Regression Equations for Preliminary Dimensioning of Axial Compressor Discs

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Abstract: In compressor design today, the process of obtaining disc parameters at the first steps is poorly formalised. This process is often determined by the experience of the designer and is not very predictable. As a result, it is possible to estimate compressor efficiency during design calculation, but determination of compressor mass and strength factors is often difficult or is performed using approximate formulas. The authors of the paper proposed to use regression formulae derived from statistical processing of a database of dimensionless parameters of more than 20 different gas turbine engines from different countries. Relying on the obtained regularities, it is possible to get a sketch of the disc in the meridional plane and to estimate its mass and strength using a semi ring model by means of the design gas-dynamic calculation model. As a result, even at the first steps of compressor design the engineer has an opportunity to screen out obviously unacceptable variants and choose the best option not only by efficiency criteria. The selected option will require fewer refinements in the future, which will reduce the number of iterations in the design process and minimise design time and costs.

1 INTRODUCTION

In all methods for designing axial compressors of gas turbine engines (GTE) known to the authors the criterion for optimizing the design at the 1D calculation stage is their gas-dynamic efficiency (efficiency). [Belousov A. (2006), Cumpsty A. (2004), Falck N. (2008), Gallimore S. J. (1999)] However, as is known, when designing compressors, they are subject to not only efficiency requirements, but also weight, strength, design, manufacturability, etc. [Inozemtsev A. (2022), Mattingly, J. D. (2002)]. Existing 1D compressor design methods take into account other criteria, usually at the level of size limitations and statistical dependencies, which is not enough for a reliable assessment of these criteria.

For this reason, a proposal was made to generate sketches of the discs in the first (rough) approximation for the compressor flow path variants obtained in the design calculation. The known shape of discs will allow not only to estimate their mass more accurately, but also to carry out a simplified assessment of strength criteria using a semi ring model. Thus, during the initial design steps, the designer will receive not only the gas-dynamic properties of the compressor together with the flow path shape, but will also be able to estimate their masses and strength factors quite accurately (more accurately than existing regression dependencies). As a result, it is possible to perform multi-criteria optimisation of compressors at the design step and to screen out high mass and low reliability (high stress) options at an early step. This will result in a high quality variant that will be an excellent starting point for further 3D calculations, reducing the number of necessary calculations performed with high-level models that require a large amount of computation time and computational resources.

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2 A DATABASE OF EXISTING COMPRESSOR DISCS

Information search in available technical publications did not reveal any formal recommendations on forming the compressor disc geometry in the first approximation. For this reason, it was decided, relying on the existing designs of high-pressure compressors of gas turbine engine, to obtain regression dependences, with the help of which it is possible to obtain the expected shape of the disc in the first approximation. In the following steps, this shape will be refined to reduce mass and stresses. However, even the first approximation discs (based on statistical data) will help to screen out obviously unacceptable compressor designs.

To obtain regression dependencies for disc shape calculation, a database of GTEs of different generations, design firms, types and purposes was formed. The database was created by measuring quality drawings - meridional cross-sections of GTEs, obtained from reliable sources (usually directly from partner engine-building enterprises, reference books and operational literature), the quality of which is not questionable. A total of 27 gas turbine engines from various countries designed over the last 40 years were reviewed (including recent models such as PW1100, Leap GE NX, etc.).

Based on the analysis of available GTE compressor drawings, a generalized disc shape was determined (Figure 1) and the dimensions whose statistical information was to be collected were outlined.

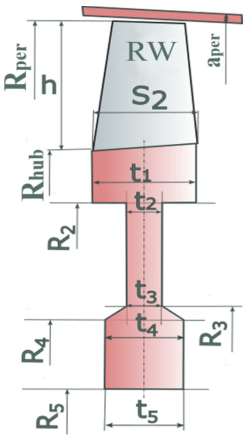

Figure 1: Principal generalized diagram of an axial compressor disc with main dimensions.

The disc dimensions shown in Figure 1 were measured and then disproportioned. The axial and diametral dimensions of the discs were related to the axial chord of the RW blades (S_2), characteristic diameters (hub diameter R_{hub}, middle R_{mid}, peripheral diameter R_{per}) and blade heights h. (a total of 11 dimensional and 20 dimensionless parameters).

In the future it is planned to expand this database and to refine the obtained regression equations.

3 DISC SHAPE DEPENDENCIES

The regression dependencies for determining the main dimensions of discs (indicated in Figure 1) were searched in the following sequence:

1. All parameters (mostly dimensionless) that can influence the value of the size of interest were collected in a single table. This included both universal parameters that are likely to influence any size (stage number, relative hub diameter (R_{hub}/R_{κ}) etc.) and size-specific parameters (e.g., dimensions of neighboring elements) in all possible combinations.
   This creates a table of possible dimensionless parameters with probable influencing factors.
2. For the generated table the preliminary data cleaning from the values falling out of the general array is carried out. Their occurrence can be caused by errors in the measurement of the prototype, data input into the database, as well as by the unusual design of the measured sample. To screen out the "uncharacteristic" values, the mathematical expectation of the parameter (arithmetic mean) X and its standard deviation σ, Walpole, R., Montgomery, D. (2012), were determined for each data column in the table formed in step 1. All points whose values were outside the range (X±2σ) were excluded from consideration. On average, no more than 5% of the points in each column were rejected. Often a point was out of the sample range for several parameters.
3. Correlation analysis was performed for the pre-cleaned data set. Using the Excel data analysis package, pairwise calculation of statistical correlation of all parameters of the array is carried out, Montgomery, D. (2012). Correlation shows to what extent one value systematically changes when another or several others differ. If the compared pair of parameters is statistically interrelated, the correlation coefficient tends to 1. If two quantities are independent, the correlation coefficient approaches 0.

4. Using the Excel data analysis package, a linear regression equation was found for the parameters pre-selected in step 3:

$$y = a_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + \cdots + b_n \cdot x_n$$

For some parameters we also considered non-linear regression equations such as:

$$y = a_0 + b_1 \cdot x_1 + \cdots + b_n \cdot x_n + b_{12} \cdot x_1 \cdot x_2 + b_{13} \cdot x_1 \cdot x_3 + \cdots + b_{ij} \cdot x_i \cdot x_j$$

The values of the coefficients were found using the Excel data analysis package and the parameters characterizing the ‘quality’ of the equation were calculated there. Special attention was paid to the following parameters, Montgomery, D. (2012):

R_2 - coefficient of determination, which shows to what extent the calculated model explains the dependence and changes of the studied parameter - Y on the studied factors - X. If this parameter tends to 1, then the obtained equation fully (maximally) describes the available data array. If R^2 tends to zero, it means that the equation poorly describes the data set. Values above 0.6...0.7 are considered acceptable.

P-value - the probability that allows to determine the significance of the found regression coefficient. In cases where $P>0.05$, the coefficient can be considered zero, which means that the corresponding independent variable does not affect the dependent variable. In other words, this parameter should be less than 0.05 and tends to zero. This coefficient was determined for each coefficient.

F is the observed (empirical) value of the F-statistic, by which the hypothesis of equality of all model coefficients to zero simultaneously is tested. F significance is a theoretical probability that under the hypothesis that all model coefficients are equal to zero simultaneously, the F-statistic is greater than the empirical value of F. This parameter should tend to zero.

Regression coefficients and values of the above regression parameters were found for the variables pre-selected in step 3. If the value of parameter P was 0.05, it was excluded from consideration. At first, the variables with the highest P value were excluded, then the search for coefficients and parameters was repeated. The variables with the largest P value were removed again and the process was repeated until P was reduced to the required level. At the same time, special attention was paid to the value of R_2. It had to be maximized. Thus, by varying all possible combinations of Y and influencing variables, the equations that had the maximum value of R_2 and minimum P for all coefficients were found. If the values of regression coefficients with acceptable statistical parameters were not found for the linear regression equation, the values of coefficients of nonlinear regression equations were searched using a similar algorithm.

Using the equations found, the values of the parameters of interest were calculated from the database values and the calculated values were compared with the real database parameters. As a result of the comparison, the mean and maximum error values of the available data set were estimated. These values also helped in the selection of equations. Obviously, the calculation errors using the equations found should be as small as possible.

4 RESULTS AND DISCUSSION

When deriving the formulae, the following assumptions were made for simplification: t_1=S_2; t_2-=t_3; t_4=t_5 (Figure 1). These assumptions are made to reduce the number of variables. It should also be noted that such a simplification is justified due to the fact that the article deals with the initial stages of design, and it is important to obtain a disk shape close to reality with a minimum number of initial data available at this stage. The obtained regression equations for calculating axial dimensions of axial compressor flow path and their statistical parameters are given in Table 1.

Table 1: Obtained regression formulas for calculating axial compressor disc sizes.

Formula	R^2
$\frac{R_5}{h} = -4.9428 + 8.8525 \cdot \bar{d}$	0.4873
$\frac{R_4}{h} = -7.0514 + 12.0374 \cdot \bar{d}$	0.6319
$\frac{R_2}{h} = -14.1513 + 4,8883 \cdot \frac{i}{z} + 21.1433 \cdot \bar{d}$	0.8732
$\frac{t_2}{R_{\text{hub}}} = 0.0976 - 0.1089 \cdot \bar{d} + 0.027 \cdot \frac{S_2}{h}$	0.5346
$\frac{t_4}{h} = -2.015 + 0.0854 \cdot i + 2.3505 \cdot \bar{d} + 4.1073 \cdot \frac{S_2}{R_{mid}}$	0.5325
$\frac{(R_1-R_2)}{(R_1-R_5)} = 0.5579 - 0.0141 \cdot i - 0.2887 \cdot \bar{d}$	0.4619
$\frac{a_{per}}{h} = 0.0308 - 0.1831 \cdot \frac{i}{z} - 0.0096 \cdot \bar{d} \cdot i + 0.4103 \cdot \frac{i}{z} \cdot \bar{d}$	0.7186

To check the adequacy of the found regression equations, they were used to calculate the dimensions of high-pressure compressor discs of some engines from the database. From engine drawings the dimensions of flow paths were taken and then using the found formulas the disc sizes are calculated. Comparison of original and calculated shapes of compressor discs is shown in Figures 2-3.

As can be seen from the results shown in Figures 2 and 3, the developed formulas allow to determine the expected shapes of axial compressor discs quite reliably. In the future it is planned to supplement the database, on the analysis results of which the equations were obtained and to carry out thinning of regression dependences.

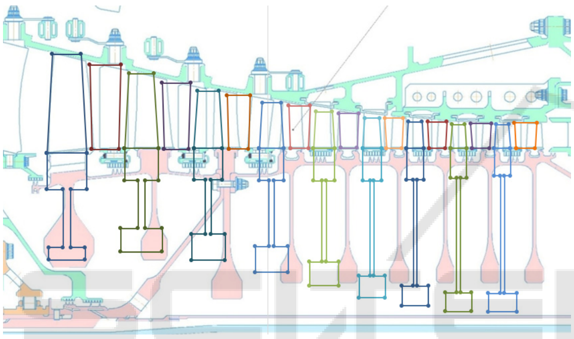

Figure 2: Comparison of real dimensions of CFM-56 high-pressure compressor discs with the results of calculation by regression equations from Table 1.

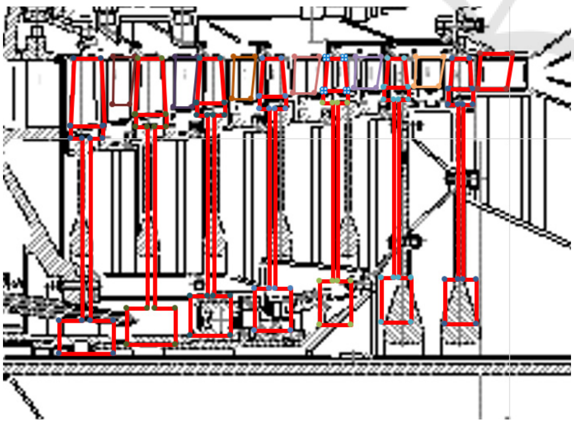

Figure 3: Comparison of real dimensions of D-36 high-pressure compressor discs with the results of calculation by regression equations from Table 1.

5 CONCLUSIONS

The presented article provides a statistical analysis of a database collected by the authors on the sizes of 27 different multi-stage axial high-pressure compressors of various aircraft engines. The database contained information about more than 200 stages. This data array was subject to statistical analysis, because of which empirical equations were found that allow, as a first approximation, to obtain the shape of the compressor disk at the initial design stages. The resulting formulas showed good agreement with real compressor designs. As a result, having the results of the design calculation of the compressor, the engineer can quickly obtain the expected disk shape, which will allow the mass of the compressor to be assessed and taken into account during optimization at the early stages and find an excellent starting point for the compressor design and its strength analysis. In the future it is planned to supplement the database, on the analysis results of which the equations were obtained and to carry out thinning of regression dependences.

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