D3 Divisor Cordial Labelling for Butterfly Graph with Shell Order

A. Sudha Rani and S. Sindu Devi*
SRMIST, Chennai, India

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Abstract: In this paper we introduce D3 Divisor Cordial labelling for butterfly graph with shell order, which is based on the concept of divisor cordial labelling. For this define a function β:V(G)→D3 such that for each edge uv, we assign the label 0 if o(u)/o(v) or o(v)/o(u) assign the label 1 if o(u) not divides o(v). Let us define the new function μα (β) which represents several edges of G having label α under the mapping β. Now β is called D3 divisor cordial labelling if |μ1 (β) −μ0 (β)| ≤ 1. The graph which satisfies the above condition is called the D3 divisor cordial labelling graph. Here we discuss the shell graph, butterfly graph, and butterfly graph with shell order graphs undergoing Dihedral group divisor Cordial Labelling.

1 INTRODUCTION

The butterfly graph is a graph with two vertices and two edges, where each vertex is connected to the other vertex by an edge, and each vertex has a loop (an edge that connects it to itself). The shell order refers to the placements of vertices in concentric shells or layers around a central vertex.

The field of graph theory is very important in many domains. One of the main applications of graph theory is graph labelling which is utilized in several disciplines such as database management, astronomy etc.

Graph labelling is the process of assigning values to vertices, edges, or both under a specific condition or conditions. A graph labelling is a map connecting the graph's elements to a collection of numbers, typically a collection of nonnegative or positive integers. Edge labelling is used when the domain is the set of edges. Total labelling is the term used when labels are applied to both vertices and edges.

This essay focuses on the finite, simple, undirected graph G with p nodes and q edges. G is also known as a (p, q) graph. For information on the ideas of graph theory and Abstract Algebra, see the references (Bondy and Murty 1976), (Harary 1972), and (Dummit and Footy 2004). Gondalial 2020, proved ring sum of the helm with star graph, gear with star graph, double wheel with star graph, jellyfish with star graph, and gem with star graph is a cordial graph divisor for information on the ideas of divisor cordial labelling we refer (Varatharajan et al. 2020) introduced the divisor cordial labelling approach. We refer to (Burton David 1980) for elementary number theory, Pair sum labelling's a theory that Ponraj et al 2010. Labelling is essential in areas like radar, networks, coding theory, and signal processing. We refer to Maya. et.al 2014 for Some New Families of Divisor Cordial Graph. (Lawrence Rozario Raj. et.al 2014) proved "Divisor Cordial Labelling of Some Disconnected Graphs. In this paper we discussed the D_3Dihedral divisor cordial labelling undergoes shell graph S_n, n≥ 6 (excluding the apex vertex), butterfly graph, butterfly graph with shell order (q,q).

2 PRELIMINARIES

2.1 Butterfly Graph

The butterfly graph $BF_{m,n}$ is a two even cycles of the same order say C_n, sharing a common vertex with m pendant edges attached at the common vertex is called a butterfly graph.

2.2 Shell Graph

A shell S_n is the graph obtained by taking (n-3) concurrent chords in a cycle C_n. The vertex at which

* Corresponding author.

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all the chords are concurrent is called the apex. The shell is also called fan f_{n-1}.

2.3 Multiple Shell Graph

A multiple shell is a collection of edge disjoint shells that have their apex in common. Hence a double shell consists of two edge disjoint shells with a common apex.

2.4 Bow Graph

A bow graph is a double shell in which each shell has any order.

2.5 Divisor Cordial Labelling

A divisor Graph G with vertex set V is cordial labelled by a bijection f from V to {1, 2,..., | V |} such that if each edge uv is given the label 1 if f(u) divides f(v) or f(v) divides f(u), and 0 otherwise, then the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1 then it is called divisor cordial labelling.

3 MAIN RESULTS

3.1 D_3 Divisor Cordial Labelling

Let β: V(G) →D_3 be a mapping such that for each edge ab we assign the label 0 if order of u divides order of v or order of v divides order of u and assign the label 1 if order of u not divides order of v. Let us define the new function $\emptyset_\alpha(\beta)$ which representthe number of edges of G having label α under the mapping β. Now β is called Dihedral group divisor cordial labelling if $|\emptyset_1(\beta) - \emptyset_0(\beta)| \leq 1$ and $\left|V_\beta(a) - V_\beta(b)\right| \leq 1$.where $V_\beta(a)$ represents number of vertices having label a under β.The graph which satisfies above condition is called Dihedral group divisor cordial labelling graph.

3.2 Order of an Element

Let G be an undirected graph without loops and multiple edges. Let us consider Dihedral group whose elements are e, a,a^2,b,ab,a^2b whose structure is given below e = (1)(2)(3), a = (123), a^2 = (132), b=(12), ab = (13), a^2b = (23) Order of each element is given by O(e) =1,O(a)=3,O(a^2)=3,O(b)=2, O(ab)=2,O(a^2b)=2.

In this section we discuss the shell graph, butterfly graph and butterfly graph with shell order graphs undergoes Dihedral group divisor Cordial Labelling.

Theorem: 3.1

The shell graph S_n, n ≥ 6 (excluding the apex vertex) is D_3 divisor cordial labelling graph.

Proof:

G should be a shell graph. The vertices and edges of the graph G are defined as V(G) ={k, m_j: j varies is from 1 to n and E(G) ={e'_j = k m_j: j varies is from 1 to n; e_j = $m_j m_{j+1}$:1 ≤ j ≤ (n − 1)}

For n ≥ 6 we discuss six cases, consider the apex vertex k = a

Case 1: n is a multiple of 6s

Let $n = 6p, p \geq 1, s \geq 1.$

Let us define the function as $\beta: V(S_n) \to D_3$ by assigning as

$\beta(m_j) = \{ab : j - 1 = 6s,$

$\beta(m_j) = \{a^2b : j - 2 = 6s$, $\beta(m_j) = \{a^2 : j - 3 = 6s$, $\beta(m_j) = \{b : j - 4 = 6s$, $\beta(m_j) = \{a : j - 5 = 6s$, $\beta(m_j) = \{e : j - 6 = 6s.$

In this instance the vertices labelled as e, a^2, b,a^2b, ab will appears p times and the vertex a will appear 2p times in the D_3 Dihedral group, and each edge labelled as 0 will appear 6p-1 times and 1will occur 6p times respectively. As a result, in this instance we obtain β as Dihedral group D_3divisor cordial labelling.

Case 2: $n - 1$ is a multiple of 6s

Let $n = 6p + 1, p \geq 1, s \geq 1.$ For $1 \leq n \leq 6p$ we assign the same labelling as in case 1,

For thr remaining vertices we assigning as $\beta(m_j) = \{ab, j - 1 = 6s$

In this instance, the vertices e, a^2, $b. a^2b$ will appears p times and the vertices a, ab will appear 2p times. in the D_3 Dihedral group, and each edge labelled as 0 will appear 6p times and 1will occur 6p+1 times respectively. As a result, in this instance we obtain β as Dihedral group D_3divisor cordial labelling.

Case 3: $n - 2$ is a multiple of 6s

Let $n = 6p + 2, p \geq 1, s \geq 1.$ For $1 \leq n \leq 6p$ we assign the same labelling as in case 1

For the remaining vertices we assigning as

$$\beta(m_j) = \begin{cases} a^2; j-1 = 6s \\ a^2b; j-2 = 6s \end{cases}$$

In this instance, the vertices a,a^2,a^2b will appears 2p times and the vertices e, b, ab will appear p times in the D_3 Dihedral group, and each edge labelled as 0will appear 6p+1 times and 1 will occur 6p+2 times respectively. As a result, in this instance we obtain β as Dihedral group D_3 divisor cordial labelling.

Case 4: $n-3$ is a multiple of 6s

Let $n = 6p+3, p \geq 1, s \geq 1$. For $1 \leq n \leq 6p$ we assign the same labelling as in case 1

For the remaining vertices, we assign as

$$\beta(m_j) = \begin{cases} b; j-1 = 6s \\ a^2; j-2 = 6s \\ e; j-3 = 6s \end{cases}$$

In this instance, the vertices e, a,a^2, b will appear 2p times and the vertices ab,a^2b will appear p times in the D_3 Dihedral group, and each edge labelled as 0 will appear 6p+3 times and 1 will occur 6p+2 times respectively. As a result, in this instance we obtain β as Dihedral group D_3divisor cordial labelling.

Case 5: $n-4$ is a multiple of 6s

Let $n = 6p+4, p \geq 1, s \geq 1$. For $1 \leq n \leq 6p$ we assign the same labelling as in case 1

For the remaining vertices we assigning as

$\beta(m_j) = \{a^2 : j-1 = 6s,$

$\beta(m_j) = \{ab: j-2 = 6s$, $\beta(m_j) = \{e: j-3 = 6s$, $\beta(m_j) = \{a^2b : j-4 = 6s$

In this instance, the vertices e,a^2, a.a^2b,ab will appear 2p times and the vertex b will appear p times in the D_3 Dihedral group, and each edge labelled as 0will appear 6p+4 times and 1 will occur6p+3 times respectively. As a result, in this instance we obtain β as Dihedral group D_3divisor cordial labelling.

Case 6: $n-5$ is a multiple of 6s

Let $n = 6p+5, p \geq 1, s \geq 1$. For $1 \leq n \leq 6p$ we assign the same labelling as in case 1

For the remaining vertices we assigning to

$\beta(m_j) = \{a^2 : j-1 = 6s,$

$\beta(m_j) = \{ab: j-2 = 6s$, $\beta(m_j) = \{e: j-3 = 6s$, $\beta(m_j) = \{a^2b : j-4 = 6s$, $\beta(m_j) = \{b : j-5 = 6s,$

In this instance, all the vertices e,a^2, a., b, ab, b will appear 2p times in the D_3 Dihedral group, and each edge labelled as 0will appear 6p+5 times and 1will occur 6p+4 times respectively. As a result, in this instance we obtain β as Dihedral group D_3divisor cordial labelling.

Table 1.

Nature	V_β(e)	V_β(a)	$V_\beta(a^2)$
$6p$	P	2p	P
$6p+1$	P	2P	p
$6p+2$	p	2P	2p
$6p+3$	2p	2p	2p
$6p+4$	2p	2p	2p
$6p+5$	2p	2p	2p

Table 1 shows number of times the vertices V_β(e), V_β(a), $V_\beta(a^2)$ will appear.

Table 2.

$V_\beta(ab)$	V_β(b)	$V_\beta(a^2b)$	$\emptyset_1(\beta)$	$\emptyset_0(\beta)$
P	P	P	6p	6p-1
2P	P	P	6p+1	6p
p	P	2p	6p+2	6p+1
P	2p	P	6p+2	6p+3
2p	P	2p	6p+3	6p+4
2p	2p	2p	6p+4	6p+5

Table 2 shows number of times the vertices $V_\beta(ab)$, V_β(b), $V_\beta(a^2b)$ will appear and also shows the number of times edges labelled as 0 and 1.

Hence all butterfly graphs $BF_{q,q}$ are Dihedral group divisor cordial labelling graph.

Theorem 3.2

The butterfly graph $BF_{q,q}$ is D_3 divisor cordial labelling for all n ≥ 3.

Proof: Let $G = BF_{q,q}$is a butterfly graph whose vertices are given by $m_1, m, \ldots m_q$, $n_1, n_2, \ldots n_q$ and edge set is E $(BF_{q,q}) = \begin{cases} m_j m_{j+1}, 1 \leq j \leq n \\ n_j n_{j+1}, 1 \leq j \leq n \\ n_0 m_j, 1 \leq j \leq n \\ n_0 j, 1 \leq j \leq n \end{cases}$ For $n \geq 3$, we discuss three cases, we consider $n_0 = a^2$ as apex vertex for all the remaining cases.

Case 1: In is a multiple of 3s

Let $n = 3p, p \geq 1, s \geq 1$.

Let us define the function as $\beta: V(BF_{q,q}) \to D_3$ by assigning as

$$\beta(m_j) = \begin{cases} b; j-1 = 3s \\ a^2b; j-2 = 3s \\ a; j-3 = 3s \end{cases}$$

$$\beta(n_j) = \begin{cases} a^2; j-1=3s \\ ab; j-2=3s \\ e; j-3=3s \end{cases}$$

In this instance, all the vertices e, a, b, ab,a^2b will appear p times, the vertex a^2 will appear 2p timesin the D_3 Dihedral group and each edge labelled as 0 will appear 4p+1 times and 1will occur 4p+1 times respectively. As a result, in this instance we obtain β as Dihedral group D_3divisor cordial labelling.

Case 2: $n-1$ is a multiple of $3s$

Let $n = 3p+1, p \geq 1, s \geq 1$. For ,$1 \leq n \leq 3p$ we assign the same labelling as in case 1,

For the remaining vertices we assigning to

$$\beta(m_j) = \{b, j-1=3s$$

$$\beta(n_j) = \{e, j-1=3s$$

In this instance, the vertices e, a^2,b will appear 2p times and a, ab, a^2b will appear p timesin the D_3 Dihedral group, and each edge labelled as 0 will appear 4p+3 times and 1 will occur4p+3 times respectively. As a result, in this instance we obtain β as Dihedral group D_3divisor cordial labelling.

Case 3: $n-2$ is a multiple of $3s$

Let $n = 3p+2, p \geq 1, s \geq 1$. For ,$1 \leq n \leq 3p$ we assign the same labelling as in case 1,

For the remaining vertices we assigning as

$$\beta(m_j) = \begin{cases} b; j-1=3s \\ a^2b; j-2=3s \end{cases}$$

$$\beta(n_j) = \begin{cases} e; j-1=3s \\ ab; j-2=3s \end{cases}$$

In this instance, the vertex a will appear p times and a^2, b,a^2b ,e,ab will appear 2p timesin the D_3 Dihedral group, and each edge labelled as 0 will appear 4p+5 times and 1 will occur 4p+5 times respectively. As a result, in this instance we obtain β as Dihedral group D_3divisor cordial labelling.

Table 3.

Nature	$V_\beta(e)$	$V_\beta(a)$	$V_\beta(a^2)$	$V_\beta(b)$
3p	P	P	2p	P
3p + 1	2p	P	2p	2p
3p + 2	2p	P	2p	2p

Table 3 shows number of times the vertices $V_\beta(e)$, $V_\beta(a)$, $V_\beta(a^2)$, $V_\beta(b)$ will appear.

Table 4.

$V_\beta(ab)$	$V_\beta(a^2b)$	$\emptyset_1(\beta)$	$\emptyset_0(\beta)$
P	P	4p+1	4p+1
P	P	4p+3	4p+3
2p	2p	4p+5	4p+5

Table 4 shows number of times the vertices $V_\beta(ab)$, $V_\beta(a^2b)$will appear and number of times the edges labelled 0 and 1.

Hence all butterfly graphs $BF_{q,q}$ are Dihedral group divisor cordial labelling graph.

Theorem 3.3

The butterfly graph with shell order (q, q) (order excludes the apex) is Dihedral group divisor cordial labelling.

Proof:-

Make G a butterfly graph by omitting the apex from the shell of order (q,q). Define V(G) = {k_0, k_1, k_2, m_j , n_j : $1 \leq j \leq y$} and E(G) = $e'_1 = k_0k_1$,$e'_2 = k_0k_2$, $e_j = k_0m_i$ and $e_{2y-1+j} = k_0n_j$: $1 \leq j \leq y$, $e_{y+j} = m_jm_{j+1}$ and $e_{3y-1+j} = n_jn_{j+1}$: $1 \leq j \leq (y-1)$}are the vertices and edges of the graph G.

Here For $n \geq 3$ we discuss 3 cases by keeping k_0 = a, k_1 = b, k_2= e as apex vertices.

Case 1: n is a multiple of $3s$

Let n = 3p, $p \geq 1$, $s \geq 1$.

$$\beta(m_j) = \begin{cases} a; j-1=3s \\ a^2b; j-2=3s \\ ab; j-3=3s \end{cases}$$

$$\beta(n_j) = \begin{cases} e; j-1=3s \\ a^2; j-2=3s \\ b; j-3=3s \end{cases}$$

In this instance, the vertices labelled as b, a, e will appear 2p times, the vertices labelled as a^2, ab, a^2b will appear p times in the D_3 Dihedral group, and each edge labelled as 0 appearing 6p times and 1 appearing 6p times respectively. As a result, in this instance, we obtain β as Dihedral group D_3 divisor cordial labelling.

Case 2: $n-1$ is a multiple of $3s$

Let n = 3p+1, $p \geq 1$, $s \geq 1$. For $1 \leq j \leq 3p$ we assign the same labelling as in case 1

The remaining vertices are labelled as

$$\beta(m_j) = \{ab, j-1=3s$$

$$\beta(n_j) = \{a^2, j-1=3s$$

In this instance, the vertices labelled as e, a,a^2,b will appear 2p times, the vertices labelled as ab, a^2b will appear p times in the D_3 Dihedral group, and each edge labelled as 0 appearing 6p+2 times and 1 appearing 6p+2 times respectively. As a result, in this instance, we obtain β as Dihedral group D_3 divisor cordial labelling.

Case 3: $n - 2$ is a multiple of $3s$

Let n = 3p+2, p ≥ 1, s ≥ 1.

For 1 ≤ j ≤ 3p we assign the same labelling as in case 1. The remaining vertices are labelled as

$$\beta(m_j) = \begin{cases} a^2b; j-1=3s \\ ab; j-2=3s \end{cases}$$

$$\beta(n_j) = \begin{cases} b; j-1=3s \\ a^2; j-2=3s \end{cases}$$

In this instance, the vertices labelled as b will appear 3p times, the vertices labelled as e, a,a^2, ab, a^2b will appear 2p times in the D_3 Dihedral group each edge labelled as 0 appearing 6p+4 times and 1 appearing 6p+4 times respectively. As a result, in this instance, we obtain β as Dihedral group D_3 divisor cordial labelling.

Hence the butterfly graph with shell order (q, q) is Dihedral group D_3 divisor cordial labelling.

4 CONCLUSIONS

In this paper, we show that butterfly graph, butterfly with the shell order (q,q), and butterfly with the shell order (q, 2q) undergoes Dihedral group D_3 divisor cordial labelling In the future, we intend to exhibit numerous graph labels of graphs connected to shells.

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