The Robustness of a Twisted Prisoner's Dilemma for Incorporating Memory and Unlikeliness of Occurrence

Akihiro Takahara[1] and Tomoko Sakiyama[2][a]

[1]*Information Systems Science, Graduate School of Science and Engineering, Soka University, Hachioji, Japan*

[2]*Department of Information Systems Science, Soka University, Hachioji, Japan*

Keywords: Spatila Prisoner's Dilemma, Memory, System-Size Analysis.

Abstract: In classical game theory, because players having Defector (D) strategy tend to survive, many studies have been conducted to determine the survival of players with Cooperator (C) strategy. Recently, we have tackled the problem of the evolution of cooperators by proposing a new model called the twisted prisoner's dilemma (TPD) model. In the proposed model, each player is given a memory length. In situations where neighbors had the same strategy as a player and a higher score than that of the player, the player updated their strategy by ignoring the classical SPD update rule. This new strategy was difficult to choose before the update. Consequently, cooperators could survive even if their memory length was small. In this study, by focusing on the system sizes, performance of the TPD model was determined. Similar results were obtained for various system sizes, except when the system size was extremely small.

1 INTRODUCTION

Cooperative behavior is the characteristic present in a population as per the game theory (Smith & Price, 1973). In game theory, propagation as a population in the interaction of cooperative and defective behavior is described (Nowak & May, 1992; Jusup *et al*., 2022). In classical game theory, there are two strategies, Cooperator (C) and Defector (D), both of which interact to obtain a payoff. The earned payoff differs depending on the owner and opponent's payoff. Therefore, a player's strategy with a high payoff is easily passed onto the next generation. However, in classical game theory, cooperators have difficulty surviving and are sensitive to the parameters.

The payoff matrix parameter in classical game theory significantly affects system evolution (Killingback and Coebeli, 1996; Smith and Price, 1973; Szabó and Toké, 1998). Thus, many studies have been conducted on the survival of cooperators (Qin *et al*., 2018; Sakiyama & Arizono, 2019; Sakiyama, 2021). Among them, the prisoner's dilemma is particularly used. Recently, the twisted prisoner's dilemma (TPD) model, which considers the player's memory of their past strategy and sometimes ignores the conventional strategy update rule, has been developed (Takahara & Sakiyama, 2023). This model calculates the frequency of strategies' appearance using each memory. Then, the strategy of low adoption rate is easily adopted by ignoring the classical strategy update rule of the spatial prisoner's dilemma (SPD) model. Several studies have focused on player's past information or the time delay effect (Deng *et al*., 2017; Danku *et al*., 2019). However, most of these studies assume that players can access the "long past." Conversely, unlike previous studies, our model assumes that players can access only recent memories. Thus, our proposed TPD model is more realistic than the classical SPD model. In our previous study using this model, we found that it was insensitive to the payoff matrix parameter and could maintain the cooperators (Takahara & Sakiyama, 2023). In this study, the model's performance was further investigated by focusing on the system size. Many studies on spatial game theory have investigated the effect of varying system sizes (Sakiyama & Arizono, 2019; Frey, 2010).

[a] https://orcid.org/ 0000-0002-2687-7228

Takahara, A. and Sakiyama, T.
The Robustness of a Twisted Prisoner's Dilemma for Incorporating Memory and Unlikeliness of Occurrence.
DOI: 10.5220/0012537800003708

In *Proceedings of the 9th International Conference on Complexity, Future Information Systems and Risk (COMPLEXIS 2024)*, pages 13-16
ISBN: 978-989-758-698-9; ISSN: 2184-5034

2 METHODS

2.1 Simulation Environments

A lattice space was formed with players in every square. The system size of the lattice space could be changed: $10 \times 10, 30 \times 30, 100 \times 100, 100 \times 200, and\ 200 \times 200$ sizes were used in this study. All squares of any system size were initially assigned either the Cooperator (C) or Defector (D) strategy. The initial distribution of the strategies was set to 0.5 for the initial defector density. Thus, both strategies were distributed with the same probability.

The payoffs were arranged as T=b, R=1, S=P=0, according to the payoff matrix depicted in Table 1, where T>R>P=S. The parameter b determining T was set to $1 < b < 2$ (Nowak & May, 1992). If a neighboring player adopted strategy C, a player employing strategy D would receive T as a temptation. Conversely, if a neighboring player employed strategy D, a player using strategy C would receive S as a sucker. A player received P as a punishment if both strategies were D. A player received R as a reward if both strategies were C. We used the Neumann neighborhood and periodic boundary conditions. Each trial included 1000 time steps.

Table 1: Payoff matrix.

		neighbor	
		C	D
Player	C	$R(1)$	$S(0)$
	D	$T(b)$	$P(0)$

2.2 Description of Spatial Prisoner's Dilemma Model

After assigning strategies to the players, the iteration began, during which the players compared their strategies with those of their neighbors based on the payoff matrix and calculated their scores. Then, the players compared their own scores with their neighbors' scores and memorized the neighbor's strategy with the highest score. All players' strategies were then synchronously updated to their memorized strategies. Their strategies remained unchanged in cases where multiple neighbors attained the highest score while employing different strategies.

2.3 Description of the Twisted Prisoner's Dilemma Model

In a previous study, the TPD model has been described (Takahara & Sakiyama, 2023). Every player was allocated a constant memory length value, denoted as θ, which remained unchanged across trials. After every player calculated their scores, they reviewed their previous strategies. The past duration considered spanned from t (current) to $t - \theta$, and the parameter n_c represented the count of cooperative strategies experienced during that period.

If their neighbor's strategy was the same as theirs, while their own score was lower, the player updated the strategy to either C or D using the following two probabilities:

For C:

$$1 - (n\_c)/\theta$$

For D:

$$(n\_c)/\theta$$

If the aforementioned conditions were not met, the strategy update rule of the SPD model was implemented. The strategy of each player is synchronously updated. In the proposed TPD model, the strategy update rule, which uses the p-values excluded from the SPD model, was not executed until $t > \theta$.

3 RESULTS

One hundred trials were performed using thousand-time steps as one trial. The defector density at the 1000-time steps for each trial was calculated and averaged over 100 trials.

First, the proposed TPD model was compared with the conventional SPD model. The system size was 100 × 100. Figure 1 shows the results. The proposed model had a defector density higher than that of the conventional model for $1.0 < b < 1.5$. However, after the parameter b passed 1.5, the proposed model had a lower value than the conventional model, suggesting that the proposed model contributed to the maintenance of the cooperator (Takahara & Sakiyama, 2023).

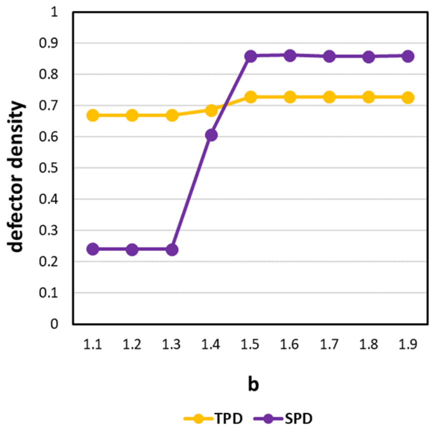

Figure 1: Defector density for the two models—SPD and TPD.

Next, the system-size effects were evaluated by comparing the various system sizes of the TPD model with its 100×100 size. Figure 2 shows the results. Most of the system sizes had similar defector density values. The results indicate that the TPD model is unaffected by changes in the system size and that a certain number of cooperators are maintained even at a certain small system size. However, the defector density of the system size with 10×10 was higher than that of the other system sizes.

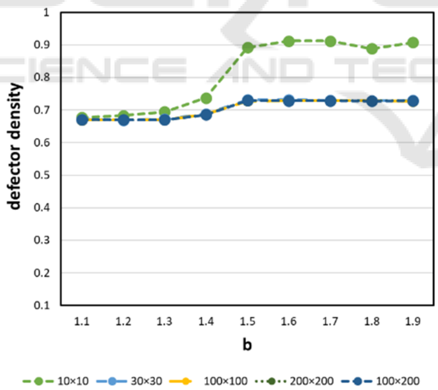

Figure 2: Defector density of the proposed TPD model for various sizes.

Hereafter, the spatial distribution of the small system size was checked to investigate why an extremely small system size affects the performance of the TPD model.

In this study, two different system sizes were investigated. The system size was either 10×10 or 30×30. The spatial distribution was displayed for several time steps ($t = 9, t = 10, t = 11, and\ t = 1000$).

Figure 3 shows the results. Given that $p = 10$ in this case, the C was maintained at $t = 9$ as in the SPD model. In this model, the C is characterized by a form that is maintained as a two-column cross, which is similar to the classical SPD model. However, the player near a cooperator then updated their strategy to C at $t = 10$. It also spread like a wave with each time step. Since a defector in the neighborhood of a cooperator had a smaller score than another defector in the same neighborhood, they had a chance to become cooperators. Also, some of the players who had their original strategy as C updated their strategies to D according to the SPD rules. These strategy updates extended further by forming a characteristic pattern. Finally, C was maintained as sparsely as a 1000-time step.

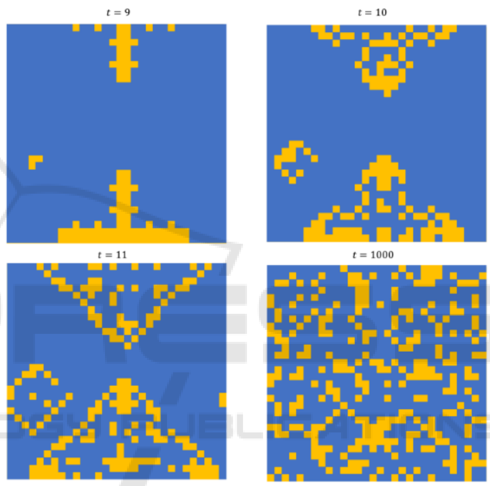

Figure 3: Spatial distribution for mutiple times in the 30×30 system size.

The 10×10 system size results are shown in Figure 4. Similarly, $p = 10$ was set for this system size. Two patterns were found for this system size. In Figure 4A, some cooperators survived until the end. However, wavy spreading could not be observed at $t = 9$, $t = 10$, and $t = 11$. Therefore, it was considered more difficult for C to survive in than in other system sizes. In addition, the cooperators did not appear at all times in Figure 4B, which is supposedly related to the initial placement as per C. Supposedly, they did not form clumps to survive, as shown in Figure 4B. Some trials created a spatial pattern that resembled those in Figure 4A and 4B, resulting in the high defector density shown in Figure 1.

The pattern of early C extinction was observed for small system sizes such as 10×10, whereas it was rarely observed for other larger system sizes.

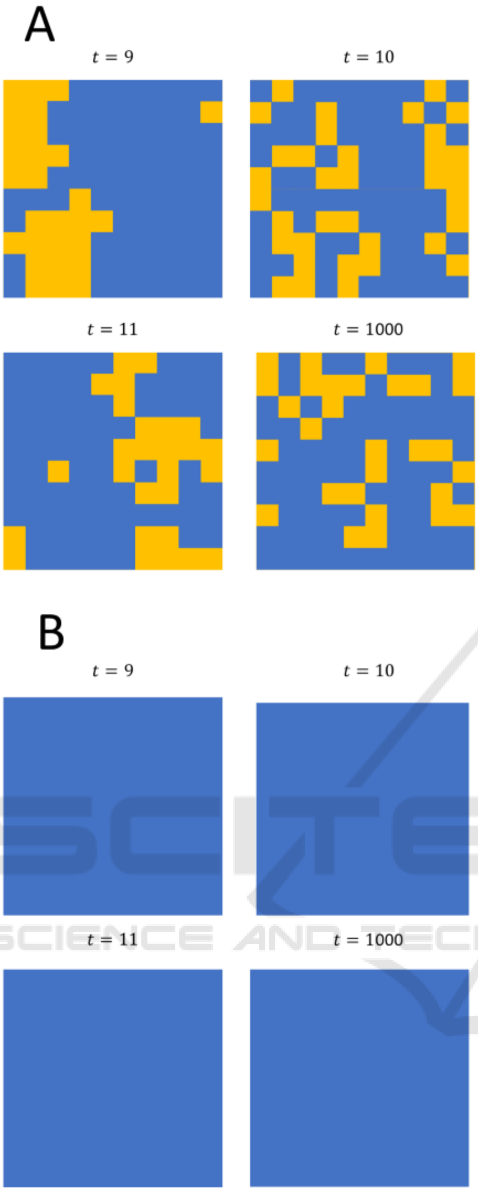

Figure 4: Spatial distribution for mutiple times in the 10×10 system size. Two different examples are shown in A and B panels.

4 CONCLUSIONS

In this study, the TPD model was compared with the conventional SPD model, and the effect of system size on the proposed TPD model was investigated. The system sizes of $10 \times 10, 30 \times 30, 100 \times 100, 100 \times 200, and\ 200 \times 200$ were compared, and the spatial distributions of the two smaller system sizes were compared. Consequently, the defector density results for all system sizes differed insignificantly except for the 10×10 system size, and the strategy C is maintained. In this model, the spatial distribution shows that the C spreads like a wave in a diamond shape (Takahara & Sakiyama, 2023). Even with a spatial distribution of the 30×30 system size, the C spreads like a diamond shape. However, the spatial distribution of the 10×10 system size makes it difficult to form such a wave. This leads to the results shown in Figure 2. In summary, it is found that the proposed model is inventive for various system sizes.

In the future, we will confirm the impact on the model by increasing the system size and changing the network topology.

REFERENCES

Deng, Z., Ma, C., Mao, X., Wang, S., Niu, Z., Gao, L. (2017). Historical payoff promotes cooperation in the prisoner's dilemma game. *Chaos, Solitons & Fractals*. 104, 1–5.

Danku, Z., Perc, M., Szolnoki, A. (2019). Knowing the past improves cooperation in the future. *Scientific Reports*. 9, 262.

Frey, E. (2010). Evolutionary game theory: Theoretical concepts and applications to microbial communities. *Physica. Part A*. 389, 4265–4298.

Killingback, T., Coebeli, M. (1996). Spatial evolutionary game theory: Hawks and Doves revisited. *Proceedings of the Royal Society of London. Series B*. 263, 1135–1144.

Jusup, M., Holme, P., Kanazawa, K., Takayasu, M., Romić, I., Wang, Z., Geček, S., Lipić, T., Podobnik, B., Wang, L., Luo, W., Klanjšček, T., Fan, J., Boccaletti, S., Perc, M. (2022). Social physics. *Physics Reports*. 948, 1–148.

Nowak, M. A., May, R. M. (1992). Evolutionary games and spatial chaos. *Nature*. 359, 826–829.

Qin, J., Chen, Y., Fu, W., Kang, Y., Perc, M. (2018). Neighborhood diversity promotes cooperation in social dilemmas. *IEEE Access*. 6, 5003–5009.

Sakiyama, T. (2021). A power law network in an evolutionary hawk–dove game. *Chaos, Solitons & Fractals*. 146, 110932.

Sakiyama, T., Arizono, I. (2019). An adaptive replacement of the rule update triggers the cooperative evolution in the Hawk–Dove game. *Chaos, Solitons & Fractals*. 121, 59–62.

Smith, J. M., Price, G. R. (1973). The logic of animal conflict. *Nature*. 246, 15–18.

Szabó, G., Toké, C. (1998). Evolutionary prisoner's dilemma game on a square lattice. *Physical Review. Part E*. 58, 69–73.

Takahara, A., Sakiyama, T. (2023). Twisted strategy may enhance the evolution of cooperation in spatial prisoner's dilemma. *Physica. Part A*, 129212.