Univariate GARCH Model for Futures Option Pricing: Application to Silver Mini Futures in Indian Commodity Market

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Abstract: This research investigates the pricing of options related to silver commodity futures within the Indian market, employing a standard univariate Generalized Autoregressive Conditional Heteroscedastic (GARCH) model with a symmetric normal distribution for return modelling. The study evaluates the performance of this option pricing model specifically for silver mini futures options traded on the Multi Commodity Exchange. Furthermore, it compares the option prices determined using the GARCH model parameters with those calculated using the Black-76 model. The findings demonstrate that the option prices derived from the GARCH model fall consistently within the bid-ask price range and significantly outperform the Black-76 model in terms of option pricing accuracy. This underscores the practical utility of GARCH models in the context of the Indian commodity market. To the best of our knowledge, this research marks the pioneering attempt to incorporate parameters generated by the GARCH model for futures option pricing within the Indian commodity market.

1 INTRODUCTION

Commodity markets play a vital role in the global economy by providing a means for investors to mitigate risks and safeguard the long-term value of their assets. This market holds particular significance for manufacturing nations, given that these nations heavily rely on a stable and efficient supply of raw materials, like metals, energy resources and agricultural products to sustain their industrial production processes. The Indian commodity market encompasses the trading of various commodities, including agricultural commodities market, bullion market, energy market, and base metal market (AngelOne, 2023). In India, where the manufacturing sector contributes approximately 16–17% to the GDP, the demand for metals has surged alongside the growth of manufacturing industries (Kakade et al., 2022). With a history spanning centuries, the Indian commodity market has undergone significant growth and is currently one of the fastest-growing markets globally. Regulatory oversight of the commodity market, which was formerly handled by the Forward Markets Commission (FMC) (Masood and Chary, 2016), had been merged with the Securities and Exchange Board of India (SEBI) in 2015. As a result, the SEBI now oversees the commodity derivatives market in India (Dubey and Shankar, 2020).

Commodity trading in India occurs on organized exchanges, like Multi Commodity Exchange of India and the National Commodity and Derivatives Exchange, where futures and options contracts are traded (Hariharan and Reddy, 2018). Participants in the Indian commodity market include producers, processors, traders, speculators, and hedgers. With its capacity for price discovery and risk management, the Indian commodity market holds significant potential for the agricultural, energy, and metal sectors, making it an integral component of the Indian economy (Pani et al., 2022).

The commodity derivatives market has seen substantial growth, highlighting the rising significance of commodities in global financial markets (Dwyer et al., 2011). In India, the commodity options market has become a crucial component of the commodity derivatives market, attracting more participants who utilize options for price risk management (Govindasamy, 2019) and speculative trading on commodity prices. This growth emphasizes the increasing importance of commodity derivatives as both risk management tools and investment opportunities.

Gold and silver options, as well as energy options based on crude oil and natural gas, are typically the

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most actively traded option contracts in the Indian commodity market (IIFLSecurities, 2023). This is primarily due to the significant size and volatility of the bullion and energy markets, which create a demand for risk management tools like options. Bullion commodity options pertain to options contracts based on the price of precious metals, such as gold and silver, which are widely traded on commodity exchanges globally (Bullion, 2023). These options provide the holder with the right, but not the obligation, to buy or sell a specific quantity of bullion or bullion futures at a predetermined price (strike price) within a specified timeframe (Commodity, 2023).

In India, metals possess significant strategic and economic importance, representing a vital and quantifiable element of economic development. They play a crucial role as primary raw materials for a diverse range of essential industries, deeply influencing economic growth (Kakade et al., 2022). Silver is one of the most actively traded commodities in the Indian market due its unique characteristics and wide range of applications including industrial demand and leveraging capability. In this paper, we focus on pricing silver mini commodity futures options in the Indian market using the univariate Generalized Auto-Regressive Conditional Heteroscedastic (GARCH) model. The GARCH model is calibrated using historical silver returns data, and Monte Carlo simulation is used to price the futures options. The model's pricing performance is contrasted with that of the Black-76 model, a flexible framework for valuing futures options. The primary focus of this research is to understand how well the GARCH model captures the time-varying nature of volatility in the commodity market. GARCH models are known for their ability to adapt to changing market conditions, reflecting periods of heightened and subdued volatility. This work aims to investigate whether this dynamic representation of volatility provides a more accurate depiction of the underlying risk factors influencing commodity futures options compared to the constant volatility assumption in the Black-76 model. To the best of our knowledge, this study represents a pioneering approach to incorporate GARCH model-derived parameters for option pricing in the Indian commodity market.

The remainder of the paper is structured as follows: Section 2 presents the literature review related to commodity option pricing, Section 3 presents the detailed methodology employed for pricing options using GARCH model-generated parameters, Section 4 presents the analysis on the performance of GARCH option pricing model and Black-76 model in pricing silver mini futures options and Section 5 presents the conclusion and scope for future research.

2 LITERATURE REVIEW

Several parametric models are made available to price options, with Black-Scholes model being the most used (Sapna and Mohan, 2023; Luo et al., 2022; Sapna and Mohan, 2022). However, the assumption of constant volatility in the Black-Scholes model does not accurately capture real-world dynamics where volatility is dynamic and unpredictable. Stochastic volatility models and GARCH models address this issue by allowing for time-varying volatility, with GARCH models focusing on conditional variance modelling and stochastic volatility models incorporating random changes in volatility over time. Considering the significance of India's commodity market, it becomes imperative to comprehend the ever-changing volatility dynamics in order to develop a robust understanding of the market's inherent risk. These volatility models, often combined with Monte Carlo Simulation, have proven effective in pricing options.

(Berhane et al., 2019) priced Ethiopian commodity options (Coffee and Sesame seeds) using a jump diffusion process. Model parameters were estimated using maximum likelihood estimation, and pricing was done using Monte Carlo simulation. The double exponential jump diffusion model was found to be the most suitable. (Hou et al., 2020) explored Bitcoin's stochastic properties using a stochastic volatility with correlated jump model and priced Bitcoin options based on these properties. They emphasized the inclusion of jumps in volatility and returns for accurate pricing and observed a negative correlation between jumps in volatility and returns.

(Srivastava and Shastri, 2020) examined the suitability of the Black-Scholes model in the Indian Capital Markets using ten popular stocks listed on the National Stock Exchange. The study compared the option prices obtained from the Black-Scholes model with the actual option prices and found a significant mispricing. (Venter et al., 2020) examined the effectiveness of the univariate GARCH model in pricing Bitcoin options and observed that the predicted market price fell within the bid and ask price limits of the option. (Venter and Maré, 2021) used the Heston-Nandi model to price Bitcoin-based Options on Futures. They also introduced a method for pricing multivariate Bitcoin Spread Options. The symmetric Heston-Nandi model was determined to be the most suitable for pricing options on futures. (Venter and Maré, 2022) examined the suitability of the GARCH model for pricing Volatility Index options. The symmetric GARCH (1,1) model with skewed Student-t distribution demonstrated the best performance. (Venter et al., 2022) examined the impact of symmetric and asymmetric GARCH models on the pricing of collateralized and non-collateral options in the South African market. The study revealed that the asymmetric GARCH model had a greater influence on the option price for longer expiration periods. The literature presents consistent findings on the effectiveness of parametric models, particularly GARCH models, in option pricing.

Although options trading has been present in India for more than 15 years, the introduction of commodity options in 2017 was driven by the increasing volatility of commodity prices (Options, 2023). However, there is limited exploration of commodity option pricing in the Indian market, creating opportunities for further investigation using both parametric and non-parametric models. Existing literature frequently employs the Duan model for pricing options based on spot prices or volatility indices (Venter et al., 2020; Duan, 1995; Venter and Maré, 2020). This research focuses on assessing the applicability of the Duan Model specifically for pricing futures options in the Indian commodity market. The study evaluates the Duan model's ability to capture volatility skew in the Indian commodity derivatives market and provides insights into the accuracy of its pricing predictions. The findings can benefit investors and traders engaged in futures options trading by informing their pricing and hedging strategies. Moreover, the results can potentially contribute to the ongoing development and refinement of options pricing models, a crucial area of research in finance and economics.

2.1 Contributions of the Proposed Work

- Duan model is a well known GARCH model used to perform volatility estimation and option pricing. This model has been used effectively to price options based on spot market. However, the applicability of Duan Model in the Futures Options market is yet to be evaluated. The major contribution made by this research work is to test the applicability of Duan Model for the pricing of Futures Options in the Indian commodity market.
- This work evaluates the effectiveness of the Duan model in capturing the volatility skew in the Indian commodity derivatives market and provide insights into the accuracy of the model's pricing predictions.
- This work compares the performance of the proposed model and the traditional Black-76 model with respect to their pricing performance to determine which model aligns well with the observed market behavior and is more suitable for real-world applications.

3 METHODOLOGY

3.1 Black-76 Model

The Black-76 model, also known as the Black model or Black-Scholes model for futures options, is an option pricing model used to determine the theoretical value of European-style options on futures contracts. It is an extension of the Black-Scholes model, which is primarily used for pricing European-style options on stocks. This model determines the option price based on the following parameters: Futures Price (F) which represents the current market price of the futures contract, Strike Price (K) which represents the price at which the option holder has the right to buy (for a call option) or sell (for a put option) the underlying futures contract, Time to Expiration (T) which indicates the remaining time until the option's expiration, risk-free interest rate (r) which indicates the continuously compounded interest rate for the time to expiration of the option. The Black-76 model assumes that the underlying futures contract follows geometric Brownian motion and that option prices are normally distributed. It is widely used in financial markets for pricing options on futures contracts and provides a theoretical framework for valuing these derivatives (Clark, 2014). However, it assumes constant volatility throughout the option's life, which may not always reflect real-world market conditions (Janková, 2018). Equation (1)-(3) represents the Black-76 formula to compute the price of call/put (C/P) futures option with $\delta = 1$ representing call option, $\delta = -1$ representing put option and $N(.)$ represents cumulative normal distribution.

$$V^{C/P} = \delta e^{-rT}[FN(\delta d_1) - KN(\delta d_2)] \tag{1}$$

$$d_1 = \frac{\ln \frac{F}{K} + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}} \tag{2}$$

$$d_2 = \frac{\ln \frac{F}{K} - \frac{\sigma^2}{2}T}{\sigma\sqrt{T}} \tag{3}$$

3.2 Univariate GARCH(1,1) Model

GARCH model is a statistical framework used to analyze and model the volatility of financial time series data. Developed as an extension of the ARCH model, GARCH introduces a more flexible and generalized approach to capturing time-varying volatility (Bollerslev, 1987). The model accounts for the conditional variance of the data, allowing it to adapt to changing market conditions. GARCH models are widely employed in finance to forecast and understand the persistence of volatility, making them valuable tools for risk management, option pricing, and portfolio optimization. In this work, a GARCH model-based parametric approach has been employed to develop a model for pricing commodity futures options by explicitly modelling the fluctuations in the commodity futures price. The behaviour of the futures price movement is captured by fitting a model to the log-returns of the commodity futures price. Maximum Likelihood Estimation is used to determine the parameters of the model. A variety of GARCH models is considered with different error distributions to represent conditional variance. The model that fits best to the available data is chosen to represent the futures price process. Finally, the approximate value of the option is determined by simulating future paths using Monte Carlo Simulation. The architecture diagram for pricing of Futures Options is shown in Figure 1.

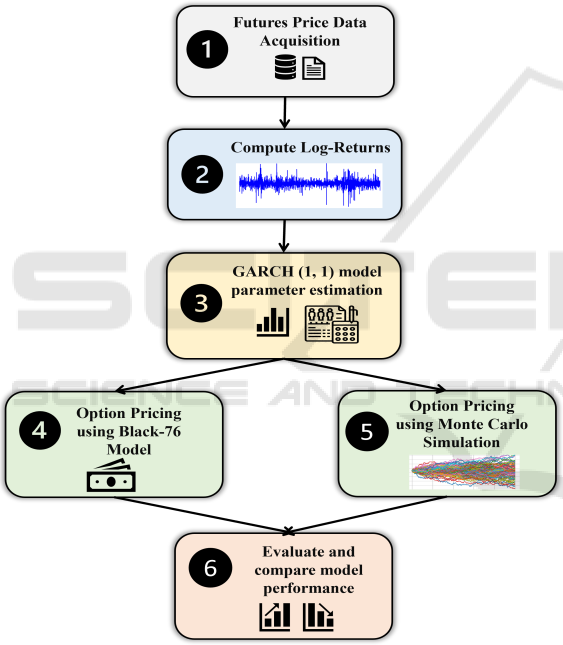


Figure 1: General architecture diagram for pricing silver mini futures options.

In this study, the log-returns of the Silver Mini Futures are modelled using the Duan model (Duan, 1995) for GARCH option pricing with dynamics of the underlying asset under real-world measure represented by Equation (4).

$$R_t \equiv \ln(\frac{F_t}{F_{t-1}}) = r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \varepsilon_t \tag{4}$$

where F_t represents the future price of the commodity at time t, r is the risk-free interest rate considered with continuous compounding, λ is the constant unit risk premium, σ_t^2 represents the conditional variance driven by a GARCH process represents the error term, ε_t represents the error term, i.e., $\varepsilon_t \sim N(0, \sigma_t^2)$, following a symmetric normal distribution.

GARCH models are widely used in finance and economics due to its ability to capture the time-varying volatility in financial data, where the volatility is modelled as a function of past returns and past residuals. In this study, GARCH (1, 1) model is considered to model the conditional variance as given by Equation (5), where σ_t^2 is the conditional variance of the return series at time t, ω is a constant, α is the autoregressive coefficient, and β is the moving average coefficient.

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{5}$$

It is well known that the price of the contingent claim is computed as the expected value of discounted payoff considered under risk-neutral measure (Oosterlee and Grzelak, 2019). However, Equation (4) and Equation (5) provides the representation for log-returns and conditional variance under the real-world measure. Thus, the GARCH dynamics for log-returns and conditional variance under risk-neutral measure is given by Equation (6) and Equation (7) respectively as suggested in (Duan, 1995).

$$R_t \equiv \ln(\frac{F_t}{F_{t-1}}) = r - \frac{1}{2}\sigma_t^2 + \varepsilon_t \tag{6}$$

$$\sigma_t^2 = \omega + \alpha(e_{t-1} - \lambda\sigma_{t-1})^2 + \beta\sigma_{t-1}^2 \tag{7}$$

$$\ln L = -\frac{1}{2}\sum_{t=1}^{N}(\ln\sigma_t^2 + \frac{[\ln\frac{F_t}{F_{t-1}} - r - \lambda\sigma_t + \frac{1}{2}\sigma_t^2]^2}{\sigma_t^2}) - \frac{N\ln(2\pi)}{2} \tag{8}$$

The GARCH model parameters are estimated using maximum likelihood estimation as given in Equation (8). Given the estimated parameters, different realization of the Silver Mini Future price can be simulated with Monte Carlo Simulation using the Equation (6) and Equation (7), and the price of the call and put Options can be computed across the multiple realizations using Equation (9) and Equation (10) respectively, where $T - t_0$ indicates the time to maturity, F_T indicates the underlying asset price at expiry and K indicates the strike price. The algorithm followed for Monte Carlo Simulation is described in Algorithm 1.

$$V_{call} = e^{-r(T-t_0)}\mathbb{E}^{\mathbb{Q}}[max(F_T - K, 0)] \tag{9}$$

$$V_{put} = e^{-r(T-t_0)}\mathbb{E}^{\mathbb{Q}}[max(K - F_T, 0)] \tag{10}$$

Algorithm 1: Monte Carlo simulation-based option pricing.

Data: F – Current futures price, K – strike price, r – rate of interest, T – time to expiry

Result: V(t_0,F) - Price of the option

1 Partition the time interval [0, T], 0 = $t_0 < t_1 <...< t_m = T$ where T represents the time to maturity;

2 Generate asset values, $f_{k,j}$, taking the risk-neutral dynamics of the underlying model with k representing time points and j representing Monte Carlo path;

3 Compute the N payoff values, V_j, where $V_j = V(T, f_{m,j})$, for all Monte Carlo paths;

- Payoff for call option is calculated as $V_j = max(F_T - K, 0)$
- Payoff for put option is calculated as $V_j = max(K - F_T, 0)$

4 Compute the average as $\mathbb{E}^{\mathbb{Q}}[V(T,F) \mid \mathcal{F}(t_0)] \approx \frac{1}{N}\sum_{j=1}^{N} V_j = V_N$;

5 Compute the option value as $V(t_0, F) \approx e^{-r(T-t_0)} V_N$;

Table 1: Descriptive statistics of log-returns.

Measure	Value
Mean	9.255361 x 10^{-5}
Maximum	0.09388484
Minimum	-0.1182392
Standard Deviation	0.01520359
Skewness	-0.2915862
Kurtosis	8.50794
No. of observations	2964

4 RESULTS AND DISCUSSION

In this paper, silver mini options on futures expiring on 17 February 2023 are considered for pricing, with silver mini representing silver traded in lots of 5 kilograms. The historical prices corresponding to Silver Mini futures is considered to build the model. The daily close price of Silver Mini Futures is recorded from 19 December 2011 to 01 February 2023 and is obtained from (Futures, 2023) which derives data from Multi Commodity Exchange of India (MCX, 2023). The graph showing the Futures price movement and log-return for Silver Mini from 19 December 2011 to 01 February 2023 is as shown in Figure 2 and Figure 3 respectively. The descriptive statistics of log-returns of Silver Mini futures price is as shown in Table 1. As per the descriptive statistics, we can see that the mean value of futures return is almost zero representing the stationarity of the time series. Also, the skewness value indicates that the returns are slightly negatively skewed, and kurtosis indicates the leptokurtic nature of the distribution, which is in line with the stylized facts associated to financial return series (McNeil et al., 2015). Normal distribution and skewed Normal distribution were considered to implement the error term in Equation (6). The Akaike Information criterion (AIC) value for model considering normal distribution was lower than that when considering skewed normal distribution. Thus, normal distribution seemed to be a better fit for the implementation, i.e., $\varepsilon_t \sim N(0, \sigma_t^2)$.

Figure 2: Silver Mini futures price movement from 19 December 2011 to 01 February 2023.

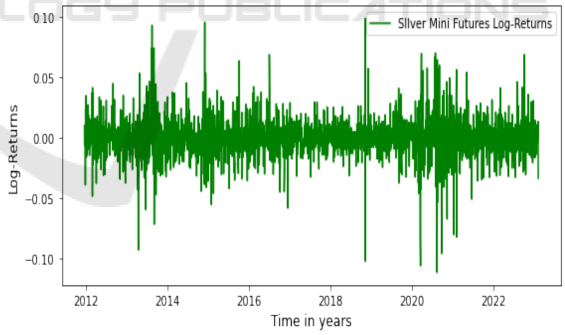


Figure 3: Silver Mini futures price log-returns from 19 December 2011 to 01 February 2023.

The GARCH model was used with a zero-mean process, and the parameters ω, α and β were estimated using maximum likelihood estimation under the real-world measure. The optimal parameter estimates can be found in Table 2. The t-test resulted in a p-value of zero for all parameters, indicating their statistical significance. These parameter values were utilized as the initial values to calculate the value of λ under the risk-neutral measure.

The GARCH (1,1) model parameters were used

Table 2: GARCH (1, 1) model parameter values.

Parameter	Estimate	Standard Error	t-Value	Pr($>|t|$)
ω	0.000007	0.000000	14.359	0
α	0.064425	0.004297	14.993	0
β	0.905420	0.005755	157.339	0

Table 3: Performance metrics for call option pricing.

Expiry	Black-76 Model				GARCH(1,1) Model			
	Bid Price		Ask Price		Bid Price		Ask Price	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
12 days	540.26	330.36	597.47	419.24	68.07	62.04	33.25	28.78
8 days	454.47	309.95	513.85	387.46	36.39	34.40	57.88	56.42
6 days	318.60	214.24	380.95	281.84	70.11	64.54	25.72	22.13
2 days	155.21	107.73	196.42	142.32	40.61	38.23	50.22	48.83

Table 4: Performance metrics for put option pricing.

Expiry	Black-76 Model				GARCH(1,1) Model			
	Bid Price		Ask Price		Bid Price		Ask Price	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
12 days	902.94	863.11	995.06	959.49	43.60	40.60	64.91	55.79
8 days	519.60	401.33	590.53	489.56	36.75	33.01	58.22	55.31
6 days	317.45	214.99	394.35	300.41	76.73	70.23	33.70	29.61
2 days	142.93	96.48	185.37	130.04	65.45	61.43	28.05	24.97

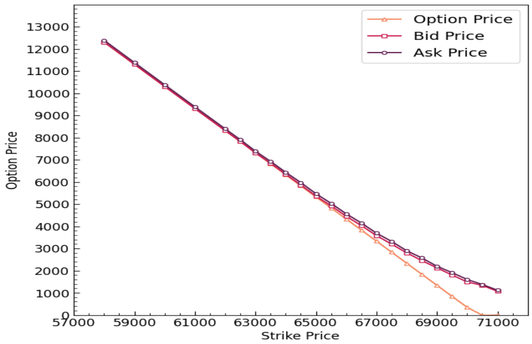
to simulate Monte Carlo paths for the Silver Mini futures price, with the intention of generating a series of potential future price trajectories based on the volatility dynamics captured by the model. A total of 10,000 Monte Carlo paths were simulated for experimentation, focusing on futures options expiring on 17 February 2023. An interest rate of 6.42% was considered from the Reserve Bank of India's 91-day treasury bill rate. Option pricing data from various days to expiry were analyzed to assess the model's accuracy in pricing Silver Mini Futures Options. Figure 4 and Figure 5 display the plot of strike price vs option price for call options and put options respectively, considering the option price generated by Black-76 model as well as GARCH model. It can be clearly seen from Figure 4 and Figure 5 that option price determined by GARCH model is very close to the bid and ask price limits quoted by MCX. Figure 5 display the relationship between strike price and option Price for call Options and put Options, respectively. The performance of Black-76 model for call and put option pricing is seen to be the worst when its far from expiry. However, a significant improvement is observed in its performance near to expiry.

Option pricing performance of the models under consideration is numerically realised using the Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) since the actual proximity of the ask price and bid price to the anticipated option price is not clearly evident in Figure 4 and Figure 5. RMSE measures the average magnitude of the errors between predicted values and observed values. MAE is a measure of the average absolute errors between predicted and observed values. The equations for RMSE and MAE are given by Equation (11) and Equation (12) respectively, where p indicates the model determined option price, $\hat{p}$ indicates the bid/ask price and N indicates the number of strike prices considered.

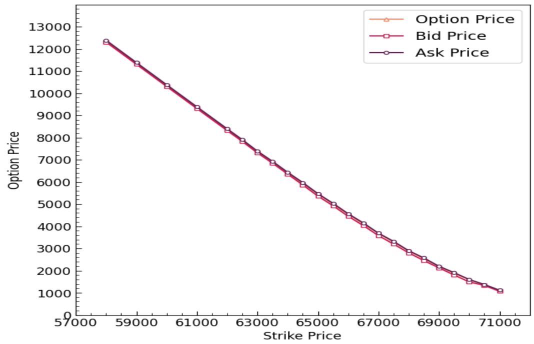
$$RMSE = \sqrt{\frac{1}{N}\sum(p-\hat{p})^2} \tag{11}$$

$$MAE = \frac{1}{N}\sum \mid p-\hat{p} \mid \tag{12}$$

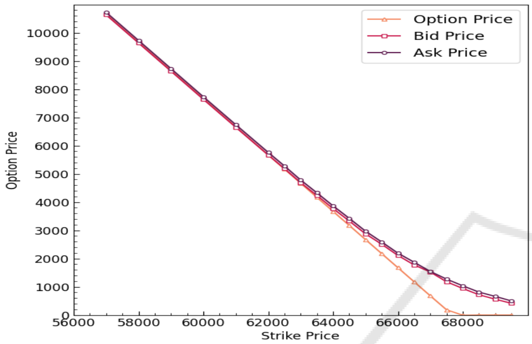
Table 3 compares the performance metrics for bid and ask prices between the Black-76 model and the GARCH(1,1) model at different expiry periods for call options. For the 12-day expiry period, the Black-76 model exhibits higher RMSE and MAE values for bid and ask prices compared to the GARCH(1,1) model. As the expiry period shortens to 8 days, both models show improvements in performance. The Black-76 model's RMSE and MAE values decrease, indicating enhanced predictive accuracy. However, the GARCH(1,1) model continues to outperform, showcasing lower RMSE and MAE for bid and ask prices. In the 6-day expiry period, the GARCH(1,1) model significantly outshines the Black-76 model,



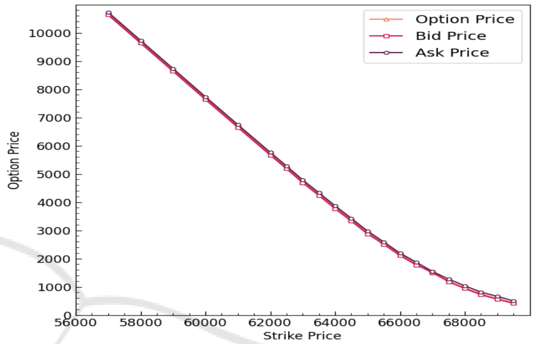
(a) Black-76 model option price - 12 days to expiry.



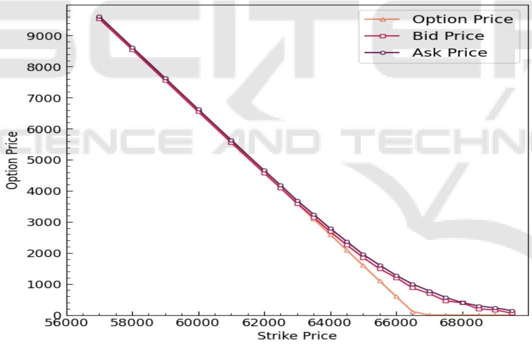
(b) GARCH model option price - 12 days to expiry.



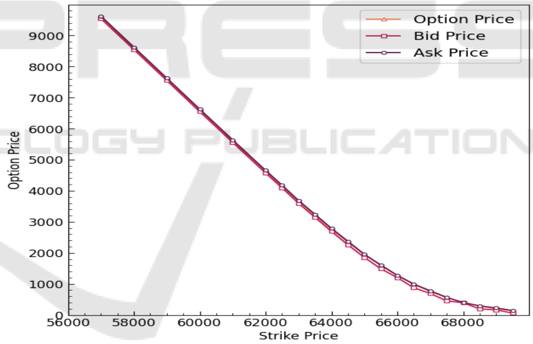
(c) Black-76 model option price - 8 days to expiry.



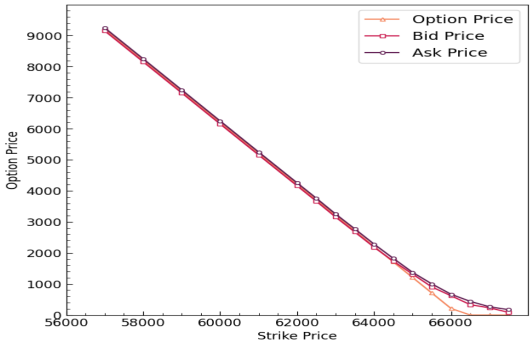
(d) GARCH model option price - 8 days to expiry.



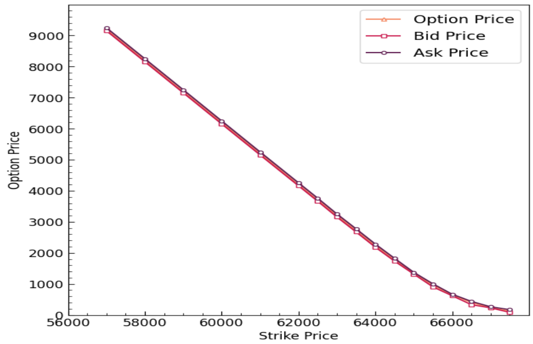
(e) Black-76 model option price - 6 days to expiry.



(f) GARCH model option price - 6 days to expiry.

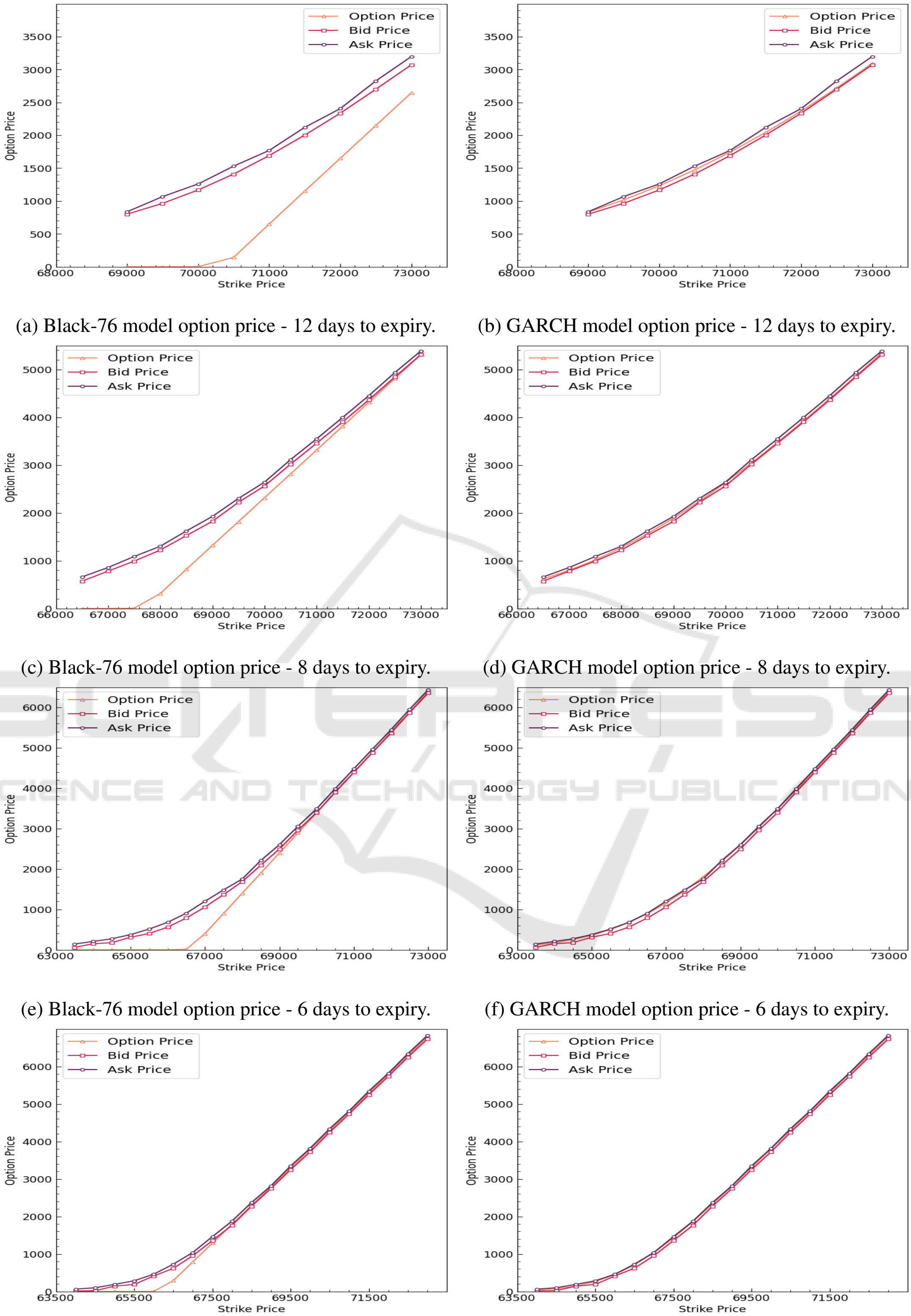


(g) Black-76 model option price - 2 days to expiry.



(h) GARCH model option price - 2 days to expiry.

Figure 4: Comparative analysis of call option pricing performance of Black-76 model and GARCH model.



(a) Black-76 model option price - 12 days to expiry.

(b) GARCH model option price - 12 days to expiry.

(c) Black-76 model option price - 8 days to expiry.

(d) GARCH model option price - 8 days to expiry.

(e) Black-76 model option price - 6 days to expiry.

(f) GARCH model option price - 6 days to expiry.

(g) Black-76 model option price - 2 days to expiry.

(h) GARCH model option price - 2 days to expiry.

Figure 5: Comparative analysis of put option pricing performance of Black-76 model and GARCH model.

demonstrating notably lower RMSE and MAE values for bid and ask prices. For short-term 2-day expiry, the Black-76 model and the GARCH(1,1) model show relatively lower RMSE and MAE values compared to longer expiry periods. In summary, the GARCH(1,1) model consistently outperforms the Black-76 model across various expiry periods, especially excelling in capturing short-term volatility dynamics, for call options. Table 4 compares the performance metrics for bid and ask prices between the Black-76 model and the GARCH(1,1) model at different expiry periods for put options. The trends observed for call options continue for put options in the 12-day expiry scenario. The GARCH(1,1) model consistently outperforms the Black-76 model, showcasing its effectiveness in capturing option pricing dynamics. Similar to call options, in the 8-day expiry, the GARCH(1,1) model maintains its superiority, providing more accurate predictions compared to the Black-76 model. As the time to expiry decreases to 6 days, the GARCH(1,1) model maintains its consistent performance, while the Black-76 model shows larger errors. In the extremely short 2-day expiry, both models show competitive performance for put options, with the GARCH(1,1) model maintaining its accuracy advantage. It can be particularly noted that, for the put options, as the options near maturity, the price of the option approaches the ask price. In conclusion, the GARCH(1,1) model consistently demonstrates superior pricing performance compared to the Black-76 model across various expiry periods for both call and put options. The GARCH(1,1) model's ability to capture short-term dynamics makes it a robust choice for commodity option pricing in the Indian market compared to the Black-76 model. The GARCH model's superior performance in terms of lower RMSE and MAE also indicates that it captures the volatility dynamics of the underlying commodity prices effectively.

5 CONCLUSIONS

The Indian Commodity Market has undergone significant changes in recent years, particularly in commodity derivatives trading. Silver holds a prominent position in this market due to its distinct qualities, such as industrial demand, volatility, diversification benefits, inflation hedging properties, and leverage opportunities. This study focuses on pricing silver mini options on futures utilizing the GARCH(1,1) model. In this research, we have conducted an analysis of commodity option pricing using two widely recognized models: the Black-Scholes model and the well known GARCH(1,1) model. Our study aimed to provide insights into the performance and suitability of these models for option pricing in the context of the Indian commodity market.

In this work, options on futures were priced by simulating Monte Carlo paths using the GARCH model parameters and its performance was also compared with the traditional Black-76 pricing model. Option pricing performance was tested considering different maturity periods of the same option until expiry. It was found that the GARCH model prices the options relatively well, with model-predicted option price sandwiched between the bid and ask price of the option. The closeness of the bid-ask price to the GARCH option price proves the realistic pricing performance shown by the GARCH model in option pricing context. The GARCH model consistently outperformed the Black-76 model in terms of predictive accuracy for commodity option pricing in the Indian commodity market, showing lower RMSE and MAE values across various expiration periods. The lower RMSE and MAE values exhibited by the GARCH model indicated its ability to capture and forecast the inherent volatility in commodity prices more effectively, making it a valuable tool for option pricing. The superior performance of the GARCH model in option pricing can have significant implications for risk management and investment decision-making in the commodity market. Investors and market participants can benefit from more accurate option pricing to make informed choices and mitigate risks effectively.

Future research can expand on this study by exploring the performance of these models in different commodity markets. The findings from the current study, which focuses on Silver Mini commodity options, might exhibit variations when applied to diverse commodities with unique market characteristics. Analyzing how the identified models perform across various commodity markets could unveil insights into the generalizability and adaptability of GARCH models. Different commodities, such as agricultural products, metals, or energy resources, often possess distinct price dynamics influenced by factors specific to each market. Evaluating model performance across this spectrum would provide a more comprehensive view of their effectiveness. Moreover, commodity markets are known for their susceptibility to changing economic conditions, geopolitical events, and other external factors. Evaluating the robustness of the identified models across different market conditions, including periods of high volatility or economic downturns, would contribute valuable insights. This approach would shed light on the models' adaptability and highlight potential areas for improvement.

Incorporating more advanced modelling techniques represents another avenue for future research. Exploring cutting-edge modelling techniques, such as machine learning algorithms or neural networks, could enhance the precision of option pricing models in commodity markets. These techniques have demonstrated success in capturing complex patterns and nonlinear relationships, potentially providing a more accurate representation of commodity price dynamics. Furthermore, future research could delve into factors beyond the traditional ones considered in option pricing models. For instance, incorporating the impact of jumps in commodity prices, which are abrupt and significant price movements, could refine the models' ability to capture extreme market events. This expanded scope of research would contribute to the continuous evolution of option pricing methodologies and their applicability in dynamic commodity markets.

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