# Job Block Scheduling in a Two-Stage, No-Idle Flow Shop with Job Weighting to Reduce Total Machine Rental Costs

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Abstract: The handling interval of the jobs is connected with likelihoods and the two of the jobs have stood together as a block in the current paper's study of a flow shop scheduling model in two stages under no idle restriction. Weight of Jobs is also introduced due to its practicality and significance value in the actual world scenarios. The objective of the study is to present a heuristic algorithm that, when used, provides an ideal or nearly optimal schedule to reduce the amount of downtime and lower rental prices. The effectiveness of the proposed approach is demonstrated through a numerical sample. This work can also be extended by considering various parameters like breakdown effect, fuzzy trapezoidal numbers, set up time etc.

## 1 INTRODUCTION

Scheduling is an indispensable process that focuses on the challenges of allocating resources to carry out a series of operations with the objective to identify the optimum solution in light of the need to optimize a function. Scheduling problems arise daily in several production units. The well-known flow shop scheduling problem conforms evaluating the best sequence for two or more jobs to be performed on two or more pre-ordered machines to optimize some measure of effectiveness. The critical constraint in an industrialized flow shop scenario is the no-idle time on machines or the inability to halt a machine after it has been started. As a result, there can be no downtime for the machines as they must run continually.

In the past five decades, there has been considerable attention paid to solve the problem of scheduling. However, (Johnson 1954) prepared the first triumphant mathematical model that successfully acquired an optimal solution for the two and three stage flow shop scheduling problem. The efficacy of Johnson's model garners significant attention from numerous researchers, who are inclined to explore this avenue. Further, in a scheduling paradigm, the weight of each job indicates its position among other jobs in terms of importance. The weight of each job increases with how significant it is for processing in relation to other jobs. From the groundbreaking research conducted by Johnson in 1954, the available scholarly literature pertaining to scheduling models exhibits a notable absence of any discussions regarding the concept of job weightage prior to the year 1980. The first investigation of the m-machine no-idle condition in a flow shop was conducted by (Adiri and Pohoryles 1982). An approach to reduce rental cost for the no idle two-stage flow shop scheduling problem that takes job weighting into account was provided by (Gupta et al. 2021). The comparative analysis of the subsystem failed simultaneously was discussed by (Shakuntla et al. 2011). (Shakuntla et al 2011) discussed the behavior analysis of polytube using supplementary variable procedure. PSO was used by (Kumari et al. 2021) to research limited situations. Using a heuristic approach, (Rajbala et al. 2022) investigated the redundancy allocation problem in the cylinder manufacturing plant.

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# 2 PRACTICAL SITUATION

Numerous practical and empirical scenarios are prevalent in our routine engagement within manufacturing and fabrication environments, wherein diverse tasks necessitate processing on a range of distinct machinery. The cotton industry, leather manufacturing unit, textile factory, etc., are possible practical examples of the weightage of jobs. Different varieties of cotton, shoes, jackets, and fabric of varying sizes or qualities are manufactured with varying degrees of importance in various production facilities. Due to a lack of finances in his early profession, one needs to rent the machines. For example, to start a pathology laboratory, much expensive equipment like a microscope, water bath, lab incubator, glucometer, blood cell counter, organ bath, haematology analyzer, urine analyzer, centrifuge, coagulometer, autoclave, tissue diagnostics, etc., one does not buy these machines but instead take on rent. Medical equipment rentals are quick and reasonably priced, which is the better option. Renting enables saving capital investments, helping choose the right equipment for the job and access the latest technology.

**Assumptions**

Two machines, J and K, process the jobs independently of one another in the following order: JK with no allowance of any inter-machine transfer.

There is no way for two machines to process on the same job at the same time.

Calculating utilization time does not take machine breakdown or setup times into account.

**Rental Policy (P)**

The machines will be rented out as needed and returned as soon as they are no longer needed. i.e., the first machine will be rented when processing jobs begins, and the second machine will be rented when the first work is finished on the first machine.

# 3 NOTATIONS

I: Jobs sequence 1,2,…, n
$S_1$: Optimal sequence using Johnson's technique
J: 1st machine
K: 2nd machine
$P_i$: Probability allied with $j_i$
$Q_i$: Probability allied with $k_i$
$W_i$ : Weightage of job i
$u_1(s_1)$: Utilization time required for machine M1 in sequence $s_1$
$u_2(s_1)$: Utilization time required for machine $M_2$ in sequence $s_1$

# 4 PROBLEM FORMULATION

Assume that two machines J and K are to process certain jobs i (1, 2… n). Finally, let Wi be the ith job's weightage. The matrix-formatted mathematical representation of the model may be expressed as in Table 1. Our objective is to identify the sequence of job {$s_1$} which helps to keep machines' rental costs down.

Table 1: Matrix-Formatted Mathematical Formulation.

| Job | Machine J | | Machine K | | Weight |
|---|---|---|---|---|---|
| *I* | $j_i$ | $p_i$ | $k_i$ | $q_i$ | $W_i$ |
| 1 | $j_1$ | $p_1$ | $k_1$ | $q_1$ | $W_1$ |
| 2 | $j_2$ | $p_2$ | $k_2$ | $q_2$ | $W_2$ |
| 3 | $j_3$ | $p_3$ | $k_3$ | $q_3$ | $W_3$ |
| n | $j_n$ | $p_n$ | $k_n$ | $q_n$ | $W_n$ |

# 5 ALGORITHM

**Step 1:** Calculate the expected processing times, named as $J_i$ & $K_i$, for the machines J & ***K*** respectively:

$$J_i = j_i \times p_i \quad (1)$$

$$K_i = k_i \times q_i \quad (2)$$

**Step 2:** If min $(J_i, K_i) = J_i$, then

$$J_i' = \frac{J_i - W_i}{W_i} \quad (3)$$

and $K_i' = \frac{K_i}{W_i}$

**If min $(J_i, K_i) = K_i$, then**

$$J_i' = \frac{J_i}{W_i} \quad (4)$$

And $K_i' = \frac{K_i + W_i}{W_i}$

**Step 3**: Consider jobs k and m are working in a job block 'α' with fix order of jobs in which priority is given to job k over m. The concept of a job block can be considered as being equivalent to a single job, denoted as α, where α is defined as (l, m).:

$$J'_\alpha = J'_l + J'_m - min\,(J'_m, K'_l)$$

$$K'_\alpha = K'_l + K'_m - (J'_m, K'_l)$$

**Step 4:** Replace jobs $\boldsymbol{l}$ and $\boldsymbol{m}$ with a single job α to transform the given problem into a new one.

**Step 5:** Get the optimum sequence $s_1$ while reducing the overall elapsed time by utilizing Johnson's method (Johnson, 1954).

**Step 6:** For schedule $s_1$, create a flow in- out table and determine total elapsed time $\boldsymbol{T_{i2}}$.

**Step 7:** Calculate

$$l_2 = T_{i2} - \sum_{n=1}^{\infty} k_i \tag{7}$$

**Step 8**: Construct flow in-flow out table for the machines using the most recent time $\boldsymbol{l_2}$ for machine K to begin processing.

Step 9: Calculate utilization time $u_1(s_1)$ and $u_2(s_1)$ of machines J and K by

$$u_1(s_1) = \sum_{n=1}^{\infty} j_i \tag{8}$$

$$u_2(s_1) = T_{i2} - l_2$$

**Step 10:** Finally, calculate

$$r(s_1) = u_1(s_1) * c_1 + u_2(s_1) * c_2 \tag{9}$$

**Numerical Illustration**

Consider Five Jobs and Two Machines With No-Idle Flow Shop Scheduling Problems In Which Processing Times Associated With Probabilities And Job Weightage, Are Given In table 2. Machines J And K Have Rental Costs Per Unit Time of Four and Six Units, Respectively. Our Goal Is To Acquire The Best Possible Job Sequencing at The Lowest Feasible Amount By Considering Jobs 2,4 In A Block (2,4) That The Machines May Be Rented Out For.

Table 2: Data Set for the Indicated Problem.

| Jobs | Machine J | | Machine K | | Weight |
|---|---|---|---|---|---|
| I | $j_i$ | $p_i$ | $k_i$ | $q_i$ | $W_i$ |
| 1 | 16 | 0.2 | 26 | 0.2 | 2 |
| 2 | 26 | 0.2 | 18 | 0.1 | 3 |
| 3 | 14 | 0.3 | 24 | 0.2 | 1 |
| 4 | 7 | 0.2 | 3 | 0.3 | 5 |
| 5 | 16 | 0.1 | 6 | 0.2 | 4 |

**Solution:** Table 3 presents, in accordance with Step 1, the expected processing times for machines J and K are as follow:

Table 3: Expected Process Time on Machines.

| I | $J_i$ | $K_i$ | $W_i$ |
|---|---|---|---|
| 1 | 3.2 | 5.4 | 2 |
| 2 | 5.4 | 1.8 | 3 |
| 3 | 4.2 | 4.8 | 1 |
| 4 | 1.4 | 0.9 | 5 |
| 5 | 1.6 | 1.0 | 4 |

The weighted flow shop times $J_i'$ & $K_i'$ are displayed in table 4 according to Step 2.

Table 4: The Weighted Flow Shop Times.

| Jobs I | Machine J IN-OUT | Machine K IN-OUT | Wi |
|---|---|---|---|
| 5 | 0-1.6 | 6.7-7.7 | 4 |
| 1 | 1.6-4.8 | 7.7-13.1 | 2 |
| 2 | 4.8-10.2 | 13.1-14.9 | 3 |
| 4 | 10.2-11.6 | 14.9-15.8 | 5 |
| 3 | 11.6-15.8 | 15.8-20.6 | 1 |

Designating it by α, as per **step 3.** Equation (5)(6)is used to calculate how long a single job α will take to process on the two machines:

$$J_\alpha' = J_2' + J_4' - min\,(J_4', K_2') = 1.8$$

$$K_\alpha' = K_2' + K_4' -(J_4', K_2') = 2.5$$

Table 5 Presents, in accordance with Step-4, the two processing times $J_i'$and $K_i'$.

Table 5: Portable Process Times for An Equivalent Job.

| I | $J_i2$ | $K_i2$ |
|---|---|---|
| 1 | 0.6 | 2.7 |
| α | 1.8 | 2.5 |
| 3 | 3.2 | 4.8 |
| 5 | 0.4 | 1.25 |

**As per Step 5;** Adopting Johnson's method, the order of the optimum sequence with minimum elapsed time is

$s_1$= 5– 1 – α – 3 = 5 – 1 – 2 – 4 – 3.

For schedule $s_1$, according to **Step 6,** a flow in- flow out table 6 is depicted below:

Table 6: Flow In-Out Table for Schedule $S_1$

| Jobs | Machine J | Machine K |
|---|---|---|
| 5 | 0-1.6 | 1.6-2.6 |
| 1 | 1.6-4.8 | 4.8-10.2 |
| 2 | 4.8-10.2 | 10.2-12.0 |
| 4 | 10.2-11.6 | 12.0-12.9 |
| 3 | 11.6-15.8 | 15.8-20.6 |

Total elapsed time =20.6

As per **Step-7;** $l_2$ = 20.6 – 13.9 = 6.7

As per **Step 8,** Create the IN-OUT table as indicated in table 7to solve the updated scheduling problem

Table 7: Flow In-Out Table for Route J→ K With Zero Idle Time.

| **Jobs I** | $J_i'$ | $K_i'$ |
|---|---|---|
| 1 | 0.6 | 2.7 |
| 2 | 1.8 | 1.6 |
| 3 | 3.2 | 4.8 |
| 4 | 0.28 | 1.18 |
| 5 | 0.4 | 1.25 |

As per **Step-9;** $u_1(s_1)$ = 15.8; $u_2(s_1)$ = 20.6 - 6.7 =1 3.9

As per **Step-10;** $r(s_1) = u_1(s_1) * c_1 + u_2(s_1) * c_2$= 15.8 * 4 + 13.9 * 6 = 146.6 units

Hence the above calculated results obtained for machine route J →K of the optimal sequence $s_1$={5, 1, 2, 4, 3} are described in table 8

Hence from the above table 8, we conclude that the proposed heuristic algorithm created for machine route J →K provides the minimum utilization time and rental cost for optimum solution $s_1$.

Table 8: Comparative Analysis of Results.

| Machine Route J → *K* | Utilization Time of K | Rental Costs |
|---|---|---|
| Proposed Algorithm | 13.9 units | 146.6 units |
| Johnson Algorithm | 19.0 units | units |

## 6 CONCLUSION

The proposed heuristic algorithm in this paper provides an efficient solution to no-idle two stage flow shop scheduling problem considering various factors such as processing time, job weightage and job block criteria by simultaneously optimizing the rental cost and utilization time. This work can also be extended by considering various parameters like breakdown effect, fuzzy trapezoidal numbers, set up time etc.

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