Sum Divisor Cordial Labelling of Sunflower Graphs

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Abstract: Consider the simple graph G with vertex set W, let g: W→ {1, 2 . . . |W|} be a bijective function of G. The function f is known as SDC labeling if the distinction between the number of lines categorized with 0 and the number of lines categorized with 1 is less than or equal to one such that a line xy is categorized 1 if 2 divides sum of f(x) and f(y), and categorized 0 otherwise for every line. A graph that is having SDC labeling is referred to as an SDC graph. This paper shows that the sunflower graph is an SDC graph for all n≥ 3.

1 INTRODUCTION

Graph theory is the study of relationships between objects. Graph theory is an ancient subject with numerous exciting modern applications. Graph theory is an important part of many different fields. (Chakraborty et al., 2018) demonstrated the use of graphs in social networks, whose complexity is increasing as social media advances. Graph theory is related to chemistry. Most theoretical chemists used mathematics to crunch numerical data until recently, but graph theory has influenced a shift toward non-numerical techniques. Labeling is one of the topics in graph theory. It has many applications in pure, applied mathematics and natural science. Some of the fields where graph labeling applies include coding theory, x-ray, crystallography, astronomy, network theory, etc. (Prasanna, 2014) demonstrated how graph labeling applies in network security, the numerical network portion of an IP address, the channel assignment process, and social media. (Kumar & Kumar vats, 2020) explained how graph labeling applies in crystallography. Graph theory principles are also used in several computer science areas such as database management systems, software architecture, algorithm design, multiprocessing, data structure, and so on. (Vinutha, 2017) has discussed how graph coloring and labeling applies in computer science. Graph coloring is used in GSM networks, aircraft scheduling etc. Also graph labeling is applied in many areas in computer science. For example, signal interference from different radio station is avoided by assigning channel to each station through the use of radio labeling.

There are many labeling in graph theory. They are prime labeling, magic labeling, graceful labeling, edge labeling, radio labeling and many others. In graph theory, one of the labeling is sum divisor cordial labeling. Sum divisor cordial graphs have thrilling features which are captivating to discover as it isn't every graph that allow sum divisor cordial labeling (Lourdusamy St Xavier & Patrick St Xavier, 2016).

Graph labeling involves assigning integers to nodes(vertices) or lines(edges) or both, according to certain rules. Labeling is referred to as vertex labeling if the domain of the function is the set of nodes. Labeling is known as edge labeling if the domain of the function is the set of lines. Total labeling is labeling where labels are assigned to both vertices and edges of the graph. For more information about this, we can refer (Gallian, 2018). (Thomas et al., 2022) have proved that integer cordial labeling admits on some graphs like olive tree, jewel graph, and crown graph. (Mitra & Bhoumik, 2022) have proved that a few graphs are tribonacci cordial graphs. (Abhirami et al., 2018) have mentioned even sum cordial labeling that admits a few graphs like the crown graph, comb graph, and many others. The topic of divisor cordial labeling became started out the way of (Varatharajan et al., 2011). (U. Prajapati & Prerak, 2020) have proved that friendship-related graphs are divisor cordial graphs. Also (Barasara & Thakkar, 2022) have proved that ladder-related graphs are sum

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divisor cordial graphs. Recently (Sharma & Parthiban, 2022) have proved that the Lilly graph is a divisor cordial graph. (Kanani & Bosmia, 2016) have proved that the flower graph Fln satisfies the axioms of the cube divisor cordial graph for all n. The idea of sum divisor cordial labeling was initiated with the aid of (Lourdusamy St Xavier & Patrick St Xavier, 2016). Currently (Adalja, 2022) has proved that a few bistar-related graphs are sum divisor cordial graphs. Also (Lourdusamy & Patrick, 2022) have proved that the axiom of sum divisor cordial labeling has been satisfied for all transformed tree. (U. M. Prajapati & Patel, 2016) have shown that an edge product cordial labeling admits in the sunflower graph for $n \geq 3$. The graph used here is simple and finite.

2 DEFINITIONS

Consider the simple graph G with vertex set W, let g: W→ {1, 2 . . . |W|} be a bijective function of G. The function f is known as DC labeling if the distinction between the number of lines categorized with 0 and the number of lines categorized with 1 is less than or equal to one such that a line xy is categorized 1 if f(x) is a divisor of f(y) or f(y) is a divisor of f(x), and categorized 0 otherwise for every line. A graph that is having DC labeling is referred to as an DC graph.

Consider the simple graph G with vertex set W, let g: W→ {1, 2 . . . |W|} be a bijective function of G. The function f is known as SDC labeling if the distinction between the number of lines categorized with 0 and the number of lines categorized with 1 is less than or equal to one such that a line xy is categorized 1 if 2 divides sum of f(x) and f(y), and categorized 0 otherwise for every line. A graph that is having SDC labeling is referred to as an SDC graph.

A wheel graph is a graph which is formed by cycle and a vertex at the center which connects to all vertices of the cycle. Let W_n be a wheel with x_0 as the center vertex and $x_1, x_2, \ldots\ x_n$ as the nodes of its cycle. The sunflower graph G is formed by adding new vertices $y_1, y_2 \ldots y_n$ such that y_i is connected to x_i, xi_{+1} (mod n) (Ponraj et al., 2015).

2.1 Theorem

The graph SF_n is an SDC graph for all $n \geq 3$.

Proof:

Let G = SF_n

Let $W(SF_n) = \{x_0, x_j, y_j : 1 \leq j \leq n\}$ and $E(G)=\{x_0x_j : 1 \leq j \leq n;\ x_jy_j : 1 \leq j \leq n;\ y_jy_{j+1} : 1 \leq j \leq n-1;\ y_jx_{j+1} : 1 \leq j \leq n-1;\ x_1y_n;\ y_1y_n\}$.

Then the order and size of the graph G are 2n+1 and 4n respectively.

Define g: W(G)→ {1, 2, 3 . . . 2n+1} by:

$g(x_0)= 1$

$g(x_j)= 2j+1,\ 1 \leq j \leq n$;

$g(y_j)= 2j,\ 1 \leq j \leq n$;

Then the induced edge labels are

$g^*(x_0x_j)= 1,\ 1 \leq j \leq n$;

$g^*(x_jy_j)= 0,\ 1 \leq j \leq n$;

$g^*(y_jx_{j+1})= 0,\ 1 \leq j \leq n-1$;

$g^*(y_jy_{j+1})= 1,\ 1 \leq j \leq n-1$;

$g^*(x_1y_n)= 0$;

$g^*(y_1y_n)= 1$;

We notice that, $e_g(0)= 2n$ and $e_g(1)= 2n$.

Thus $|e_g(0)-e_g(1)|= |2n-2n|= 0 \leq 1$

Hence, the sunflower graph SF_n is an SDC graph for all $n \geq 3$.

2.2 Example

The sunflower graph SF_n, in which n=3 is shown in figure 1.

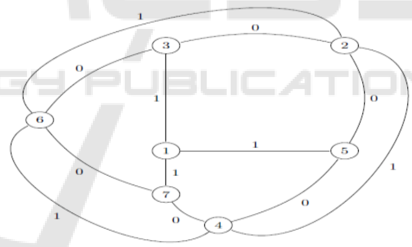

Figure 1.

From figure 1, $|e_g(0)-e_g(1)|= |6-6|= 0 \leq 1$.

So, we conclude that the sunflower graph SF_n, where n= 3 is having SDC labeling.

Hence the sunflower graph SF_n, in which n=3 is an SDC graph.

2.3 Example

The sunflower graph SF_n, in which n=4 is shown in figure 2.

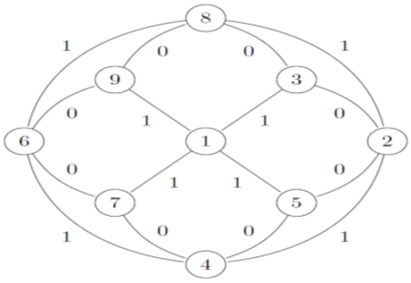

Figure 2.

From figure 2, $|e_g(0)-e_g(1)|= |8-8|= 0\leq 1$.

So, we conclude that the sunflower graph SF_n, where n= 4 is having SDC labeling.

Hence the sunflower graph SF_n, in which n=4 is an SDC graph.

2.4 Example

The sunflower graph SF_n, in which n=5 is shown in figure 3.

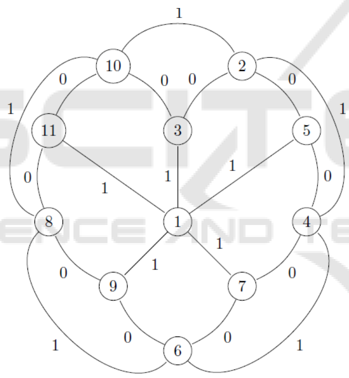

Figure 3.

From figure 3, $|e_g(0)-e_g(1)|= |10-10|= 0\leq 1$.

So, we conclude that the sunflower graph SF_n, where n= 5 is having SDC labeling.

Hence the sunflower graph SF_n, in which n=5 is an SDC graph.

3 CONCLUSION

In this paper, we have shown that the sunflower graph is an SDC graph for all $n\geq 3$.

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