs is not explicitly supported. This paper presents a method to extend the functionality of PSAT to also compute the trajectory sensitivities with respect to control inputs, which ultimately forms the basis for real-time protection schemes such as MPC. The proposed method is validated using direct time-domain simulation results.

1 INTRODUCTION

In today's deregulated market scenario, power utilities are compelled to do trade-off between cost and design, which results in most power systems operating close to their capacity, making them susceptible to disturbances. To cope with this as well as the ever increasing load demand and meet customer satisfaction index, it is imperative to adopt measures to avert any large scale shutdown following the occurrence of severe fault and disturbances. Thus a basic requirement is to set up certain real time protection schemes which can take necessary control actions upon detecting any potential instability in the system.

Model Predictive Control (MPC) is a promising control strategy for tackling voltage instability following contingencies in a power system. Briefly, MPC works on the principle of receding horizon control and computes optimal control strategies depending upon the dynamics of the system that can be represented by the trajectories of its states. Trajectory sensitivity provides a valuable insight into the behavior of a dynamic system: It estimates how the system trajectory would change when there is a slight change in input, state, or output, which would not be otherwise obvious only from its nominal trajectory (Hiskens and Pai, 2000).

In (Zima and Andersson, 2003), MPC based control using trajectory sensitivity is discussed but only a preliminary idea of calculating trajectory sensitivity is given. Trajectory sensitivity based Model Predictive Control protection scheme for power systems is presented in (Jin et al., 2010) and (Jin et al., 2007). These papers utilize shunt capacitors, i.e., reactive power compensation technique for control purposes and determine capacitor switching sequence by minimizing an objective function which includes the trajectory deviation of voltage and cost of control. Trajectory sensitivities are used to estimate the effect of controls on the voltage behavior in a linearized manner. An MPC based voltage control strategy is also proposed in (Hiskens and Gong, 2005), where the objective is to find optimized control of load depending upon the trajectory behavior of bus voltages.

Trajectory sensitivity based analysis for nonlinear and hybrid systems, such as power systems, is well studied in control domain. According to (Hiskens and Pai, 2000), the approach is based upon linearizing the system around a nominal trajectory rather than around an equilibrium point. It is therefore possible to determine directly the change in a trajectory due to (small)

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changes in initial conditions, parameters, and/or controls. Trajectory sensitivities have a potential for enabling both preventive and emergency control. A survey on application of trajectory sensitivity in power system is provided in (Tang and McCalley, 2013), and importance is given to the accuracy refinement of the calculation. In (Ferreira et al., 2004) some basics of trajectory sensitivity based analysis for transient stability, dynamic voltage stability and influence of load shedding are mentioned, and different case studies are shown. These papers relied on the software package EUROSTAG for their study. In (Ghosh et al., 2004), trajectory sensitivity analysis is used in determining stable operating range of TCSC and in (Chatterjee and Ghosh, 2007), trajectory sensitivity analysis is used to study the effect of the FACTs controller on the transient stability; in both of these papers, sensitivity with respect to system parameters is calculated numerically using a much simpler method than analytical computation. (Nasri et al., 2014), (Nasri et al., 2013b), and (Nasri et al., 2013a) has presented trajectory sensitivity based analysis for optimal location of series, shunt capacitors, and FACTs devices respectively; these papers calculated trajectory sensitivities of rotor angles with respect to line reactance and reactive power injected at different nodes, and for simulation the commercial software SIMPOWER is used. In (Abdelsalam et al., 2017), trajectory sensitivity with respect to a system parameter is used to design the LQR based voltage control of wind generation but detailed architecture of trajectory sensitivity computation is not discussed. The theory of calculating trajectory sensitivities is comprehensively described in the papers (Hiskens and Pai, 2000), (Laufenberg and Pai, 1997) and (Hiskens and Pai, 2002), and there are other works where this derivation of sensitivities are used for designing of different control strategies.

The main contribution of our paper is to calculate trajectory sensitivities with respect to control input in PSAT/MATLAB platform. The computed trajectory sensitivity values are validated with simulation results for an example run of a power system.

The rest of the paper is organized in 5 sections. First, the background on basics of trajectory sensitivity calculation is presented, and the issues in computing trajectory sensitivity in PSAT framework with respect to control inputs are discussed. Next, the proposed solution of the problem is presented. This is followed by the implementation details and the extension to PSAT, together with the validation results. Finally, the paper is concluded.

2 BACKGROUND

2.1 Trajectory Sensitivity Overview

A power system can be modeled using Differential Algebraic Equations (DAEs) of the form:

$$\dot{x} = f(x,y,u), \tag{1}$$

$$0 = g(x,y,u), \tag{2}$$

where x is a vector containing dynamic state variables, y is a vector of algebraic variables, and u is a vector of control input variables. The solution of these two equations provide the trajectories of state and algebraic variables for a given initial state vector, and control trajectory. To device the impact of changing control actions on system behavior, a main interest is to determine the effect of change of control input u on state variables x and algebraic variables y. From this point of view the derivation of trajectory sensitivities of state and algebraic variable with respect to control becomes important.

Trajectory Sensitivity of x and y with respect to u is defined as the rate of change of x and y around the nominal trajectory with respect to an infinitesimal change in control input u. Then, using Taylor series approximation, the trajectory sensitivity of x and y with respect to u is given by their 1st-order approximations, $x_u(t) = \frac{\partial x(t)}{\partial u}$ and $y_u(t) = \frac{\partial y(t)}{\partial u}$ respectively.

The dynamics of x_u and y_u can be obtained by differentiating equations (1) & (2) with respect to control input u, resulting in,

$$\dot{x_u}(t) = f_x x_u(t) + f_y y_u(t) + f_u, \tag{3}$$

$$0 = g_x x_u(t) + g_y y_u(t) + g_u, \tag{4}$$

Note the Jacobian matrices f_x, f_y, g_x, g_y, f_u, g_u are all time varying. Thus, the calculation of trajectory sensitivities x_u and y_u requires the knowledge of all the above 6 Jacobians at each sampling instant, along with a time domain simulation of equations (3) & (4). More details on trajectory sensitivity can be found in (Hiskens and Pai, 2000) and (Hiskens and Pai, 2002).

2.2 MPC for Voltage Stability

In general, Model Predictive Control (MPC) refers to a class of algorithms that compute a sequence of control variable adjustments in order to optimize the future behavior of a plant. The principle of MPC is graphically depicted in Figure 1 (Jin et al., 2010), which shows that the control is recomputed at each sample instant for the remaining control horizon by using a model prediction over a prediction horizon. The latest measurements are used to better estimate the current state and thereby improving the prediction, and the trajectory sensitivities with respect to the control are used for quickly estimating the trajectories in the prediction horizon. Only the computed control of the first instant is applied, and then the process is repeated at the next sample instant.

In context of power systems, MPC can be used to exercise optimal control action upon any severe disturbance, following which different complications may arise; one most common occurrences is the drop in bus voltages. To mitigate this risk of voltage instability, the two major techniques are: a) Reactive Power Compensation by switching on shunt capacitors and b) Under Load Tap-changer (ULTC) Control. Apart from these, in some severe contingencies, exercising load-shedding becomes essential to maintain the overall stability of the network.

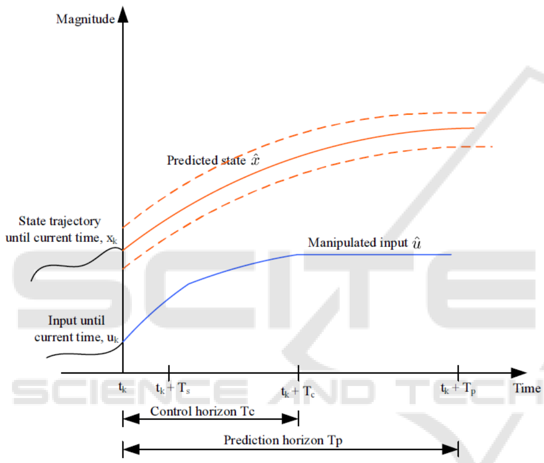

Figure 1: Principle of MPC.

3 PROPOSED EXTENSIONS TO PSAT TO ENABLE TRAJECTORY SENSITIVITY COMPUTATION

3.1 Enhancing PSAT to Store the Jacobians that It Already Computes

Equations (3) & (4) explicitly imply that for calculation of trajectory sensitivities x_u and y_u, the knowledge of f_x, f_y, g_x, g_y, f_u, g_u is required at each time instants. Out of these 6, the first 4 Jacobians, f_x, f_y, g_x, g_y are the components of an unreduced Jacobian $J = \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix}$, which are calculated by PSAT at each time instant in course of the time domain simulation of any system. However, due to a limited capacity of storage, PSAT stores f_x, f_y, g_x, g_y only for the final time instant. We implemented a slight additional coding within PSAT in its time domain integration subroutine to alleviate this problem.

3.2 Our Approach to Computing Trajectory Sensitivities wrt Controls

One significant limitation of PSAT is that during time domain simulation, the values of the Jacobians f_u, g_u are not computed. In particular, if shunt capacitors are to be used as a control input, computing the values of f_u and g_u during the time domain simulation would require major changes in the original PSAT code. On the other hand, in case of Under Load Tap-changer, its model in PSAT has its own state variable, and that can be used to compute f_u and g_u by treating controls as additional state variables with zero-dynamics, as explained Section 3.3. Once all 6 Jacobians are obtained, the trajectory sensitivities x_u and y_u can be obtained by solving equations (3) & (4) numerically, as also explained below.

We first propose to compute the Jacobians f_u and g_u by treating u as a state variable, having zero-dynamics ($\dot{u} = 0$), so it remains held constant at its current value, and the trajectory sensitivity with respect to control around that nominal u gets computed. A similar idea was proposed in (Hiskens and Pai, 2002) for computing trajectory sensitivity with respect to a parameter (rather control).

With the control variables augmented as part of the state variables, the equations (1) & (2) also get augmented:

$$\dot{x} = f(x,y,u) \tag{5}$$

$$\dot{u} = 0 \tag{6}$$

$$0 = g(x,y,u) \tag{7}$$

We denote the control-augmented state variables as, $\bar{x} = \begin{pmatrix} x \\ u \end{pmatrix}$, algebraic variables as y, and note the dimensions of the various variables as: $x \in R^n, y \in R^m, u \in R^p$.

Combining equations (5), (6) & (7) we have,

$$\dot{\bar{x}} = \begin{pmatrix} \dot{x} \\ \dot{u} \end{pmatrix} = \begin{pmatrix} f(x,y,u) \\ 0 \end{pmatrix} = \bar{f}(\bar{x},y) \tag{8}$$

$$0 = g(\bar{x},y) \tag{9}$$

Differentiating equations (8) & (9) with respect to control input u, results in,

$$\dot{\bar{x}}_u(t) = \bar{f}_{\bar{x}}\bar{x}_u(t) + \bar{f}_y y_u(t) \tag{10}$$

$$0 = g_{\bar{x}}\bar{x}_u(t) + g_y y_u(t) \tag{11}$$

Note $\overline{f}_{\bar{x}} = \begin{pmatrix} f_x & f_u \\ 0 & 0 \end{pmatrix}$, $\overline{f}_y = \begin{pmatrix} f_y \\ 0 \end{pmatrix}$, and $g_{\bar{x}} = \begin{pmatrix} g_x & g_u \end{pmatrix}$. As discussed earlier, with the above control-augmented states, in PSAT at each time instant the Jacobians $\overline{f}_{\bar{x}}$, $\overline{f}_y$, $g_{\bar{x}}$ and g_y are calculated in course of the time domain simulation from which we can extract the desired f_u and g_u from $\bar{f}_{\bar{x}}$ and $g_{\bar{x}}$ respectively.

Once the Jacobians $\bar{f}_{\bar{x}}, \bar{f}_y, g_{\bar{x}}, g_y$ are obtained from PSAT, equations (10) and (11) can be numerically solved to obtain the required trajectory sensitivities x_u and y_u. At an initial time t_0, $\bar{x}_u(t_0)$= $\begin{pmatrix} x_u(t_0) \\ u_u(t_0) \end{pmatrix}$ is a $(n+p)\times p$ matrix where $x_u(t_0) = 0_{n\times p}$ and $u_u(t_0) = I_{p\times p}$, so that $\bar{x}_u(t_0)$= $\begin{pmatrix} 0_{n\times p} \\ I_{p\times p} \end{pmatrix}$. This is because a change in any control only changes that control with rate 1, whereas none of the initial states or the other controls are affected by it. Then the initial value of $y_u(t_0)$ can be obtained using equation (11), resulting in:

$$y_u(t_0) = -g_y(t_0)^{-1} g_{\bar{x}}(t_0)\bar{x}_u(t_0) \qquad (12)$$

Now, using trapezoidal integration, $\bar{x}_u$ and y_u in equations (10) and (11) can be approximated as,

$$\begin{pmatrix} \frac{h}{2}\overline{f}_{\bar{x}}^{k+1} - I & \frac{h}{2}\overline{f}_y^{k+1} \\ g_{\bar{x}}^{k+1} & g_y^{k+1} \end{pmatrix} \begin{pmatrix} \bar{x}_u^{k+1} \\ y_u^{k+1} \end{pmatrix} = \begin{pmatrix} \frac{h}{2}(-\overline{f}_{\bar{x}}^{k}\bar{x}_u^{k} - \overline{f}_y^{k} y_u^{k}) \\ 0 \end{pmatrix} \qquad (13)$$

In equation (13), the superscript k is used for the variable values at that kth sampling instant. Thus starting from t_0, by solving the linear equation (13) at each time instant, trajectory sensitivities for all state and algebraic variables with respect to control input, x_u^k and y_u^k, can be calculated at each sample instant k.

3.3 Case of SVC Control

The modeling of control input as a new state variable is not straightforward in PSAT. Here, we first discuss the modeling for shunt capacitors, which we have been able to do by utilizing the TYPE-1 Static VAR Compensator (SVC) model in PSAT (Milano, 2005). The SVC model itself is shown in Figure 2.

Equations (14) & (15) provide the DAEs representing this model,

$$\dot{b}_{SVC} = \frac{K_r(V_{ref} - V) - b_{SVC}}{T_r} \qquad (14)$$

$$Q = b_{SVC}V^2 \qquad (15)$$

From equations (14) & (15), it is clear that this TYPE-1 SVC itself has a dynamic and can generate output

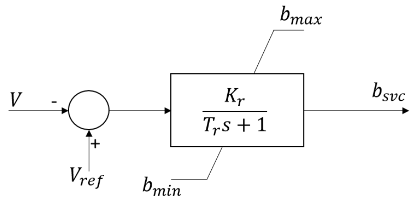

Figure 2: SVC TYPE-1 block.

controls b_{SVC} to mitigate the risk of voltage collapse. Here, it is important to mention one specific feature of this model, that the regulator has an anti-windup limiter, so the output susceptance b_{SVC} saturates if one of the maximum or minimum limits is reached. Basically, if the output b_{SVC} is lower than b_{min} (resp., higher than b_{max}), then the output will take the value b_{min} (resp., b_{max}).

In order to make this device behave like an ordinary shunt capacitor, we make an appropriate selection of the parameters of TYPE-1 SVC model. In doing this, we choose the time-constant T_r very high whereas make the gain K_r very low, so $\dot{b}_{SVC}$ is always near 0 and consequently b_{SVC} always takes the constant value b_{min}. Thus treating u as a state variable in TYPE-1 SVC model, and setting large T_r, small K_r, and b_{min}, we are able to ensure u has zero rate of change, and remains initialized at b_{min}, maintaining that constant value. The desired f_u & g_u are then extracted from the Jacobian J of controls-augmented states as explained in the previous section and hence, trajectory sensitivities x_u and y_u can be calculated.

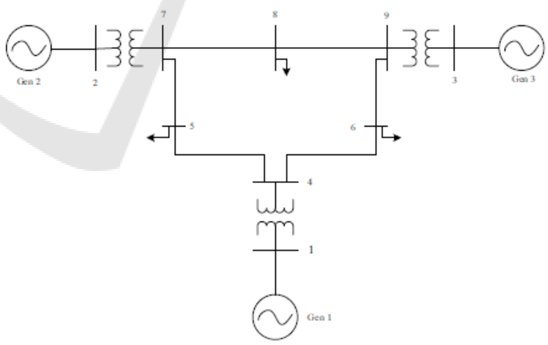

Figure 3: WECC 3-generator 9-bus test system.

To validate that our above scheme works, a 9-bus 3-generator WECC system of Figure 3 was simulated in PSAT. We considered a three-phase fault at bus 5 at t = 1.0 second, which gets cleared at t = 1.15 seconds by the tripping of the line between buses 4 and 5. The system is comprised of shunt capacitors at buses 5, 7, and 8, each having 0.01 initial value. To compute the trajectory sensitivities with respect to these shunt capacitors around the said nominal values, those are replaced by SVC blocks with large T_r and small K_r.

The parameters of the SVC block are chosen as, V_{ref}= 1 p.u.; b_{max} = 0.8 p.u.; b_{min} = 0.01 p.u.; K_r = 10^{-7}; T_r = 10^5 sec. Figure 4 & 5 confirm that the thus modeled SVC blocks replicate the behaviors of the shunt capacitors. Also, the value of state variables b_{SVC} obtained from PSAT database is constant equaling 0.01, demonstrating that the output of SVC block remains fixed at b_{min}.

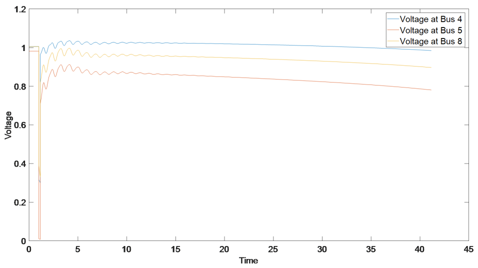

Figure 4: Plot of Voltages with Shunt block.

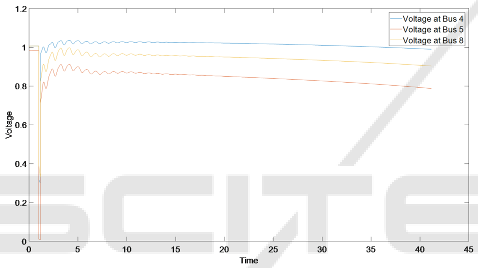

Figure 5: Plot of Voltages with SVC block modeled like Shunt block.

3.4 Case of ULTC Control

Next, for the case of ULTC, its continuous model (Milano, 2011) for voltage control, shown in Figure 6, is quite similar to the model representing the SVC. For continuous control action, the dynamics of the model is represented by the equation (16),

$$\dot{m} = -K_d m + K_i(V_m - V_{ref}) \tag{16}$$

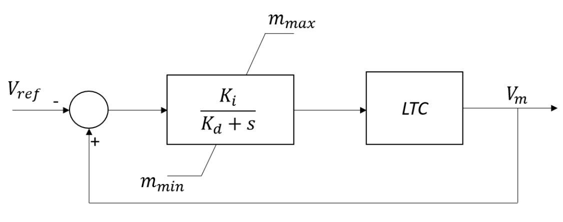

Figure 6: ULTC Block for Voltage Control.

Thus, for the ULTC based control scheme as well, an appropriate assignment of the model parameters K_d and K_i can be done for making its inherent dynamics zero as desired, to obtain the desired Jacobians with respect to the controls and use those to compute the desired trajectory sensitivities.

3.5 Case of Load Control

Loads of any power network can be of different types, e.g. static loads, voltage dependent loads, frequency dependent loads, exponential recovery loads, ZIP loads etc. To study voltage instability related issues, it is reasonable to consider exponential recovery loads. The dynamics of exponential recovery load of any bus is represented by the following DAEs.
For active power,

$$\dot{x}_P = -x_P/T_P + P_0(V/V_0)^{\alpha_s} - P_0(V/V_0)^{\alpha_t} \tag{17}$$

$$P = x_P/T_P + P_0(V/V_0)^{\alpha_t} \tag{18}$$

$$P_0 = K_1 P_B \tag{19}$$

For reactive power,

$$\dot{x}_Q = -x_Q/T_Q + Q_0(V/V_0)^{\beta_s} - Q_0(V/V_0)^{\beta_t} \tag{20}$$

$$Q = x_Q/T_Q + Q_0(V/V_0)^{\beta_t} \tag{21}$$

$$Q_0 = K_2 Q_B \tag{22}$$

where, P and Q are the active and reactive power consumption at the respective bus, x_p and x_q are state variables related to active and reactive power dynamics, T_P and T_Q are time constants of the exponential recovery response, α_s and β_s are exponents related to the steady-state load response, α_t and β_t are exponents related to the transient load response, V and V_0 are current and initial bus voltages, respectively.

Now, $P_0 = K_1 \times P_B$ and $Q_0 = K_2 \times Q_B$ depend on the base active power (P_B) and reactive power (Q_B) of the respective buses. These P_B and Q_B serve as the control parameters, for to exercise load-shedding, a reduction in the base load ($P_B + jQ_B$) of a particular bus is required. Thus, to measure the impact on bus voltages for infinitesimal change of base load, we have to compute the trajectory sensitivity of bus voltages with respect of P_B and Q_B.

In doing so, we have introduce two new state variables $x_{PB} = P_B$ and $x_{QB} = Q_B$ with zero dynamics and modified the load dynamics equations accordingly as follows. For active power,

$$\dot{x}_P = -x_P/T_P + K_1 x_{PB}(V/V_0)^{\alpha_s} - K_1 x_{PB}(V/V_0)^{\alpha_t} \tag{23}$$

$$P = x_P/T_P + K_1 x_{PB}(V/V_0)^{\alpha_t} \tag{24}$$

$$\dot{x}_{PB} = 0 \tag{25}$$

For reactive power,

$$\dot{x}_Q = -x_Q/T_Q + K_2 x_{QB}(V/V_0)^{\beta_s} - K_2 x_{QB}(V/V_0)^{\beta_t} \tag{26}$$

$$Q = x_Q/T_Q + K_2 x_{QB}(V/V_0)^{\beta_t} \tag{27}$$

$$\dot{x}_{QB} = 0 \tag{28}$$

Next using the technique detailed in Section 3.2, the desired Jacobians with respect to controls are computed and stored for calculating trajectory sensitivities. Note, to introduce new state variables, the corresponding sub-routine of PSAT for exponential load requires certain modifications that we have also performed.

4 ARCHITECTURE OF EXTENDED PSAT FOR TRAJECTORY SENSITIVITIES

A proposed architecture for computing tarjectory sensitivities with respect to control input is shown in Figure 7 & 8; it consists of two blocks: the basic PSAT algorithm (Block-1), and the Trajectory Sensitivity Calculation block (Block-2). The PSAT algorithm of Block-1 includes the power flow solution and the modified time domain integration subroutines for controls-augmented states to yield $\bar{f}_{\bar{x}}, \bar{f}_y, g_{\bar{x}}, g_y$ at all sampling instants from which all the 6 Jacobians are extracted. The Trajectory Sensitivity Calculation block (Block-2) then imports all the required elements from Block-1 and computes and stores the trajectory sensitivity values $\bar{x}_u$, y_u for each sampling instant from which x_u and y_u can further be extracted.

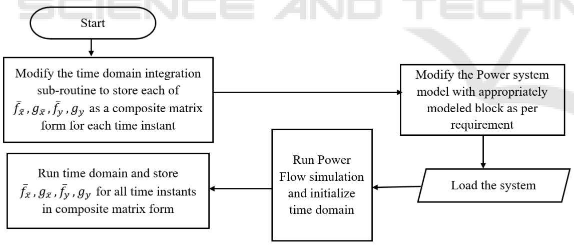

Figure 7: Block-1.

4.1 PSAT Implementation & Validation

For validation, the same 9-bus 3-generator WECC test system shown in Figure 3, with 3 appropriately modeled SVC blocks at bus 5, bus 7, and bus 8 is selected. These are designated as control-1 (u_1), control-2 (u_2), control-3 (u_3). The entire system is simulated in PSAT and for each sampling instant, the trajectory sensitivities of bus voltages with respect to u_1, u_2, and u_3 are computed, using the algorithm mentioned in the previous section.

Letting S_{nj} denote the trajectory sensitivity of nth bus voltage at its nominal value, V_n, with respect the jth control input as, u_j, when the jth control input is

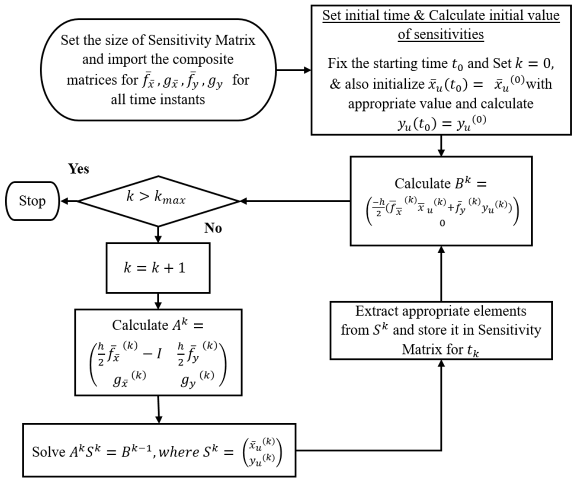

Figure 8: Block-2.

changed by Δu_j amount, the resulting voltage $\widehat{V}_n$ of the nth bus can be approximated by,

$$\widehat{V}_n = V_n + \sum_j S_{nj} \Delta u_j \tag{29}$$

In the simulation of the nominal system, the control inputs are chosen as, u_1 = 0 p.u., u_2 = 0 p.u., and u_3 = 0 p.u. This system is simulated in PSAT and for analysis purposes, the voltages of buses 4, 5, and 8, at 3 different sampling instants are tabulated in Table 1, along with the corresponding trajectory sensitivities. Using equation (29), the predicted voltages $\widehat{V}_n$ are calculated for the change in control inputs Δu_1 = 0.01 p.u., Δu_2 = 0.01 p.u., and Δu_3 = 0.01 p.u. The values of $\widehat{V}_n$ are listed in Table 2.

Table 1: Simulation Results under Base Load.

Time-instant	Bus no.	V_n(in p.u.)	S_{n1}	S_{n2}	S_{n3}
$t = 1.15s$	Bus 4	0.9652	0.0248	0.0298	0.0334
$t = 1.15s$	Bus 5	0.7098	0.1511	0.1150	0.0977
$t = 1.15s$	Bus 8	0.8204	0.0755	0.0907	0.1014
$t = 22.75s$	Bus 4	1.0172	0.0781	0.0718	0.0868
$t = 22.75s$	Bus 5	0.8403	0.2233	0.1427	0.1355
$t = 22.75s$	Bus 8	0.9426	0.1283	0.1208	0.1472
$t = 40.05s$	Bus 4	0.9852	0.2437	0.2019	0.2210
$t = 40.05s$	Bus 5	0.7781	0.5483	0.3987	0.3941
$t = 40.05s$	Bus 8	0.8956	0.3861	0.3238	0.3533

To validate the estimated results, the system is now simulated with u_1 = 0.01 p.u., u_2 = 0.01 p.u. & u_3 = 0.01 p.u., while the simulated bus voltages V_n' are also depicted in Table 2. The percentage error of simulated versus estimated values of bus voltages are determined by $\%error = \frac{\widehat{V}_n - V_n'}{V_n'} \times 100$. The last column of Table 2 shows the values of percentage errors which are all well below 0.5%.

Table 2: Percentage Error of Estimated results and Simulated results for Δu_1 = 0.01 p.u.,Δu_2 = 0.01 p.u. & Δu_3 = 0.01 p.u.

Time instant	Bus no.	$\widehat{V}_n$ (in p.u.)	V_n'(in p.u.)	% error
$t = 1.15s$	Bus 4	0.9661	0.9662	-0.0073
$t = 1.15s$	Bus 5	0.7134	0.7131	0.0503
$t = 1.15s$	Bus 8	0.8231	0.8229	0.0261
$t = 22.75s$	Bus 4	1.0196	1.0197	-0.0007
$t = 22.75s$	Bus 5	0.8454	0.8453	0.0017
$t = 22.75s$	Bus 8	0.9466	0.9465	0.0012
$t = 40.05s$	Bus 4	0.9918	0.9915	0.0341
$t = 40.05s$	Bus 5	0.7916	0.7907	0.1025
$t = 40.05s$	Bus 8	0.9063	0.9056	0.0691

In order to validate our approach of computing the trajectory sensitivity with respect to loads as the control inputs, we designate the load at bus 6 as control-4 (u_4), where $u_4 = P_B^4 + jQ_B^4$. Now, keeping other control inputs u_1, u_2 and u_3 unchanged, for $\Delta u_4 = \Delta P_B^4 + j\Delta Q_B^4 = 0.01 + j0.01$, the trajectory-sensitivity based predicted versus the simulated voltages are obtained using the process as described earlier in Table 3, followed by the calculation of percentage errors, which was found to less than 0.5% (see Table 4). Note in Table 3, in accordance to the previous notation, $S_{nj}^{P_B}$ & $S_{nj}^{Q_B}$, represents trajectory sensitivity of nth bus with respect to base active power (P_B) and base reactive power (Q_B) of the jth load.

Table 3: Simulation Results under Base Load.

Time-instant	Bus no.	V_n(in p.u.)	$S_{n4}^{P_B}$	$S_{n4}^{Q_B}$
$t = 8.50s$	Bus 4	1.0222	-0.0729	-0.0625
$t = 8.50s$	Bus 5	0.89336	-0.0514	-0.0405
$t = 8.50s$	Bus 8	0.96806	-0.0576	-0.0480
$t = 28.00s$	Bus 4	1.0089	-0.1348	-0.0888
$t = 28.00s$	Bus 5	0.86050	-0.1051	-0.0665
$t = 28.00s$	Bus 8	0.94473	-0.1084	-0.07099
$t = 38.60s$	Bus 4	0.99538	-0.1877	-0.1144
$t = 38.60s$	Bus 5	0.83686	-0.1782	-0.1068
$t = 38.60s$	Bus 8	0.92645	-0.1718	-0.1051

These results validate the proposed method of computation of trajectory sensitivities with respect to the control inputs as proposed by our control-augmented state-space method, numerical integration, and their PSAT implementation.

4.2 A Specific Application of the Implementation

The proposed implementation extends the functionality of the software PSAT for computation trajectory sensitivities with respect to any control inputs and parameters other than the system variables, enhancing the PSAT's capabilities beyond simulation, to control synthesis.

As an example, in Model Predictive based controller based voltage stability scheme, the common control actions are switching of shunt capacitors, raising/lowering of Under load tap-changers, and exercising load-shedding in a coordinated manner. The corresponding optimization problem is complex in case of large power system comprising of highly non-linear component dynamics. In this context, the computation of voltage trajectory sensitivities with respect to control inputs provides an efficient way to estimate the effect of control inputs on voltage trajectories, and selecting an optimal control strategy.

As discussed earlier, in the available version of PSAT, there is no specific sub-routine to calculate trajectory sensitivity with respect to control inputs. Further such computation also requires the computation of the Jacobian matrices with respect to controls, which are also not made available in PSAT. Our implementation extends PSAT to facilitate the computation of such Jacobians with respect to control inputs as well as parameterized loads by augmenting the respective control inputs and load parameters into the state variables, with zero dynamics. This not only helps to compute trajectory sensitivities with respect to controls, but also enables the computation of load margin sensitivity, and study of neighbouring trajectories under different control actions or variation of parameters.

Table 4: Percentage Error of Estimated versus Simulated results for $\Delta u_4 = 0.01 + j0.01$ with $\Delta u_1 = \Delta u_2 = \Delta u_3 = 0$.

Time instant	Bus no.	$\widehat{V}_n$(in p.u.)	V_n'(in p.u.)	% error
$t = 8.50s$	Bus 4	1.0235	1.02276	0.0797
$t = 8.50s$	Bus 5	0.89428	0.89364	0.07134
$t = 8.50s$	Bus 8	0.96912	0.96839	0.0755
$t = 28.00s$	Bus 4	1.0112	1.0096	0.1545
$t = 28.00s$	Bus 5	0.86222	0.8608	0.1602
$t = 28.00s$	Bus 8	0.94652	0.94511	0.1498
$t = 38.60s$	Bus 4	0.99841	0.99619	0.2219
$t = 38.60s$	Bus 5	0.83971	0.83735	0.2810
$t = 38.60s$	Bus 8	0.92922	0.92697	0.2427

5 CONCLUSION

This paper presented a way of computing trajectory sensitivity with respect to control in PSAT based on augmentation of state-space with zero-dynamics controls. Three types of control were considered:

SVC, ULTC, and Loads. With proper modification of certain parameters of available control models in PSAT, it became possible to achieve the desired zero-dynamics of control, and then to calculate the desired trajectory sensitivities. Our approach thus enables a convenient way for designing of real-time protection schemes, such as MPC, that use trajectory sensitives to quickly estimate future behaviors due to changes in inputs and initial state/algebraic variables. The paper also described the structure of the algorithm for computing trajectory sensitivities, and presented the validation of the trajectory sensitivity computation results by computing the same values through direct simulations of trajectories under various control inputs. The percentage error was found to be no more than 0.5%.

The extended PSAT implementation is further being developed for model-predictive control application.

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