Discretization Strategies for Improved Health State Labeling in Multivariable Predictive Maintenance Systems

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Abstract: In machine learning, effective data preprocessing, particularly in the context of predictive maintenance, is a key to achieving accurate predictions. Predictive maintenance datasets commonly exhibit binary health states, offering limited insights into transitional phases between optimal and failure states. This work introduces an approach to label data derived from intricate electronic systems based on unsupervised discretization techniques. The proposed method uses data distribution patterns and predefined failure thresholds to discern the overall health of a system. By adopting this approach, the model achieves a nuanced classification that not only distinguishes between healthy and failure states but also incorporates multiple transitional states. These states act as intermediary phases in the system's progression toward potential failure, enhancing the granularity of predictive maintenance assessments. The primary objective of this methodology is to increase anomaly detection capabilities within electronic systems. Through the utilization of unsupervised discretization, the model ensures a data-driven approach to system monitoring and health evaluation. The inclusion of multiple transitional states in the labeling process facilitates a more precise predictive maintenance framework, enabling informed decision-making in maintenance strategies. This article contributes to advancing the effectiveness of predictive maintenance applications by addressing the limitations associated with binary labeling, ultimately encouraging a more nuanced and accurate understanding of system health.

1 INTRODUCTION

Labels in datasets are crucial in the use of supervised machine learning, their quality directly affects the performance of prediction (Budach et al., 2022). For instance, in predictive maintenance, a crucial endeavor is reducing failures and associated costs by predicting issues (Mobley, 2002; Ran et al., 2019). The quality of labels is then directly linked to the reliability of whether or not failures are predicted before they occur and deviate from optimal states.

While machine learning techniques have shown promise in predictive maintenance (Carvalho et al., 2019), the reliance on traditional supervised labeling methods presents significant challenges. Manual annotation of data is time-consuming, often requiring expert knowledge, and can lead to limitations in scalability and efficiency. Moreover, in the context of electronic systems, the dynamic and intricate nature of data poses challenges for accurate labeling with health indicators or Remaining Useful Life (RUL). Consequently, this research aims to explore data labeling within complex electronic systems.

In response to the limitations of traditional labeling methods, the goal of this study is to propose an approach of unsupervised labeling using discretization techniques for electronic systems. Discretization methods, such as Equal Width (EW) and Equal Frequency (EF) (Catlett, 1991), provide an unsupervised method for classifying parameters into categories such as health states. By incorporating failure thresholds, a clear distinction can be established between healthy, failure, and transition states for each of the system parameters. By combining those, a global state of the system can be created, giving a more detailed system health assessment compared to common binary states in public datasets (Tan and Raghavan, 2010).

The proposed approach significantly enhances the details by determining new transition states to assess the system's health. This detailed prediction capability facilitates decision-making since it provides more

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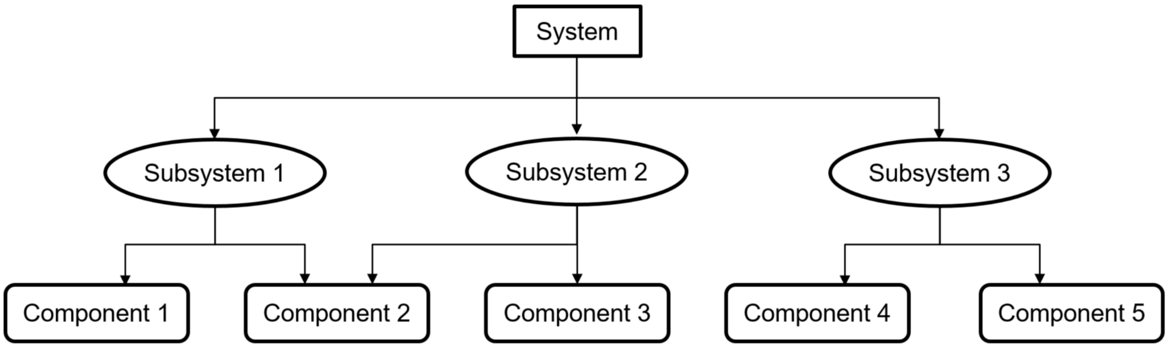


Figure 1: Example of a multilevel system's structure.

information. It enables system administrators to take proactive actions, thereby minimizing downtime and optimizing system reliability (Mobley, 2002). The proposed method is versatile and scalable. It can be generalized to handle complex, multi-component systems, broadening its applicability in system health labeling.

The article is organized as follows. First, the context of predictive maintenance and labeling is explored in the domain of electronic maintenance. Secondly, the proposed labeling methodology is explained, discussing the application of unsupervised discretization methods. Then the results of labeling are presented and discussed for a public dataset. Finally, the last part concludes and summarizes the key findings while providing recommendations for future research.

2 ELECTRONIC SYSTEM MAINTENANCE

2.1 Context

As illustrated in Figure 1, the studied system consists of several electronic subsystems that perform different functions. Each subsystem is composed of multiple components which can be part of several subsystems. Different parameters of each subsystem are monitored by collecting data at regular intervals. These measurements are organized into control runs that verify if each component is operating within its nominal range and does not exceed any failure threshold. If this threshold is reached, the component is considered as non-functioning and needs to be repaired.

A single threshold proves insufficient for a comprehensive characterization of a system's state. In many instances, the goal is to establish a more nuanced health assessment, typically manifesting as a health indicator, health state, or Remaining Useful Life (RUL) of the system (Lei et al., 2018a). The health indicator represents a numerical value evaluating the overall condition of the system. Similarly, the health state, while similar to the health indicator, adopts a categorical form rather than a numerical value. On the other hand, RUL estimation is directed towards predicting the remaining lifespan of the system. Each of these indicators offers valuable insights crucial for informed decision-making in maintenance practices.

2.2 Health Diagnosis

Heath indicators are either physics-based or virtual (Hu et al., 2012). The difference between these two types lies in the method used for their calculations. The physical indicator can be calculated using statistical methods or signal processing techniques based on measurements related to the equipment. It is often the root mean square of signals (Huang et al., 2017), but it can be calculated in many other ways depending on the data processed, such as vibrations (Soualhi et al., 2015). The virtual indicators are based on the fusion of multiple physical health indicators or several signals. Principal Component Analysis is the method generally used for this type of approach, but there are also many methods possible to determine it (Lei et al., 2018b). For instance, it can be estimated using unsupervised ML algorithms (Kurrewar et al., 2021).

Health states are often created by dividing a health indicator into multiple states by identifying trends in the indicator values. A simple strategy for two-state division involves checking if the indicator exceeds an alarm threshold. Various methods are used to determine this threshold (Lei et al., 2018b).

When degradation trends of machinery are inconsistent and cannot be expressed using a single model, multi-state division is used. This division can be achieved through various methods such as the analysis of change points in health indicators (Hu et al., 2016) or by applying clustering algorithms (Scanlon et al., 2013). Machine learning classifiers can also be applied to multi-stage division (Soualhi et al., 2015). To conclude, health states' labeling is a crucial step to precisely describe the behavior of the studied system. Yet it presents several challenges in the context of predictive maintenance.

2.3 Challenges of Data Labeling in Predictive Maintenance

Data labeling stands as a critical phase in supervised machine learning, where labeled data are imperative for training models effectively. However, in the domain of predictive maintenance, datasets often contain only binary labels indicating normal or failure states (Jovicic et al., 2023), unfortunately, transition states are frequently missing. These intermediate states represent crucial transition phases and are relevant in the context of predictive maintenance. However, the same level of certainty is not easily achievable when it comes to identifying transitional states between these two conditions.

The challenge in data labeling for predictive maintenance increases when dealing with failure events. The quantity of failure labels in databases is often limited due to preventive maintenance strategies, where components are replaced before actual failure occurs. This strategy decreases the number of recorded failures, complicating the labeling process and affecting the model's ability to generalize effectively.

2.4 Dataset

For this study, multiple public datasets were considered to benchmark the studied method such as popular datasets: CMAPSS (Saxena et al., 2008), bearing dataset (Lee et al., 2007), or milling dataset (Agogino and Goebel, 2007). However, the majority does not provide or consider failure thresholds, which is a crucial element in predictive maintenance analysis in the presented context. For this reason, the AI4I predictive maintenance dataset (Matzka, 2020) has been chosen due to its feature of providing failure thresholds.

Including 10,000 data points with five features, the dataset includes a 'machine failure' label indicating various failure modes. Notably, three of these modes are threshold-dependent: Heat Dissipation Failure (HDF), Power Failure (PWF), and Overstrain Failure (OSF). While Tool Wear Failure (TWF) and Random Failures (RNF) are based on random occurrences.

To enhance the dataset, modifications were made to introduce columns specifying the defined failure thresholds. This adjustment ensures that limits are set for individual parameters rather than combinations, facilitating the analysis of failure states. It is important to highlight that, despite its robust representation of failures, the dataset does not include explicit information on transition states. This limitation underscores the need for the proposed method, which focuses on addressing this gap.

The following sections of this paper will examine existing methodologies and propose new strategies to enhance data labeling in the context of predictive maintenance, ultimately contributing to the reliability and performance of predictive maintenance systems.

3 PROPOSED LABELING TECHNIQUES

3.1 Overview of Discretization Approaches

Discretization is a process that transforms continuous data into discrete categories, typically finite sets of distinct intervals. Several methods exist for this purpose, they can be classified as supervised or unsupervised (García et al., 2013).

Supervised methods use labeled data to guide the process of dividing continuous features into discrete categories. They generally outperform unsupervised methods due to their context-specific nature (Dougherty et al., 1995). For this reason, the most common methods for discretization are ChiMerge (Kerber, 1992), Minimum Description Length principle (Rissanen, 1986), or entropy-based techniques (Fayyad and Irani, 1993). However, they have to use data with class information. In practical cases, manual annotation of data is often used to create labeled data before using those approaches. However, in the case of the presented dataset (Section 2.4), labels for discretization have not been created, so such supervised techniques cannot be used and unsupervised methods are the only choice.

Unsupervised discretization methods, such as the EW discretization method, divide the range of continuous values into a predetermined number of intervals of equal width. This approach is straightforward to implement and computationally efficient. However, it is sensitive to outliers, as extreme values can significantly affect the width of intervals, leading to a suboptimal representation of the data distribution (Catlett, 1991). The EF discretization method partitions the data into intervals that contain approximately the same number of data points, aiming to address the sensitivity to outliers seen in EW discretiza-

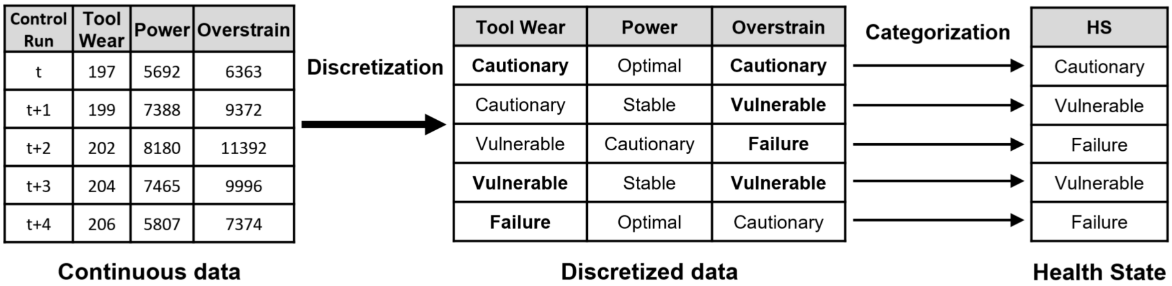
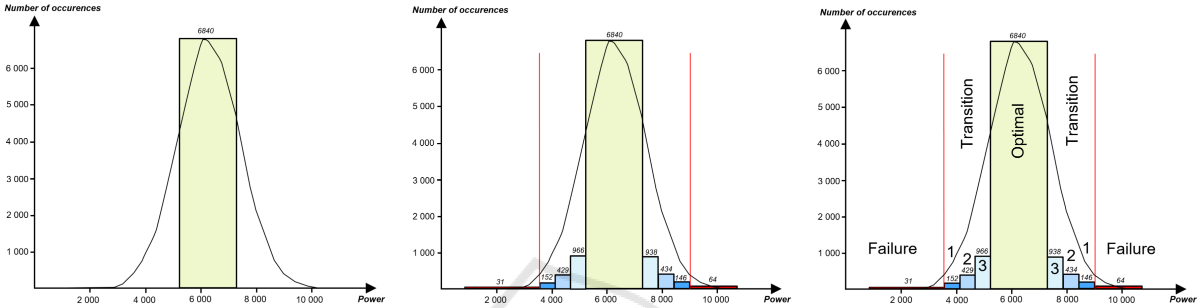


Figure 2: Labeling process by discretization.



(a) Standard deviation approach to create the *optimal* state.

(b) Addition of the others categories of health state.

(c) Labelization of the different health states.

Figure 3: Distribution of the power parameter (1 = Vulnerable state, 2 = Cautionary state and 3 = Stable state).

tion. However, it may struggle with uneven data distributions, where certain intervals may capture sparse or dense regions of data. This method is particularly useful when the goal is to ensure each category has a comparable number of instances.

Both of those approaches can be used in the context of discretization, but in the following section, the new approach is presented using discretization techniques as a way of unsupervised data labeling.

3.2 Multivariable System Labeling Through Discretization Approach

It is common for predictive maintenance databases to lack detailed health states, often merely indicating a binary state of failure or non-failure. The proposed Multivariable System Labeling through Discretization (MSLD) approach creates these health states for a multivariable system using unsupervised discretization of the acquired data. This method enhances the granularity of system health assessment, enabling more detailed predictions and effective maintenance strategies.

The proposed method consists of two main steps: discretization and categorization as shown in Figure 2.

In the discretization step, each measured parameter from control runs is converted into discrete values based on its distribution. A standard deviation-based approach has been used to identify the optimal operating range for each parameter. It corresponds to the values that fall within plus or minus one standard deviation from the mean value to identify outliers.

For example, in Figure 2, the power parameter has an average value of 6279 and a standard deviation of 1067. In this case, values between 5212 and 7347 are considered optimal (Figure 3a). The size of the optimal class is arbitrary and can be adjusted by experts based on the stability of the studied system. Any values outside this range are considered as non-optimal. Furthermore, two additional categories are also created for values that exceed the failure thresholds on either side, representing a failure state for the equipment. The failure thresholds for the power parameter are 3500 and 9000. The values between the failure threshold and the optimal state are further divided into multiple intervals using the EW discretization method. The EW method is used to have similar size bins to reflect the actual distribution of the data. This way, with the example of three transition states, values between 7347 to 7898 are categorized as the stable state, 7898 to 8449 as the cautionary state, and 8449 to 9000 as the vulnerable state. The same type of state is applied to the other side of the Gaussian curve (Figure 3b). The number of transition states on each side is determined by experts depending on the system. In the case of this article, the choice of

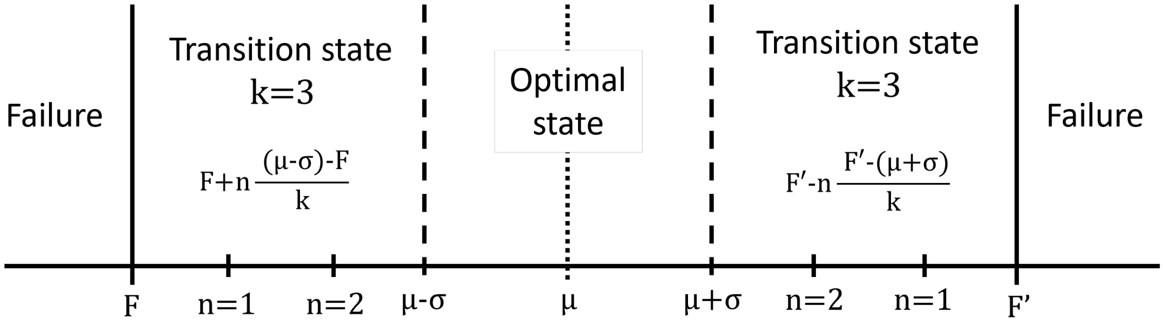


Figure 4: Labeling process by discretization.

three transition states is based on the results obtained in Sec. 4.

Figure 4 represents the discretization step. F and F' represent the failure thresholds, μ the mean value and σ the standard deviation. The optimal state is then defined by the values between $\mu-\sigma$ and $\mu+\sigma$. k is the number of transition states and n defines the limit between the different transition states with the following formula for value inferior to the optimal:

$$F+n\frac{(\mu-\sigma)-F}{k}$$

And for value superior the optimal:

$$F'-n\frac{F'-(\mu+\sigma)}{k}$$

In the categorization step, each control run is assigned a specific category. This assignment is based on the discretized values of its parameters. The category assigned to the control run corresponds to this most deviated parameter. This deviation is measured in terms of how far the parameter's value is from its normal range. This is based on the assumption that the health status of the system is determined by the state of its most degraded component. In other words, if one component of the system is in a poor state, it significantly affects the overall health of the system, regardless of the state of the other components.

With the example of three transition states, the following states can be defined as vulnerable, cautionary, and stable. They represent different levels between the failure and optimal state. The vulnerable state indicates a condition closest to the failure state, signifying a potential early warning or indication of an impending issue. The cautionary state reflects an intermediate condition between the failure state and an optimal state, suggesting a moderate level of concern. The stable state, on the other hand, is the closest to the optimal state, indicating a state with minimal risk or deviation from normal system operation (Figure 3c). These states provide a nuanced understanding of the system's health, with transitions between them serving as key indicators for effective predictive maintenance.

This approach allows for a more detailed and comprehensive understanding of the system's health. Potential issues can be identified early and appropriate corrective measures can be taken, thereby enhancing the effectiveness of the predictive maintenance strategies.

4 RESULTS AND DISCUSSION

4.1 Results

After introducing the new labeling approach, this section will discuss the results and effectiveness of this method. The presented results of the discretization step are for the power parameter from the AI4I dataset, aiming to identify distinct states of the system. This method is applied to all parameters, leading then to categorization. Figure 3 illustrates the distribution of the power parameter, categorized into different states with the previously described approach in Sec. 3.2. The optimal state, denoted by the green category, encompasses 68% of the dataset, while the transition state (blue) represents 31% and failure states (red) constitute 1%. These categories serve as crucial indicators of the system's health, with transitions playing a pivotal role in precise predictive maintenance.

Table 1 presents the distribution of the power parameter for three different discretization techniques: EW, EF, and MSLD. Depending on the discretization method used, the distribution can greatly vary. Because of the way EF works, there is a high number of values in extreme bins which leads to an unbalanced diagnosis after categorization 2. EW and MSLD are more adapted to discretize parameters because they do not alter the shape of the data distribution.

The effectiveness of various discretization tech-

Table 1: Different distributions for the "Power" parameter with different discretization techniques.

	Failure	Vulnerable	Cautionary	Stable	Optimal
EW	95	451	1415	3240	4799
EF	95	2453	2464	2453	2439
MSLD	95	298	863	1904	6840

niques for all parameters, when applying the same categorization step is provided in Table 2. The outcomes of health states after the categorization step are heavily dependent on the discretization method. It becomes evident that the simple EW and EF methods are suboptimal for the discretization step of the MSLD approach. MSLD discretization step approach provides much more details and transition states than EW and EF methods.

Table 2: Table of the different distribution of system states depending on the discretization technique.

	Failure	Vulnerable	Cautionary	Stable	Optimal
Binary	348	0	0	0	9652
EW	348	3851	5186	615	0
EF	348	7421	1928	303	0
MSLD	348	2074	3290	3290	998

As shown in Table 2, the original binary scenario, with only failure and non-failure states, lacks granularity. This could lead to missed opportunities for early intervention before a system failure occurs. The EW and EF methods provide more detailed states, which could allow for more proactive maintenance strategies. However, the absence of optimal states might indicate an over-prediction of system issues, potentially leading to unnecessary interventions. The MSLD method seems to provide a more balanced distribution across all states, including optimal ones. This could offer a more nuanced understanding of system health, allowing for targeted interventions and efficient resource allocation.

As seen with EF and EW when obtaining the diagnosis from the weakest link among all parameters, the attribution of an excessive number of values at the extremities of the binning fails to accurately depict the actual health state of the system. The MSLD method outperforms the others, offering a more balanced representation of different system states with the help of failure thresholds.

Table 3 presents the performance of different machine learning algorithms (Decision Tree (DT), Random Forest (RF), K-Nearest Neighbours (KNN), and XGBoost) using previous discretization methods (EF, EW, MSLD) and the original binary states dataset. The performance is measured by the F1 score for different numbers of classes.

From the table, it is evident that the performance generally decreases as the number of classes increases. This is expected as increasing the number of classes adds complexity to the model, making it harder to achieve high accuracy. However, the rate of decrease varies depending on the algorithm and discretization method used.

For instance, the XGBoost algorithm maintains relatively high performance across all numbers of classes and discretization methods, with the F1 score only slightly decreasing as the number of classes increases. This suggests that XGBoost is robust to the increase in class numbers and can handle the added complexity well. On the other hand, the other three algorithms, especially KNN, show a significant drop in performance as the number of classes increases, indicating that it may not be the best choice for this particular problem.

In terms of the discretization methods with XGBoost, EF, and EW perform similarly with MSLD across all numbers of classes. But, as seen previously in Table 2, the distribution of system states from MSLD is more balanced.

Considering the trade-off between performance and the number of classes, choosing nine classes seems to be a good balance. It corresponds to three transition states on each side, two failure states, and the optimal state. This choice provides more granularity than the original binary states while still maintaining relatively high performance across all algorithms and discretization methods. Specifically, the XGBoost algorithm with the more balanced MSLD discretization method is a more robust performance across different numbers of classes.

4.2 Discussion and Limitations

The proposed approach relies on an unsupervised method, which means the role of the expert is important in selecting the right number of transition states. This choice is based on the results obtained with the different configurations.

The number of transition states as well as the size of the optimal state need to be configured correctly depending on the dataset.

The MSLD approach presented here can be applied in a generalized manner to various databases with failure thresholds, to determine transition states.

The presented approach provides a more granular understanding of system transitions. By discretizing data and accounting for transition states, the precision of health state labeling is enhanced. Although this method may not necessarily yield superior prediction performance compared to other approaches,

Table 3: Results with different ML algorithms, discretization methods, and number of transition classes.

Algorithm	Discretization Method	*F1Score with n classes*			
		n=5	*n=7*	*n=9*	*n=11*
DT	EF	97.3	89.3	85.4	79.2
	EW	97.0	92.7	77.2	75.9
	MSLD	93.6	81.7	74.1	63.9
	Binary	96.7			
RF	EF	96.4	91.1	82.9	85.2
	EW	96.7	92.0	85.5	82.7
	MSLD	94.7	84.8	80.1	75.4
	Binary	99.1			
KNN	EF	53.6	50.2	47.2	45.4
	EW	66.8	60.3	60.4	59.0
	MSLD	54.1	52.6	48.9	48.1
	Binary	97.3			
XGBoost	EF	98.5	97.9	97.9	97.1
	EW	98.2	98.3	97.4	97.5
	MSLD	98.7	97.8	97.9	96.7
	Binary	98.8			

its strength lies in its ability to accurately classify a wider range of health states, thereby improving descriptive abilities without necessarily impacting overall predictive performance.

This method is limited to datasets with failure thresholds which give context for the creation of transition states. In many cases, it restricts the use of this method because failure thresholds are not always present. But, if there are no failure thresholds, expert knowledge can be used to determine them.

The discussed methods use manual tuning but it is not always optimal nor efficient. Implementing an automatic parameter optimization could enhance both efficiency and accuracy. Future research will explore these techniques for their applicability in health state labeling. This could lead to more robust health state estimation, improving system reliability and longevity.

5 CONCLUSION

In conclusion, the introduced methodology enhances predictive maintenance practices by addressing the limitations associated with binary labeling commonly found in existing datasets. The unsupervised discretization technique, guided by data distribution and failure thresholds, enables a nuanced classification of multiple transitional states. It allows the experts to rapidly decide the best discretization according to their knowledge and experience. The research underscores the versatility of the MSLD approach, emphasizing its applicability across diverse electronic systems and databases. By providing a more intricate understanding of a system's health and incorporating transitional states as vital indicators, the proposed method enhances anomaly detection. This contribution improves decision-making in maintenance strategies, contributing to the refinement of predictive maintenance applications for a more accurate and informed approach to system health assessment.

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