# Shaping an Adaptive Path on Analytic Geometry with Automatic Formative Assessment and Interactive Feedback

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Keywords: Adaptive Education, Analytic Geometry, Automatic Assessment, Formative Assessment, Digital Learning Environments, Mathematics Education.

Abstract: This paper proposes a contribution to the practice of automatic formative assessment and adaptive education to highlight how it enhances the teaching and learning of Mathematics in secondary school. In this context we developed an adaptive learning path on Analytic Geometry, whose potential is discussed in the light of a theoretical framework, by describing and analyzing some activities. The activities allowed to introduce, address, explore the circle both as a geometric locus and its relationship with real situations through contextualized problems. The effectiveness of the path to reach the learning outcomes was tested through a quasi-experimental research design. The experimentation involved 98 third-year students from two upper secondary schools in Turin, Italy. Results were monitored through pre- and post-tests whose outcomes were compared with those of a control group; in addition, a final questionnaire was administered to the treated group. The results show the effectiveness of the tested activities in improving mathematical understanding; they also suggest strategies to introduce automatic formative assessment for adaptive learning in an effective way into a learning path, but they also highlight the need of solid pedagogical foundations which require continuous training of teachers by experts.

## 1 INTRODUCTION

Adaptive learning is a personalized learning methodology that is based on the individual experiences of the learner so as to implement an educational and training path capable of adapting to his or her actual needs and aligning with his or her learning times and styles. Unlike traditional learning experiences, adaptive learning allows for a fluid and stimulating study, which progressively helps the learner to reach and practice his or her actual potential (Romano et al., 2023).

This paper presents an experiment in mathematics education that provides a contribution to adaptive learning through automatic formative assessment in upper secondary school. The motivation from which this research paper arises is to highlight how automatic formative assessment enhances the teaching and learning of Mathematics by presenting its characteristics, peculiarities, potential and its effectiveness for the involvement and interactivity of activities that allows to be designed, including the possible improvements toward which this mode of assessment remains open. This study is based on the conceptualization of automatic formative assessment proposed by the DELTA (Digital Education for Learning and Teaching Advances) research group of the University of Turin (Barana et al., 2021). In particular, we refer to two kinds of adaptive activities: guided activities, i.e., activities whose resolution appears to be conducted step by step to show a possible solving process or example of problem solving; and activities with interactive feedback that outline a path that leads the learner to the resolution of a task after one or more autonomous attempts allowed (Barana et al., 2021). Students can try the activities multiple times, and the possibility to generate algorithmic questions allows for

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Barana, A., Fissore, C., Marchisio Conte, M. and Tassone, M.
Shaping an Adaptive Path on Analytic Geometry with Automatic Formative Assessment and Interactive Feedback.
DOI: 10.5220/0012627900003693
Paper published under CC license (CC BY-NC-ND 4.0)
In *Proceedings of the 16th International Conference on Computer Supported Education (CSEDU 2024) - Volume 2*, pages 361-372
ISBN: 978-989-758-697-2; ISSN: 2184-5026
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numerically different situations at each attempt. In this context we developed an adaptive learning path whose potential is proposed and discussed in detail in light of a theoretical framework by describing and analyzing some examples of digital activities designed in the context of automatic formative assessment and involving peer collaboration. The path deals with the circle from an analytical point of view. The activities were tested through a teaching experiment which involved a total of 98 grade 11 students from two scientific lyceums in Turin (Italy), divided between treated and control classes. The experimental activities consisted of adaptive digital activities to be carried out in the classroom and at home. Results were monitored through a pre-test, a post-test and a final questionnaire (the latter administered only to the treated group). Students involved in adaptive learning were allowed to identify their level of readiness based on INVALSI (the national agency in charge of evaluation of the Italian education system) levels, merged into 3 macro levels for simplicity. The following sections will detail the methodology adopted to structure the experiment and to adapt the same activities to different learning contexts for multiple factors. This was achieved thanks to the help of the Ministry of Education and Merit's Problem Posing & Solving (PPS) Project platform (Brancaccio, et al., 2015). The second section discusses the theoretical framework in which our research fits and in light of which we have attempted to analyze and comment on our results. In the third section we outline the research questions that guided us in the planning, implementation and management of the classroom teaching experiment and in the choice of data collection and analysis techniques, while the fourth section reports and discusses the most important results obtained. Lastly, the conclusions highlight some possible future developments of the work and some limitations that emerged from the research, with attached suggestions to improve the effectiveness of automatic formative assessment in any future experiments.

# 2 THEORETICAL FRAMEWORK

## 2.1 Digital Learning Environment

The traditional learning environment is the classroom where teacher explains, the student learns individually or with peers and uses concrete tools such as paper, pen, and a blackboard. The advent of technology has introduced tablets, PCs and interactive whiteboards into these environments and shifted learning to a non-physical dimension: the Internet. Digital Learning Environments (DLEs) denote learning ecosystems in which there is a human component (teacher, students, peers), a technological one (software, web-conference tools, assessment tools, smartphones, computers, tablets or interactive whiteboards, cameras, etc.) and the interactions between the two (dialogues between members, human-technology interactions, etc.) (Barana & Marchisio, 2022). The technological apparatus of a DLE can support a learning activity in the following ways (Barana, Conte, et al., 2019; Barana, Marchisio, et al., 2019): (a) creation, management and editing of resources (e.g., interactive files, theoretical lessons, glossaries, videos), activities (e.g., discussion chats, tests, forums, questionnaires, submission of assignments) and the general learning environment by both faculty and students; (b) provision of materials; (c) collection of qualitative and quantitative data on student action, use of materials (whether a resource was used and how many times), participation; (d) analysis and processing of collected data to monitor skill development; (e) provision of feedback on the activities carried out to both students and teachers.

## 2.2 Formative Assessment

The term "formative assessment" was coined in 1967 by Michael Scriven in opposition to "summative assessment" to denote a particular practice aimed at gathering information about a course with the aim of improving the program (Scriven, 1967). The definition later given by Paul Black and Dylan Wiliam and well accepted in the literature is the following: "Classroom practice is formative to the extent that evidence about student achievement is collected, interpreted, and used by teachers, students, or their peers to make decisions about next steps in instruction" (Black & Wiliam, 2009). According to William and Thompson (2007), formative assessment can be characterized by five strategies: (1) the teacher should clarify learning intentions and criteria for success, peers should understand and share learning intentions and criteria for success, the student should understand learning intentions and criteria for success; (2) the teacher should design effective classroom discussions and other tasks that stimulate learners' understanding; (3) the teacher should provide feedback that prompts learners to progress; (4) students should be activated as mutual teaching resources; (5) students should be activated as the owners of their learning.

Feedback is probably the most distinctive and characteristic element of formative assessment and has been defined by John Hattie and Helen Timplerley as "the information provided by an agent (teacher, peer, book, parent, personal experience, etc.) regarding aspects of one's performance or understanding" (Hattie & Timperley, 2007). To be effective, feedback must answer three main questions, "Where am I going?", "How am I going?", and "Where am I going next?". In addition, a feedback can work at 4 different levels: at the activity level (how well the activity was performed); at the process level (what process is involved in the activity); at the self-regulation level (what metacognitive processes it triggers); at the personal level (what personal evaluations and affections concern the student as a person) (Hattie & Timperley, 2007). According to Sadler (Sadler, 1989), for feedback to be productive it must alter the gap between current and desired performance because if the information is not, or cannot be, processed by the student to achieve an improvement it will have no effect on his or her learning. In any curricular area where a grade or score given to a student constitutes only a number the attention of the subject concerned is diverted from such judgments and the criteria that formulated it, and the idea that learning is a product rather than an evolving process is built up, even if only on an unconscious level. In this sense, a grade can therefore be counterproductive for educational purposes (Sadler, 1989). The learner must be able to judge the quality of what he or she is learning and properly control what he or she puts into play as he or she solves a problem, performs an action, or states an opinion. If the learner generates relevant information, the procedure is part of self-monitoring; if the source of the information is external, it is associated with feedback (Sadler, 1989). Formative assessment includes both self-monitoring and feedback, and the goal of many educational systems that use them is to facilitate the transition from feedback to self-monitoring so that it occurs as naturally as possible.

## 2.3 The Power of Feedback

First, it is important to distinguish between 2 categories of feedback (Shute, 2008). The first one is elaborate feedback, which contains the explanation leading to the correct solution, allows links to other reading materials, cues, suggestions or combinations thereof. In Mathematics, it often takes the form of a guided solution proposal. It can be divided into more manageable units to avoid cognitive overload. The second one is corrective feedback, which simply says whether the answer is right or wrong. According to Shute (2008), the kind of feedback that really allows for cognitive improvement and engagement is elaborate feedback. According to Kluger and DeNisi (1996) computerized feedback is more effective than human-provided feedback because it helps the student more to generate internal feedback to close the gap between current performance and expected performance. Corbalan, Paas, and Cuypers (2010) compare 3 different levels of formative feedback: feedback at the end of the solution, feedback on all steps of the solution at the same time, and feedback on all steps of the solution successively.

For those who have not managed to reach the exact solution, feedback provides partial credit based on the correctness of their step-by-step response and acts as a motivational lever as well as providing more complete knowledge to both the student and the teacher about their skills (Barana et al., 2020). However, it is good to keep in mind that too much feedback within the same task can end up detracting from performance, and overly specific feedback can also be harmful because it leads the student to focus excessively on the specific task and not on the general strategy (Hattie & Timperley, 2007). Feedback has its greatest effect when a student expects his or her answer to be correct and instead it turns out to be incorrect because they are studying the item longer in the attempt to correct and understand the misunderstanding; conversely, if the answer is incorrect and the certainty was low, the feedback is often ignored (Ashford & Cummings, 1983). Black and Wiliam (1998) have proven that, for a teacher to use feedback to praise the student's performance in a given context and not the student as a person brings better results because it helps learners develop what Dweck in 2000 called "incremental vision", that is, the ability to grasp learning as an evolving process and not as a static, defined product (Dweck, 2000).

Peer involvements present an important source of external feedback: the dialogue with peers improves the students' sense of self-control on learning in several ways (Laurillard, 2002). First, students understand better when a concept is explained by a peer who has just learned the topic at hand, since they happen to find language more accessible when it comes from their peers. Second, peer discussion exposes students to alternative perspectives that allow them to revise, and possibly reject, their initial idea and build new knowledge through dialogue and comparison. Third, by commenting on peer work, students develop detachment from judgment in favor of evaluation of their work. Fourth, peer discussion can be motivating because it encourages students to persist by not giving up (Boyle & Nicol, 2003).

The current research challenge is to refine the principles related to interactive feedback, identify gaps and gather further evidence on the potential of formative assessment and feedback to support self-regulation.

## 2.4 Adaptive Teaching and Adaptive Learning

According to Borich (2017) adaptive teaching means "applying different instructional strategies for different groups of students so that the natural diversity prevailing in the classroom does not prevent any student from achieving success." This way teaching has gained importance because the purpose of education has shifted from the static transmission of knowledge to the building of skills that imply "personal attitudes" that allow knowledge and skills to be brought into play in solving certain situations or problems within a student-centered educational approach (European Commission, 2018). The adaptive approach is particularly effective when applied to small groups; it also helps students overcome their individual difficulties (Mascarenhas et al., 2016). It is supported by the use of technologies that can also successfully replace the physical presence of the teacher through the use of feedback and suggestions that a teacher would not be able to provide during execution to all students at the same time. An adaptive teaching/learning system, i.e. one that can adapt to the characteristics, needs and even preferences of teachers and students, presupposes the involvement of adaptive technologies that are generally controlled by computational devices. The information collected is stored within a learning model that provides a starting point for deciding how to deliver personalized content for each student (Mascarenhas et al., 2016).

## 2.5 Automatic Formative Assessment

When formative assessment is offered through an automatic assessment system that automatically evaluates responses and returns feedback, it is called automatic formative assessment (Barana et al., 2021).

Some automatic formative assessment systems allow the creation of adaptive questions, namely questions divided into sections that are shown based on the answer given in the previous section. With adaptive questions, problems can be proposed in the form of: "Guided Activities" or through activities with interactive feedback – terms suggested in (Barana et al., 2021). Guided activities mean activities whose resolution appears outlined step by step to show an alternative solution procedure or example of problem solving. These types of questions were extremely useful during the COVID-19 pandemic, when learners needed tools that could guide their cognitive processes and help them develop new skills even without the physical presence of a teacher. Activities with interactive feedback, on the other hand, represent an interactive process provided following a student's first independent attempt at solving a task, tending to be in the case of an incorrect solution or for those who failed to answer the proposed question independently, but possibly also for those who answered correctly to enable them to compare the solving strategy they used with the one suggested. In interactive feedback, the main question is broken down into sub-questions that investigate prerequisites, fundamental operations or other representations of the initial proposed problem. In each step, if an incorrect answer is still given, the correct one will be shown so that it can be used in subsequent steps. The DELTA research group of the University of Turin has proposed a model for designing activities for automatic formative assessment using an automatic formative assessment system enhanced by a mathematical engine that contemplates the following features (Barana et al., 2018): (a) always available activities, which can be tackled independently without limitations in time, duration and number of attempts; (b) algorithmic questions in which the values, parameters, graphs or formulas in the question text and feedback vary randomly with each attempt by a student and for each individual student due to the involvement of teacher-defined algorithms; (c) open-ended answers, evaluated for their mathematical correctness due to the advanced computational capabilities of the system, where students are invited to answer by adopting the most suitable register among numerical, graphical, symbolic, literal, etc.; (d) immediate feedback, or at least, short-term feedback, that can be provided, for instance, by creating tests with no more than 5 questions and feedback at the end of the test, in order to show it while the student is still focused on the task; (e) contextualization of the task in the real world or in applications that are relevant for students in order to produce greater engagement and build to deeper understanding; (f) proposing interactive feedback, which has been shown to be particularly effective in developing mathematical skills.

In this paper, we distinguish *adaptive questions* (which can take the form of guided activities or interactive feedback) from *adaptive assignments*, which are tests composed of a set number of (simple) items that are submitted to students one at a time; the student's performance on one question determines the level of the next question. Adaptive assignments are often used to determine a student's level of knowledge on a concept or topic, which can be determined precisely by solving tasks of increasing difficulty (Botta, 2021). Once the assignment is completed, feedback is returned to students depicting the progress profile of their performance from which they can extract a final level of competence, facilitating self-assessment through metacognitive effort. Examples of adaptive questions and adaptive assignment will be given in the following sections.

# 3 METHODOLOGY

## 3.1 Research Questions

The research activity was guided by some fundamental questions that clearly identify and outline the crucial aspects of the project: (RQ1) How can the same adaptive questions and adaptive assignments can adapt to students with different learning prerequisites? (RQ2) What results did the designed adaptive path have in relation to the disciplinary learning outcomes? (RQ3) How did learners respond to the proposed adaptive teaching?

With the first research question (RQ1) we want to investigate whether, and in what way, it is possible to use and/or adapt the same didactic activities in even decidedly diverse contexts in order to produce meaningful, varied, and complete learning despite the differences in the learning background and prerequisites that distinguish the subjects constituting the class group. With the second research question (RQ2) we try to examine the outcomes of the experimentation in terms of learning outcomes achieved by the students involved in order to understand whether the effect produced by the proposed activities, which can be achieved in a short time interval and with reference to the control classes, not involved in the adaptive path, can be translated into a positive, negative or null reference. Finally, with the third research question (RQ3), we also focus our attention on how learners perceive and receive our adaptive path.

## 3.2 Experiment Organization

The method adopted is quasi-experimental. The participants are students of grade 11 (which in Italy corresponds to the third year of upper secondary school, age 16) of scientific lyceum in the city of Turin (Italy). Specifically, 3 classes were involved in the treated group (54 students) and 2 classes in the control group (44 students), for a total of 98 students. The classes were recruited based on volunteer subscription of their Mathematics teachers to the project. Two of the three teachers in the treated group were used to propose activities with automatic formative assessment to their classes, while the other had never used this method before, like the two teachers of the control group. The learning outcomes of the experimentation were: identifying the equation of a circle from its geometrical properties and vice versa; define the mutual position of two circles in the Cartesian plane; solving real-world problems involving circles. The treated classes followed a 4/6 hour course, 2 or 4 entitled "The circle as a geometric locus" and 2 entitled "The circle as a physical locus". During the class meetings, one of the researchers of the team was present in class to support the teacher and the students especially with the use of the platform. They were organized as follows: students worked in pairs of internally homogeneous level; an adaptive assignment and some adaptive questions with interactive feedback or guided activities were proposed; between one meeting and the next, other activities similar to those carried out in class were assigned to the students.

The choice of using internally homogeneous level groups is motivated by the fact that if the students have approximately the same starting level, the adaptive activity can adapt to both at the same time. In this way it was possible to activate collaboration during adaptive activities. Moreover, the peer collaboration could be even more effective, since it is not influenced by a leader-follower relationship, and both the students, having approximately the same level of competence, need to actively reason on tasks (Barana et al., 2023). The students' initial level was determined through the pre-test and discussed with their teachers. Students could access the activities from the Moodle platform of the national PP&S Project (Brancaccio, et al., 2015), which is integrated with the automatic assessment system Moebius Assessment, widely used for learning Mathematics thanks to its mathematical engine (Fahlgren & Brunström, 2023). Figure 1 shows an example of a guided activity to illustrate its peculiar features, called "Cooling Towers". The learners are faced with an apparently unknown situation in which, following the suggestions provided in the guided path, they are asked to extrapolate the essential information from the prompts and carry out simple replacement operations.

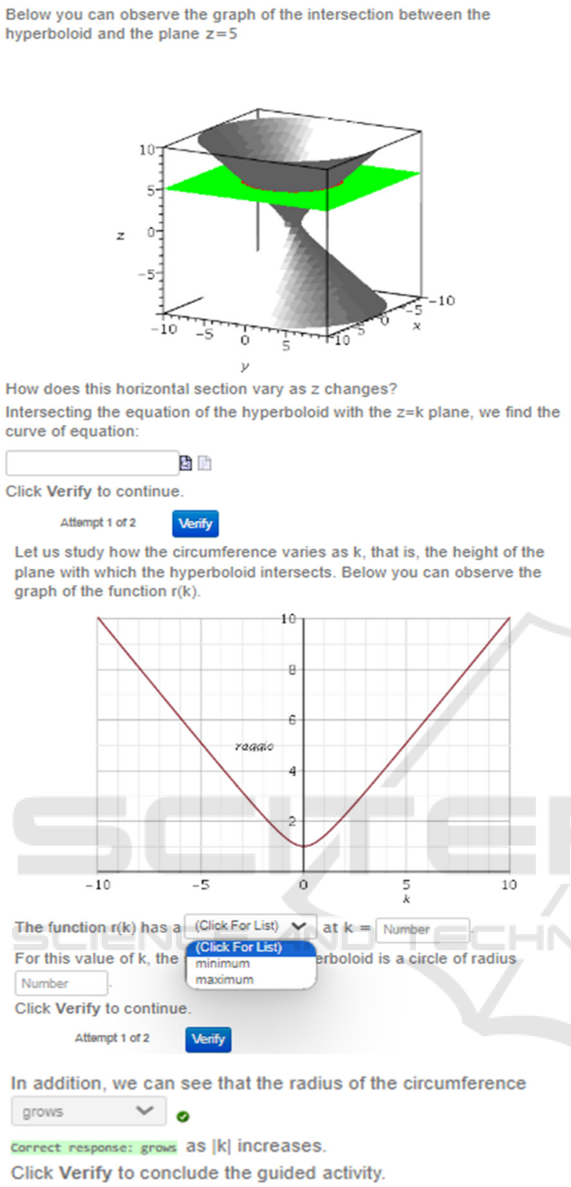



Figure 1: Guided activity "Cooling towers".

The preview function (icon depicting a P) allows the learners to visualize in a graphical register their answer and thus be able to compare the graph proposed in the text and the one generated by their answer. This function provides students with some initial feedback that allows them to self-assess their answer before submitting it to the system, thus in a moment in which they are still allowed to change it. This makes it possible to activate the fifth strategy of formative assessment conceived by Wiliam and Thompson (2007): students have the opportunity to develop responsibility in their learning path; they should be able to take advantage of all the tools that are made available to them in order to proceed adequately in the resolution, and at the same time it supports self-assessment since it makes the learners able to establish the correctness of their answer independently. Lastly, the preview also works on the psychological-affective level since it enables the learners to reassure themselves about the correctness of their actions. Figure 2 shows an example of an adaptive question with interactive feedback called "Let's catch Criminals". It allows a number of initial autonomous attempts. In the case of a correct answer, the task is successfully completed. In the case of an incorrect answer, after a few attempts allowed, a guided path starts: at each step, the learner is led to develop the intermediate results necessary to obtain the final solution.

The results of the experimentation were evaluated through a pre-test and a post-test administered to the control and treated group. The treated classes took the pre-test before starting the adaptive path; the control classes took it before starting to work on the circle with their teachers in a traditional way in order to compare this approach to the one proposed in this experiment. Their teachers were kept updated on the results of the research and the adaptive activities developed for their future use were shared with them. The post-test was administered after completing the circle with or without the adaptive path, and after a few weeks to let students the time to study the topic and, for the treated group, complete the online assignment. The pre-test included 10 questions, some of which including sub-items for a total of 25 sub-items, about analytic geometry, mostly about the line and the parabola, which are usually studied before dealing with the circle. 4 of them were contextualized in real-world situations, while the others concerned abstract mathematical situations. Most of them are adapted by the INVALSI tests of Mathematics for grade 10 or 13. The post-test included 7 questions with a total of 18 sub-items, all of them dealing with the circle. Some of them were adapted by INVALSI tests, while others were created appositely for this test. 9 items out of 18 were contextualized in real-world situations and almost all of them involved problem-solving processes, so the post-test had a higher level of difficulty than the pre-test. The experimentation was also evaluated through the review of videos taken with the aid of a camera during class meetings, with the permission of the students' families and in compliance with the rules on privacy and data processing, in an attempt to monitor the reasoning and communicative processes implemented by the students while working in pairs to carry out the activities.

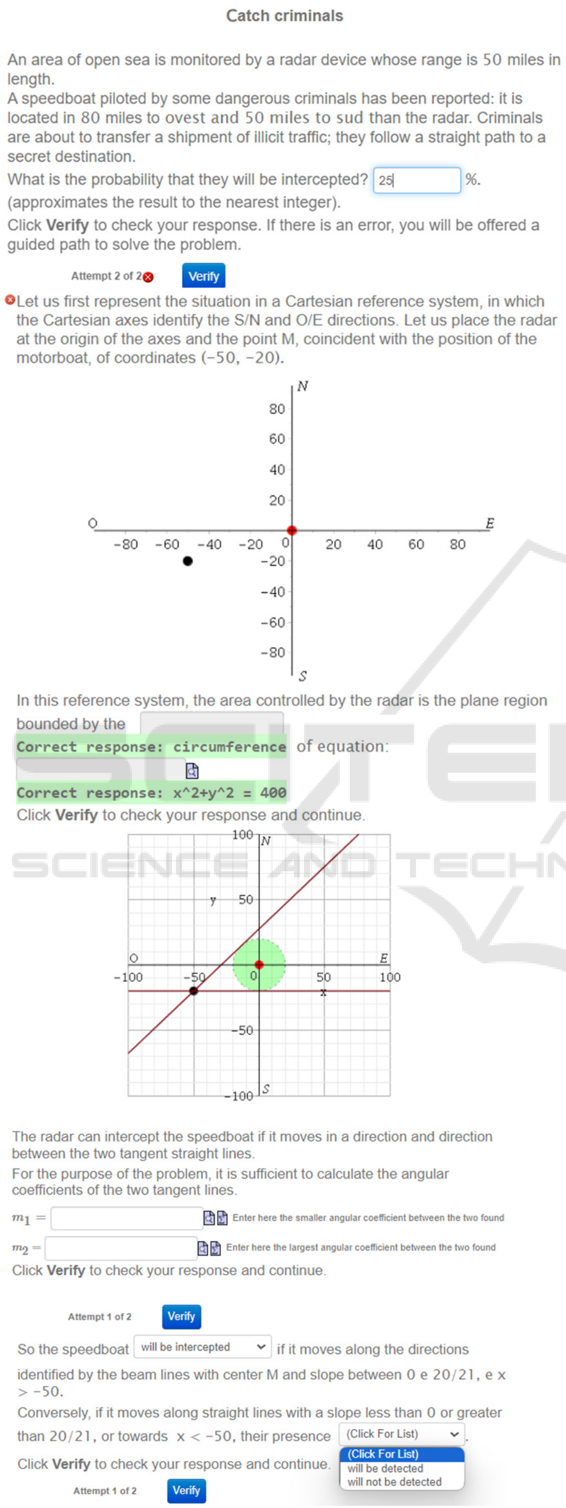



Figure 2: Activity with interactive feedback “Catch criminals”.

Lastly, students in the treated classes completed a final questionnaire. The student questionnaire was made up of 23 total items that students could answer on a 5-value scale ranging from "not at all" to "very much" divided into: 10 items to detect appreciation of the proposed activities and pair work; 13 items to test the expository clarity and effectiveness of the activities and the role of interactive feedback; 3 open-ended questions with the task of recording impressions, critical issues, but also strengths of the proposed educational offering. The project was carried out in compliance with the local, national and institutional ethical laws and requirements. All participants and their families signed a permission for using their data for research purposes. The project obtained the ethical approval from the Bioethical Committee of the University of Turin.

## 3.3 Data Collection and Analysis

The data obtained from our research were divided into 4 macro categories: data from responses to adaptive activities; data from the pre-test and post-test; data from the final questionnaire; data from monitoring pairs of students. We analyzed the results following different paths depending on the type of data. For results to adaptive activities present on the platform, we calculated the times of performance in specific activities; the number of student pairs that obtained full scores on some questions; the scores totaled in the adaptive assignment. For the post-test and pre-test results, they were related to 100 and then we performed an analysis based on 3 types of tests:

1. analysis of covariance using the ANCOVA test. This test proved to be particularly suitable for examining whether the results of the post-test (dependent variable) depended on the treated group or to the control group (categorical independent variable) while taking into account the results obtained at pre-test (covariate), which identifies an initial stage, common to all subjects involved in the survey (Creswell & Clark, 2017).

2. Chi-square test that allowed us to check whether or not the correctness of the answers to single items were dependent on membership in the experimental or control group and to make observations related to the proposed learning path.

3. Cramer's V test that allowed us to test whether the difference in scores obtained in specific post-test items by the two groups were significant and to make observations related to the proposed learning path.

For the qualitative data collected from the questionnaires, we made an initial distinction between the answers given to the Likert-scale items proposed in the questionnaire and the answers given to the open-ended questions asking for strengths and weaknesses. Response occurrence rates were calculated in an attempt to quantitatively highlight which aspects of the proposed experimentation they found most successful, which were most problematic, which new aspects were highlighted and which they considered desirable in their future daily teaching.

# 4 RESULTS

## 4.1 Research Question 1

The educational experimentation was not carried out in the same way in all the 3 treated classes. This fact was dictated by the meeting of significantly different school contexts and classroom environments, each enriched by the specific characteristics of each student who was part of it. What led to a clear distinction in methods was mainly having or not having already introduced circle before our meetings, condition that affects the students' learning background and prerequisites. For this reason, we have divided the description of each meeting into 2 sections, taking this fact into account. This enabled us to highlight the malleability of this learning path.

### 4.1.1 Circle Already Introduced Before Starting the Adaptive Path

In two out of the three treated classes, the circle had already been introduced to the students by their teachers before starting the adaptive learning path. In the first meeting, after creating the working pairs based on the pre-test results, 2 activities were proposed. Firstly, students were asked to complete the adaptive assignment to get feedback about their starting level. The adaptive assignment provides feedback that Shute (2008) calls corrective since for each question the student pairs can visualize the correctness of their answer based on the colour (green or red) that appears in the box corresponding to that question, depending whether it was correct or not. Secondly, adaptive questions split into three difficulty levels were proposed. Students were asked to start with the level they believed they belonged to based on feedback from the adaptive assignment. Adaptive questions provide students with elaborate feedback (Shute, 2008) in that in the event of an error, guided processes help students to achieve the correct solution. The adaptive assignment containing 8-10 questions depending on the initial preparation of the class. Before starting the assignment, students could view the graph shown in Figure 3 whose number of horizontal squares refers to the total questions in the test (the blue colored box corresponding to the question being asked) while the number of vertical squares refers to the difficulty level to which the question belongs. After answered a question, clicking on the Next button, the next question appears but from the graph students can visualize, by corrective feedback, the outcome of one's answer: if the box turns green the answer was correct (Figure 4) and they go up a level, if the box turns red the answer given was wrong (Figure 5) and they go down in level or continue to answering a question to Initial level.

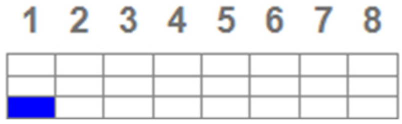



Figure 3: Adaptive assignment graph.

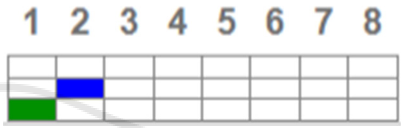



Figure 4: Correct answer.

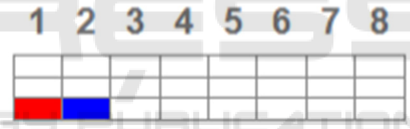



Figure 5: Incorrect answer.

Learners anxiously await immediate feedback and, in the case of a correct answer, express satisfaction and enthusiasm demonstrating how immediate feedback relating to the resolution of a question actually has an impact on a personal level. In fact, Hattie and Timperley (2007) state that it increases esteem, both toward the partner and toward oneself. On the other hand, disappointment is perceived in the case of an incorrect answer, but with the intention to improve in subsequent questions not by giving up in despair or frustration, but by proceeding with greater attention, precision, and reflection. This attitude integrates well with the "incremental view" identified by Dweck (2000). In Figure 6 we report a graph depicting a possible trend recorded by a pair performing the adaptive assignment. It represents an example of what Corbalan, Paas, and Cuypers (2010) call "formative feedback on all the steps provided later", which fits well into the third level identified by Hattie and Timperley (2007), that of self-regulation, since it allows the subjects who receive it to trigger a personal metacognitive reflection on their performance.

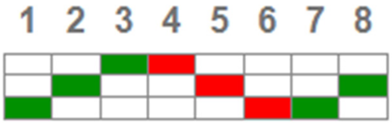



Figure 6: Hypothetical performance in adaptive assignment.

In the second meeting, contextualized problems related to real-life situations were presented in which learners are engaged in activities of exploring, contextualizing, extracting essential data, and modeling an actual situation. In one class 4 out of 8 pairs autonomously solved the first contextualized problem without the need of the interactive feedback and within a reasonable time (from 12 min to 48 min), while in the second class 3 pairs gave the correct answer autonomously without the interactive feedback. We observe from the resolution of a video-recorded pair how the possibility of multiple attempts before starting the guided path turns out to be effective because, after a first wrong answer (Figure 7), students appear more thoughtful, careful to grasp every single piece of information in order to decipher the text well and obtain all the essential data so as to reach the correct solution (Figure 8). In this case, it was necessary to start by proposing the adaptive questions to the pairs, and in particular the guided activities, in which the learners had the opportunity to receive elaborate feedback (Shute, 2008) in the form of a guided path to help them proceed gradually in solving the tasks. Thus, students were asked to go through the adaptive questions starting from the easiest task and proceed by increasing the level.

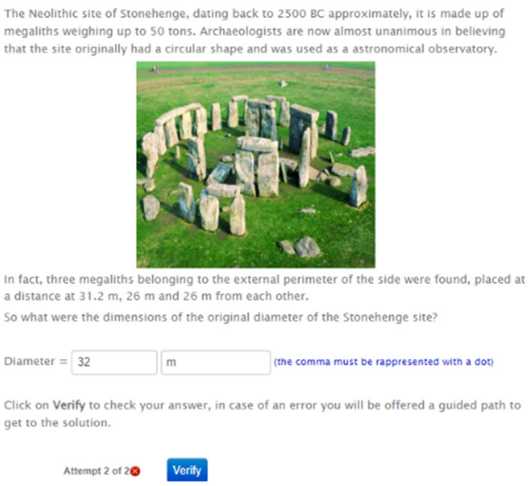



Figure 7: First attempt.


Figure 8: Second attempt.

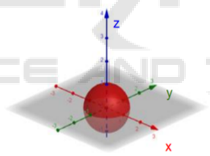
In this context, we could observe how the pairs of students, despite the possibility of guided pathways, still attempted to reason through problem solving independently, by using the knowledge of Euclidean and Analytic Geometry they already possessed.

The adaptive assignment was proposed later to help them self-assess the knowledge they had acquired up to that point. We noticed that some pairs of students, when acknowledging that the adaptive assignment was too difficult for them, went back to the guided activities and to the adaptive questions with interactive feedback to revise the contents before attempting the adaptive assignment again. Comparing the averages of the scores in terms of percentages obtained by the 3 treated classes in the adaptive assignment, we can observe that the averages in percentages totaled by the classes that had already introduced the circle in class are 52.5% and 55% respectively while the best result, 69.62%, was obtained by the class that started the experimentation without knowing anything about the circle, and whose knowledge was uniquely gained through the adaptive questions. Therefore, we can assume that our adaptive activities were effective to understand the circle given the time they had available.

## 4.2 Research Question 2

For the analyses of the results of the pre-test and the post-test, one of the treated classes was excluded from the sample since they did not perform the post-test due to time constraints. It was one of the two classes in which the circle had already been introduced before starting to work with the adaptive path. Moreover, we excluded from the sample those students who did not perform one of the two tests. Therefore, the sample considered for the analysis of the results includes 40 students from the control classes and 33 students from the treated classes. The ANCOVA test, conducted to compare the post-test ratings for the 2 groups, using the pre-test ratings as a covariate, showed an 11-point gap out of 100 between the 2 groups in favor of the treated group. Table 1 shows the estimates of the post-test ratings using the pre-test ratings as a covariate. The ANCOVA test statistic is 9.954 and its significance is 0.002, so we can state that the difference between the averages of the post-test ratings taking into account the pre-test as covariate is significant. We also analyzed by means of the chi-square test the scores obtained by the 2 groups in the individual items focusing our attention on those items that report significant statistics on the differences in the distribution of scores between the 2 groups. In particular, we focused on the results that emerged from the analysis of question 6, items b and c, (Figure 9). In Table 2 and Table 3 we report the results totaled by the 2 groups in item b and item c, respectively. From the first table, it emerges that exactly 100% of the students in the control classes did not answer or answered incorrectly while about 12% of the students in the experimental classes scored the highest.

6) A sphere in a Cartesian reference system having the center the origin can be expressed by the following Cartesian equation $x^2 + y^2 + z^2 = r^2$ where $r > 3$ represents the radius of the sphere while z represents the height.




a. A horizontal plane in space has equation of the type z=k where k denotes the height of the plane. If we intersect the sphere with the $z = 2$ plane we get a circle. What is the radius of this circle?
**Answer:** ____________________
b. What is the equation of the horizontal plane that the circle of maximum radius locates on that sphere?
**Answer:** ____________________
c. What is the equation of the circle obtained by intersecting with the $y = -3$ plane?
**Answer:** ____________________

Figure 9: Question 6 in post-test.

Table 1: Difference between the avarages of assessments at post-test taking into account the pre-test as a covariate.

| Mode | Avarage | Std. error |
|---|---|---|
| Control classes | 5.47 | 2.44 |
| Experimental classes | 66.94 | 2.69 |

Table 2: Averages of the assessment grades.

| Mode | Score 0 | Score 1 |
|---|---|---|
| Control classes | 100,0% | 0,0% |
| Experimental classes | 87,9% | 12,1% |

Table 3: Averages of the assessment grades.

| Mode | Score 0 | Score 0,75 | Score 1 |
|---|---|---|---|
| Control classes | 97,6% | 2,4% | 0,0% |
| Exp. classes | 72,7% | 3,0% | 24,2% |

The chi-square test shows a value of 5.378 with p-value of 0.02, which therefore appears to be acceptable. Cramer's V value appears to be moderate but significant (V=0.268 with significance of 0.02), meaning that the 27% of variance is due to the difference in the treatment. From Table 4 (question 6, item c) we observe that more than 97% of the students in the control classes, answered incorrectly, about 3% partially correctly but none entirely correctly while in the experimental classes we have about 24% of students achieving a full score. The chi-square test gives a value of 11.532 with significance of 0.003. Cramer’s V is found to be significant (V=0.392 with p value of 0.003), meaning that 39% of variance is due to the difference in the treatment. These results obtained in item b and c of question 6 can be justified by the fact that a similar problem was faced by the experimental classes with the guided activity “Cooling Towers” by getting them accustomed to moving from the 2-dimensional to the 3-dimensional plane and considering the circle as a curve obtained from the intersection of 2 3-dimensional geometric locus. Becoming familiar with parametric and contextualized problems, which are not usually carried out in traditional lessons, helped students in the experimental classes to be able to extrapolate essential information from the text to model the situations described. However, the score obtained by the experimental classes is still a low score; this fact can be explained considering that the reference task is not a routine task and analytic geometry in 3 dimensions is rarely used to deal with 2-dimensional objects, even if it might be effective to do so.

## 4.3 Research Question 3

### 4.3.1 Analysis of Video Recording

From the video recordings of some pairs of students during classroom work, some relevant factors emerged: the ability to divide the tasks to be performed to reach the final solution; the crucial role of external feedback coming from the other member of the pair (Laurillard, 2002); the challenging attitude towards interactive feedback (Ashford & Cummings, 1983); two possible responses to immediate feedback: satisfaction and increased esteem, both towards the partner and towards oneself (Hattie & Timperley, 2007) in the case of a correct answer, and disappointment in the case of a wrong answer but the intention to improve in subsequent questions by not indulging in despair, but proceeding with greater attention, precision and reflection typical of an "incremental view" (Dweck, 2000). Lastly, guided pathways are not "addictive", but on the contrary, students try to "do without" to find the solutions on their own.

#### 4.3.2 Analysis of the Questionnaire

From the response percentages for each item in the final questionnaire, we can see how there is a good satisfaction with the proposed activities. The highest percentages for the "very much" answer are obtained at the items "It was helpful to have immediate feedback after each response" (54.72 %) and "It was helpful to have the step-by-step guided resolutions" (50.94 %), and in both cases no one answered "not at all". Learners found "much" in the items: "I enjoyed working actively during the meetings" (54.72 %); "The activities allowed me to better understand what my preparation level is" (62.26 %); "The activities allowed me to improve my level of preparation" (52.83 %), which thus call for teaching to be as engaging and stimulating as possible. The item in which the largest percentage of responses appears to be "not at all" (13.21 %) and "little" 49.06 %) is the one related to the habit of working in these modalities in their regular math lessons, demonstrating that this is an unusual methodology that is still under-proposed by teachers. 11.32% of students answered "not at all" to the item "I enjoyed solving problems contextualized in reality," while 13.21 % of students answered "a little" to the item "It was useful to solve problems contextualized in reality", perhaps indicating that students are not used to this type of activity and therefore put them in more difficulty. In support of this fact, we observe how 15% of students in the open-ended question "What aspect could be improved?" asked for the easier exercises.

## 5 CONCLUSIONS

In conclusion, we can state that in reference to the first research question (RQ1) we were able to observe how the designed activities were able to adapt, albeit with some limitations, to learning environments that varied considerably because of the school context, the class and the students who compose it, as well as the different levels of preparation on the topic to be treated in the experimental classes. From the analysis of the results to the adaptive activities investigated with the second research question (RQ2) we can state how the students who participated in the adaptive activities scored higher on the post-test than the students in the control classes, recording a gap of 11 points out of 100 overall. Finally, from the operative student answers to the proposed adaptive path collected through the video recordings and the questionnaire, it was possible to highlight some foundational aspects of the work students carried out pairs of homogeneous levels and their interaction with adaptive activities and interactive feedback. The questionnaire revealed general satisfaction with the activities, whose originality, versatility, clarity of exposition and relevance to reality were recognized, while the use of interactive feedback proved effective in ensuring that students used the information provided to them to improve their performance (RQ3). Thanks to that questionnaire, numerous valuable suggestions were collected to refine, expand, and enrich the involvement of automatic formative assessment in teaching Mathematics. Among them, one that is worth mentioning is the possibility to use a clearer and more functional platform and try to design, however complex, not a single and rigid path with interactive feedback, but one that can be differentiated and that is capable of taking into account the path taken by the students independently, regardless of its successful completion. This would prevent the effort of the students, who we have observed to be very stimulated and intent on doing the exercises and problems without help, from being wasted by a guide that does not take into account the variety of solution methods that a mathematical problem typically offers. It was deemed necessary to train teachers pedagogically and professionally so that they could learn to independently design meaningful adaptive questions and adaptive assignments for their classes. Finally, some further analyses that can be carried out starting from the data collected in our research are trying to model the path followed by each individual student from pre-test to post-test through the adaptive activities carried out in class, and studying the impact of the activities on students' self-assessment skills.

## ACKNOWLEDGEMENTS

The authors would like to thank the participating teachers for their commitment during the project, and all the students who took part to the experimentation. The authors would like also to thank the bank foundation Compagnia di San Paolo Foundation for their support on the OPERA project (Open Program for Education Research and Activities).

## REFERENCES

Ashford, S. J., & Cummings, L. L. (1983). Feedback as an individual resource: Personal strategies of creating information. *Organizational Behavior and Human Performance*, *32*(3), 370–398.

Barana, A., Boetti, G., Marchisio, M., Perrotta, A., & Sacchet, M. (2023). Investigating the Knowledge Co-Construction Process in Homogeneous Ability Groups during Computational Lab Activities in Financial Mathematics. *Sustainability*, *15*(18), 13466. https://doi.org/10.3390/su151813466

Barana, A., Conte, A., Fioravera, M., Marchisio, M., & Rabellino, S. (2018). A Model of Formative Automatic Assessment and Interactive Feedback for STEM. *Proceedings of 2018 IEEE 42nd Annual Computer Software and Applications Conference (COMPSAC)*, 1016–1025. https://doi.org/10.1109/COMPSAC.2018.00178

Barana, A., Conte, A., Fissore, C., Marchisio, M., & Rabellino, S. (2019). Learning Analytics to improve Formative Assessment strategies. *Journal of E-Learning and Knowledge Society*, *15*(3), 75–88. https://doi.org/10.20368/1971-8829/1135057

Barana, A., Fissore, C., & Marchisio, M. (2020). From Standardized Assessment to Automatic Formative Assessment for Adaptive Teaching: *Proceedings of the 12th International Conference on Computer Supported Education*, 285–296. https://doi.org/10.5220/0009577302850296

Barana, A., & Marchisio, M. (2022). A Model for the Analysis of the Interactions in a Digital Learning Environment During Mathematical Activities. In B. Csapó & J. Uhomoibhi (Eds.), *Computer Supported Education* (Vol. 1624, pp. 429–448). Springer International Publishing. https://doi.org/10.1007/978-3-031-14756-2_21

Barana, A., Marchisio, M., & Miori, R. (2019). MATE-BOOSTER: Design of an e-Learning Course to Boost Mathematical Competence. *Proceedings of the 11th International Conference on Computer Supported Education (CSEDU 2019)*, *1*, 280–291.

Barana, A., Marchisio, M., & Sacchet, M. (2021). Interactive Feedback for Learning Mathematics in a Digital Learning Environment. *Education Sciences*, *11*(6), 279. https://doi.org/10.3390/educsci11060279

Black, P., & Wiliam, D. (1998). Assessment and Classroom Learning. *Assessment in Education: Principles, Policy & Practice*, *5*(1), 7–74.

Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational Assessment, Evaluation and Accountability*, *21*(1), 5–31. https://doi.org/10.1007/s11092-008-9068-5

Borich, G. (2017). *Effective teaching methods* (9th ed.). Pearson.

Botta, E. (2021). Verification of the measuring properties and content validity of a computer based MST test for the estimation of mathematics skills in Grade 10. *7th International Conference on Higher Education Advances (HEAd'21)*. Seventh International Conference on Higher Education Advances. https://doi.org/10.4995/HEAd21.2021.13046

Boyle, J., & Nicol, D. (2003). Using classroom communication systems to support interaction and discussion in large class settings. *Association for Learning Technology Journal*, *11*(3), 43–57.

Brancaccio, A., Marchisio, M., Meneghini, C., & Pardini, C. (2015). More SMART Mathematics and Science for teaching and learning. *Mondo Digitale, 14*(58), 1–8.

Corbalan, G., Paas, F., & Cuypers, H. (2010). Computer-based feedback in linear algebra: Effects on transfer performance and motivation. *Computers & Education*, *55*(2), 692–703. https://doi.org/10.1016/j.compedu.2010.03.002

Creswell, J. W., & Clark, V. L. P. (2017). *Designing and conducting mixed methods research*. Sage publications.

Dweck, C. S. (2000). *Self-theories: Their role in motivation, personality, and development*. Psychology Press.

European Parliament and Council. (2018). Council Recommendation of 22 May 2018 on key competences for lifelong learning. *Official Journal of the European Union,* 1–13.

Fahlgren, M., & Brunström, M. (2023). Designing example-generating tasks for a technology-rich mathematical environment. International *Journal of Mathematical Education in Science and Technology*, 1–17. https://doi.org/10.1080/0020739X.2023.2255188

Hattie, J., & Timperley, H. (2007). The Power of Feedback. *Review of Educational Research*, *77*(1), 81–112. https://doi.org/10.3102/003465430298487

Kluger, A. N., & DeNisi, A. (1996). The effects of feedback interventions on performance: A historical review, a meta-analysis, and a preliminary feedback intervention theory. *Psychological Bulletin*, *119*(2), 254–284. https://doi.org/10.1037/0033-2909.119.2.254

Laurillard, D. (2002). *Rethinking university teaching: A conversational framework for the effective use of learning technologies* (2. ed). Routledge/Falmer.

Mascarenhas, A., Parsons, S., & Burrowbridge, S. C. (2016). Preparing Teachers for High-Need Schools: A Focus on Thoughtfully Adaptive Teaching. *Occasional Paper Series*, *2011*(25), 26.

Romano, A., Petruccioli, R., Rossi, S., Bulletti, F., & Puglisi, A. (2023). Practices for adaptive teaching in STEAM disciplines: The Project T.E.S.T. *QTimes Journal of Education Technology and Social Studies*, *1*(1), 312–328. https://doi.org/10.14668/QTimes_15123

Sadler, D. R. (1989). Formative assessment and the design of instructional systems. *Instructional Science*, *18*(2), 119–144.

Scriven, M. (1967). *The methodology of evaluation*. Lafayette, Ind: Purdue University.

Shute, V. J. (2008). Focus on Formative Feedback. *Review of Educational Research*, *78*(1), 153–189.

Thompson, M., Wiliam, D., & Dwyer, C. A. (2007). Integrating assessment with instruction: What will it take to make it work? In *The future of assessment: Shaping teaching and learning* (pp. 53–82). Erlbaum.