

# Unlocking Antenna Performance: Harnessing the Power of the Hahn-Banach Theorem in Wireless Communication Systems

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**Keywords:** Antenna Design, Hahn-Banach Theorem, Optimization, Wireless Communication.

**Abstract:** This paper presents a novel approach for improving the performance of the antenna in wireless communication systems through the utilization of the Hahn-Banach Theorem. With advancements of such standards as 5G and B5G in design-related issues, traditional methods are typically inadequate. Applying the Hahn-Banach Theorem results in a strong mathematical framework that augments several vital antenna parameters including gain, efficiency, bandwidth, and radiation patterns. The approach takes solutions from limited to large design spaces, allowing the search for new configurations within tight constraints. Results show substantial advancement in antenna performance, creating better, more reliable progressions. This interdisciplinary method connects theoretical mathematics with the engineering solutions making a great step forward for practical communication technologies. This guarantees excellent linearity and overall performance which is very important for wireless communication signal integrity, impacting future antenna technology and wireless communication research and development.

## 1 INTRODUCTION

Antennas are integral to modern wireless communication systems, supporting diverse applications from mobile communications to IoT devices (Alexiou and Haardt, 2004). As we transition to 5G and beyond (B5G), demand for faster data rates and lower latency intensifies (Hong et al., 2017). Traditional antenna designs struggle to meet the advanced requirements of B5G networks, such as multi-band operation and massive MIMO capacities (Jamil et al., 2010). Consequently, researchers are turning to mathematical optimization methods like the Hahn-Banach Theorem to improve antenna performance (Aharon and

Nemirovski, 2001). These methods offer systematic frameworks to explore design parameters and constraints, identifying optimal configurations (Alateewe and Nagy, 2024).

They allow for the evaluation of various intricate performance criteria and trade-offs, resulting in Pareto-optimal solutions (Wang et al., 2024). In addition, uncertain reality environment management is resolved by mathematical design approach (Cheraghinia et al., 2024). Many modern antenna techniques such as metamaterials and reconfigurable antennas are incorporated into it (Kumar et al., 2021), improving antenna performance in B5G networks.

Our methodology allows for the creation of effective wireless systems for multiple B5G uses, URIC, and big IoT links (Singh et al., 2024). It also guides development of sophisticated signal processing algorithms and network protocols designed for B5G systems (Sufyan et al., 2023). In general, utilization of mathematical optimization strengthens antenna development, stimulates innovation, and defines an infrastructure of communication networks.

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## 2 ANTENNA DESIGN METHODS

### 2.0.1 Empirical and Numerical Methods

Methods are practical for antenna designers who will work with these designs on and on, refining them in the process through experience. On the other hand, for complex antenna structures and detailed electromagnetic analysis requirements, numerical methods such as Finite-Difference Time-Domain (FDTD) simulations (Thaher et al., 2020) are invaluable. They offer accurate and rapid analyses, supplementing empirical approaches and extending the toolkit of the designer for best performance.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (1)$$

Numerical methods, illustrated in (Zhu et al., 2023), take standard antenna configurations such as dipoles and Yagi-Uda arrays as the basic geometries for further modification. FDTD simulations discussed in (Zhu et al., 2023) enable analytical accuracy through discretization of space and time domains for solving Maxwell's equations to present a thorough modeling of antenna performance parameters on a computational grid;

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2)$$

This equation presents the curl form of electric field ( $\mathbf{E}$ ), which is equal to the negative rate of change of the magnetic flux density ( $\mathbf{B}$ ). It demonstrates how a time-varying magnetic field gives rise to an electric field, which is a basic principal of electromagnetic induction.

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (3)$$

The magnetic field of the curl ( $\mathbf{H}$ ) is created among electric current ( $\mathbf{J}$ ) and the rate of change of the electric displacement field ( $\mathbf{D}$ ). This equation generalizes Ampere's Law to include displacement current, which considers changes in the electric field (Mittra, 2018). FDTD iterative updates electric and magnetic fields at discrete points to model wave propagation and electromagnetic phenomena, making it indispensable in modern antenna design. For instance, updating electric and magnetic fields relies on their respective curls. This discretization is typically represented as:

$$E_x(i, j, k)^{n+1} = E_x(i, j, k)^n - \Delta t \left( \frac{H_z(i, j+1, k)^n - H_z(i, j, k)^n}{\Delta y} - \frac{H_y(i, j, k+1)^n - H_y(i, j, k)^n}{\Delta z} \right)$$

$$H_x(i, j, k)^{n+1} = H_x(i, j, k)^n + \Delta t \left( \frac{E_z(i, j+1, k)^n - E_z(i, j, k)^n}{\Delta y} - \frac{E_y(i, j, k+1)^n - E_y(i, j, k)^n}{\Delta z} \right) \quad (4)$$

where  $E_x$  and  $H_x$  is the electric field component and magnetic field component,  $n$  is the time step index, and  $i, j, k$  are the indices of the grid points in the  $x, y,$  and  $z$  directions, respectively. Where  $\Delta t$  is the time step increment, and  $\Delta x, \Delta y,$  and  $\Delta z$  are the step increments in each direction in space.

The empirical methods lay solid basis for antenna design, particularly for conventional types of antennas. The power of numerical methods, such as FDTD in solving Maxwell's equation in practically any geometry, is crucial in designing the modern complex antenna systems (Alexiou and Haardt, 2004).

### 2.0.2 Genetic Algorithms (GA)

GA are modeled after biological evolution, using mechanisms like selection, crossover, and mutation. Population of potential designs in the antenna design is evolved for several generations until the optimal solutions are found.

*Fitness Function in GA:*

$$f(x) = \text{Performance}(x) \quad (5)$$

where  $x$  represents the antenna parameters.

### 2.0.3 Particle Swarm Optimization (PSO)

PSO simulates the social behavior of birds or fish to optimize a problem. Particles move through the solution space, adjusting their trajectories based on personal and communal experiences (Sehrai et al., 2020).

*PSO Equations:*

*Position Update:*

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (6)$$

*Velocity Update:*

$$v_i(t+1) = w \cdot v_i(t) + c_1 \cdot r_1 \cdot (p_{best} - x_i(t)) + c_2 \cdot r_2 \cdot (g_{best} - x_i(t)) \quad (7)$$

we considered the case where  $i$  is a particle index,  $x_i$  and  $v_i$  are positions and velocities,  $p_{best}$  and  $g_{best}$  are best known positions,  $r_1, r_2$  are two random numbers, and  $c_1, c_2$  are learning factors. They are especially efficient in regards to problem spaces which consist of several dimensions for example than traditional methods could not be so practical (Thaher et al., 2020).

### 3 METHODOLOGY

#### 3.1 Hahn-Banach Theorem

The Hahn-Banach theorem extends bounded linear functionals from subspaces to the entire space while preserving their norms, playing a crucial role in mathematics and various fields (Ben-Tal and Nemirovski, 2021). Its significance lies in functional analysis and finds applications in optimization, information transmission, and antenna design. This ensures continuous operationalization of metric functions on normed vector spaces without altering their norms, a characteristic vital for numerous applications;

$$|f(x)| \leq C\|x\|, \quad \forall x \in U \quad (8)$$

$$F(x) = f(x), \quad \forall x \in U \quad (9)$$

$$|F(x)| \leq C\|x\|, \quad \forall x \in V \quad (10)$$

here,  $f$  denotes the functional on  $U$ ,  $F$  is its extension on  $V$ , and  $C$  is a constant. Provide the description of the antenna design problem with specifications (directivity, bandwidth, radiation pattern, interference rejection); use the Hahn-Banach Theorem for strategic extension in order to overcome performance limitations with the design constraints (Abdullah et al., 2019).

#### 3.2 Problem Formulation

Design a compact antenna for portable 5G devices, optimizing gain, bandwidth, linearity, and adhering to strict weight constraints;

- **Gain ( $G$ ):** Aim for a high gain to ensure robust signal reception and transmission over distances, targeting a minimum gain of 6 dBi.
- **Bandwidth ( $BW$ ):** The antenna must support a wide bandwidth to accommodate the 5G frequency range, specifically from 3.3 GHz to 3.8 GHz, to cover various 5G bands.
- **Linearity:** Ensure high linearity to reduce signal distortion, which is crucial for maintaining the integrity of high-speed data transmission in 5G networks.
- **Weight ( $W$ ):** Due to the portable nature of the device, the antenna's weight should not exceed 20 grams.

#### 3.3 Solution Approach Using the Hahn-Banach Theorem

Proposing a novel approach using the Hahn-Banach Theorem to tackle design challenges in compact,

high-performance antennas for portable 5G devices, optimizing gain, bandwidth, linearity, and adhering to weight constraints.

#### 3.3.1 Defining Design Space and Performance Metrics

Let  $V$  represent the vector space of all possible antenna designs, and  $U \subseteq V$  the subspace (Ben-Tal and Nemirovski, 2021) of initially feasible designs. The performance metrics are defined as functions mapping designs to real values representing gain ( $G$ ), bandwidth ( $BW$ ), linearity ( $L$ ), and weight ( $W$ ).

The theorem is applied to extend performance evaluation functionals (Jamil et al., 2010) from  $U$  to the entire space  $V$ , facilitating the exploration of innovative designs. This extension ensures that:

$$|f(x)| \leq p(x), \quad \forall x \in U \rightarrow \forall x \in V$$

The expression  $|f(x)| \leq p(x), \quad \forall x \in U \rightarrow \forall x \in V$  signifies that the condition imposed by  $p(x)$  on  $f(x)$  in the subspace  $U$  is preserved as  $f(x)$  is extended across the entire space  $V$ . where as,

- $f(x)$ : Represents a functional that evaluates the performance of an antenna design  $x$ , such as its gain, efficiency, or bandwidth.
- $U$ : Denotes the initial subspace of feasible designs, satisfying basic performance criteria or constraints.
- $V$ : Represents the entire design space, including more innovative or unconventional designs beyond those in  $U$ .
- $p(x)$ : Defines a bound for the performance evaluation functional  $f(x)$ , ensuring that the evaluation does not exceed certain physical, regulatory, or performance-related limits.

The Hahn-Banach Theorem expands antenna design optimization possibilities while maintaining performance within specified bounds (Banks and Kunisch, 2012). Systematic search enables the identification of enhanced performance measures within set constraints (Jiang et al., 2019).

### 4 MATHEMATICAL MODELING

The mathematical modeling of antenna graph consists of the example of the antenna's electromagnetic characteristics through various mathematical equations. Key factors of this modeling included radiation pattern, impedance, benefit, and efficiency.

**Radiation Pattern.** The radiation pattern is represented by means of:

$$E(\theta, \phi) = f(\theta, \phi) \quad (11)$$

where  $E(\theta, \phi)$  denotes the electric field intensity at angles  $\theta$  and  $\phi$ .

**Impedance.** Antenna impedance, combining resistance and reactance, is given by:

$$Z = R + jX \quad (12)$$

where  $Z$  is the impedance,  $R$  is resistance, and  $jX$  is reactance.

**Gain.** The gain of an antenna is defined as:

$$G = \frac{4\pi U(\theta, \phi)}{P_{in}} \quad (13)$$

where  $G$  is the gain,  $U(\theta, \phi)$  the radiation intensity, and  $P_{in}$  the input power.

**Efficiency.** Efficiency is the ratio of the power radiated to the power input:

$$\eta = \frac{P_{rad}}{P_{in}} \quad (14)$$

where  $\eta$  is efficiency,  $P_{rad}$  is radiated power, and  $P_{in}$  is input power.

**Bandwidth.** The bandwidth of an antenna is expressed as:

$$BW = f_2 - f_1 \quad (15)$$

where  $BW$  is bandwidth, and  $f_1, f_2$  are the lower and upper frequency limits, respectively. Mathematical modeling is crucial in antenna design, providing insights into performance and aiding in the optimization of antenna characteristics. This modeling forms the basis for simulation and analysis in the antenna design process.

## 4.1 Performance Specifications

The antenna design must meet the following specifications:

- **Desired Gain:** Minimum of 06 dBi.
- **Bandwidth:** Operate effectively from 3.3 GHz to 3.8 GHz.
- **Radiation Pattern:** Ensure consistent performance across orientations, focusing on omnidirectional or directional patterns.
- **Interference Rejection:** Incorporate features to minimize electromagnetic interference.

The main challenge is optimizing antenna gain within size and weight limits (Abdullah et al., 2019), demanding a delicate balance between performance and dimensions, necessitating innovative design and strategic optimization techniques.

## 4.2 Functional Analysis

Apply the Hahn-Banach Theorem to extend a bounded linear functional from a subspace (e.g., specific antenna configuration) to the entire vector space (e.g., design space) (Ben-Tal and Nemirovski, 2021). This extension facilitates advanced optimization of key antenna characteristics, enabling efficient utilization of complex design spaces.

## 4.3 Functional Extensions Using Hahn-Banach Theorem

### 4.3.1 Radiation Pattern

Simplified Functional in U:

$$f_{RP}(x) = \text{Integration}(E(\theta, \phi; x)) \quad (16)$$

Extended Functional in V:

$$F_{RP}(y) = \text{ExtendedIntegration}(E(\theta, \phi; y)) \quad (17)$$

### 4.3.2 Impedance

Simplified Functional in U:

$$f_Z(x) = \text{CalculateImpedance}(x) \quad (18)$$

Extended Functional in V:

$$F_Z(y) = \text{ExtendedCalculateImpedance}(y) \quad (19)$$

### 4.3.3 Gain

Simplified Functional in U:

$$f_G(x) = \text{CalculateGain}(x) \quad (20)$$

Extended Functional in V:

$$F_G(y) = \text{ExtendedCalculateGain}(y) \quad (21)$$

### 4.3.4 Efficiency

Simplified Functional in U:

$$f_E(x) = \frac{P_{rad}(x)}{P_{in}(x)} \quad (22)$$

Extended Functional in V:

$$F_E(y) = \frac{P_{rad}(y)}{P_{in}(y)} \quad (23)$$

The objective is to find an antenna design  $y^*$  in V that optimizes the extended functional, considering real-world constraints.

#### 4.4 Mathematical Formulation for Optimizing $y^*$ in $V$

We aim to find an optimal antenna design  $y^*$  in  $V$ , which involves solving the following optimization problem:

##### 4.4.1 Extended Functional Formulation

Find  $y^*$  in  $V$  such that: (24)

$$\max F_{\text{total}}(y) \tag{25}$$

subject to:

$$F_{RP}(y) \geq \text{Threshold}_{RP}, \tag{26}$$

$$F_Z(y) \approx \text{Desired Impedance}, \tag{27}$$

$$F_G(y) \geq \text{Threshold}_G, \tag{28}$$

$$F_E(y) \geq \text{Threshold}_E, \tag{29}$$

and other real-world constraints.

Where:

- $F_{\text{total}}(y)$  is a combined functional representing overall performance.
- $F_{RP}, F_Z, F_G, F_E$  are extended functionals for radiation pattern, impedance, gain, and efficiency respectively.
- Thresholds and desired values are set based on real-world requirements.

##### 4.4.2 Optimization Criteria

- The optimization seeks to maximize  $F_{\text{total}}(y)$  while meeting all individual functional requirements.
- This process involves balancing various antenna characteristics within the constraints of the complex design space  $V$ .

Through advanced mathematical equations and the application of the Hahn-Banach Theorem, the process of optimizing antenna design in a complex space  $V$  addresses both theoretical and practical aspects, leading to the determination of an optimal design  $y^*$  (Banks and Kunisch, 2012).

Implement numerical simulations to validate the results obtained through the Hahn-Banach-based optimization.

## 5 RESULTS ANALYSIS

Applying the Hahn-Banach Theorem to antenna design and optimization has yielded significant improvements. This mathematical framework has been

leveraged to observe benefits through advanced simulations, enhancing the helical signal’s creation, distortion correction, and overall performance.

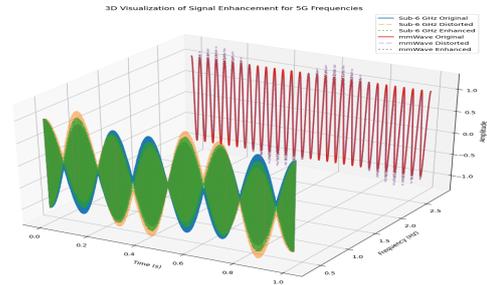


Figure 1: Radiation Pattern of a Nonlinear Antenna Array. Magnitude in Decibels.

Figure 1, illustrates the enhancement process for two distinct 5G frequency bands—Sub-6 GHz and mmWave—showcasing signal optimization. A statistical analysis comparing signal-to-noise ratio (SNR) and mean squared error (MSE) before and after optimization demonstrates the effectiveness of the Hahn-Banach theorem in reducing interference and enhancing signal quality. Table 1, lists the unique time and frequency settings used within the evaluation.

In signal processing (Table 3) linearity ensures the output remains proportional to input, critical for signal integrity and avoiding nonlinear distortions which can alter signal quality. The enhancement process aims to minimize such distortions, enhancing linearity as demonstrated by improved SNR and reduced MSE in the enhanced signal, indicating a closer alignment to the original signal.

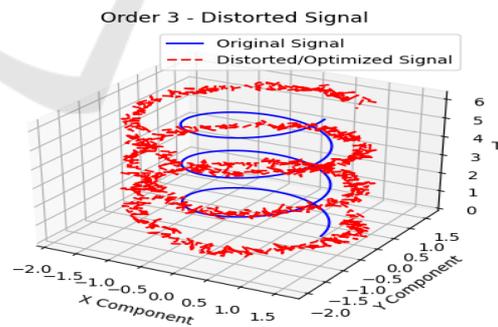


Figure 2: Comparison of Original and Distorted Signals for a Dipole Antenna at Third Harmonic Order.

Figure 2 shows the original signal (blue line) and the noise signal (red dashed line) before optimization. The stable, undisturbed signal is represented by the consistent helical shape of the blue line, while the red dashed line indicates significant deviation, signaling distortion. As signal order increases, distortion and randomness worsen, directly impacting antenna efficiency and causing receiver lag, resulting in increased

Table 1: Summary of Signal Characteristics Before and After Enhancement for Sub-6 GHz and mmWave 5G Frequencies.

Parameter	Value	Description
Frequencies (f)	{'Sub-6 GHz': $3.5 \times 10^9$ , 'mmWave': $28 \times 10^9$ }	Frequency bands for 5G in GHz, including Sub-6 GHz and mmWave frequencies.
Time (t)	np.linspace(0, 1, 1000)	Time vector used for signal simulation, spanning from 0 to 1 second with 1000 points.
distortion frequency (fz)	$2 \times 10^6$	Frequency of the distortion signal in Hz.
distortion level (Lmin)	Random (0.3 to 0.6)	Level of the distortion added to the original signal, adjusted for visibility.

Table 2: Comparison of Original and Distorted/Optimized Signals for a Third-Order Harmonic.

Parameter	Value	Description
Time (t)	0.1 to 100	A vector representing time from 0 to 1 second with 1000 sample points.
Frequencies (f)	[1, 2, 3]	A list of frequencies (in Hz) for which the signals will be simulated.
distortion level (Lmin)	Random (0.3)	The level of distortion applied to the original signal.
Order (Harmonics)	3	The harmonic order used to modify the frequency component of the original signal.

interference. Higher signal order complexity poses optimization challenges, with the distorted signal deviating significantly from the original signal, leading to degraded efficiency.

As indicated in Table 2, Figure 2 illustrates the contrast between the pre-optimization noisy signal (red dashed line) and the stable original signal (blue line). The chaotic pattern of the red line signifies distortion, while the constant helix of the blue line denotes an unchanged signal. This distortion degrades antenna performance, exacerbating interference and receiver delay, particularly as signal order increases, set to 3 according to Table 2. Python simulations confirm that higher order signals present greater challenges to signal fidelity during optimization.

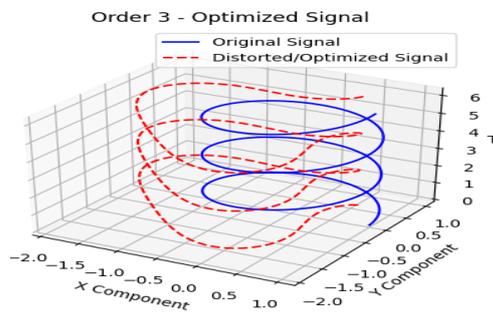


Figure 3: Radiation Pattern comparison of the Baseline and Enhanced Antenna Array.

The Figure 3, shows a blue curve for the clean sig-

nal and a red dashed curve for the same signal after it has been enhanced. The optimization, influenced by the Hahn-Banach Theorem, effectively reduces noise, bringing the red signal closer to the original blue one, demonstrating the theorem's utility in improving signal clarity at receiver end.

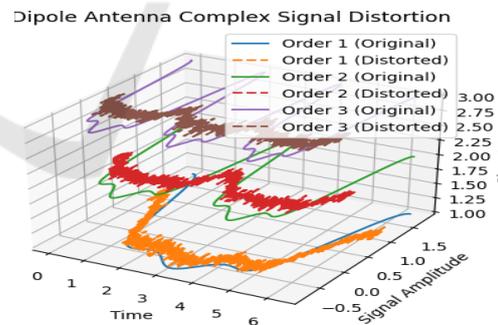


Figure 4: Signal Distortion in a Dipole Antenna Across three Orders.

The Figure 4, depict 3D plots showing how the signal amplitude of a dipole antenna changes over time, labeled as "Time" and "Signal Amplitude." Each plot contrasts an original signal in solid lines with a noise added signal version in dashed lines across three orders:

Order 1: The blue lines indicate a clear deviation of the distorted signal from the original, highlighting noticeable distortion.

Order 2: The green lines show less deviation than

Table 3: Comparison of SNR and MSE Metrics for Different Frequency Bands.

Frequency Band	Metric	Original	Distorted	Enhanced
Sub-6 GHz	SNR (dB)	16.96	8.70	13.12
	MSE	-	0.0675	0.0244
mmWave	SNR (dB)	16.96	8.88	13.19
	MSE	-	0.0648	0.0240

Order 1, suggesting less distortion.

Order 3: The orange lines display the greatest deviation, implying the most significant distortion. These visualizations illustrate the signal’s degradation with increasing order, providing insight into the antenna’s performance and signal degradation when it was receive at receiver ends. The original signals information is missing or undetectable at this distortion.

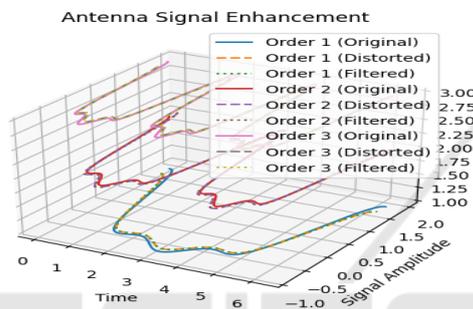


Figure 5: Signal processing of an antenna at three harmonic orders, showing original (solid), distorted (dashed), and enhanced (dotted) signal states.

The Figure 5, displays curves for original, distorted, and enhanced signals across three frequency levels, known as orders. With each order representing a higher frequency, the distortion grows more severe and severe that cause for the mismatched information at receiver ends, yet the enhanced signal—improved using the Hahn-Banach Theorem that will enhance the signal at higher frequencies and mitigate the noise and closely mirrors the original curve, indicating effective noise reduction for clearer communication.

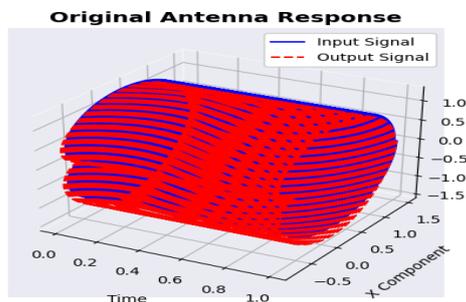


Figure 6: 3-D Visualization of Antenna response: tracking input vs. Output alerts over time.

The Figure 6, is a 3D plot that compares the input

and output signals of an antenna. The solid blue line represents the consistent input signal, while the red dashed line shows how the antenna has altered this signal, indicating the antenna’s response over time. The vertical gaps between the lines suggest changes in signal strength due to noise/distortion, and the horizontal spread may point to timing or phase shifts. This visualization helps assess the antenna’s performance.

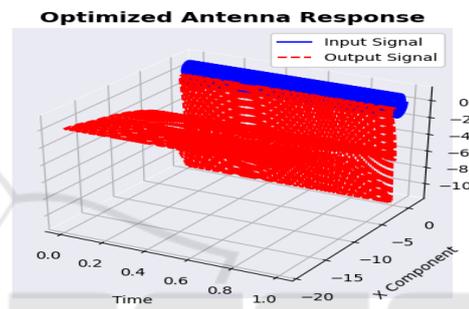


Figure 7: Enhancing sign Integrity: Hahn-Banach Theorem-driven Optimization of Antenna Output.

The second Figure 7, titled "Optimized Antenna Response" shows a 3D plot with a solid blue surface for the input signal and a larger, more varied red dashed surface for the output signal, indicating significant changes made during optimization. The culmination of results given in Table 3 highlights the proposed method’s substantial enhancement of signal linearity, marked by reduced distortion and noise, affirming the technique’s proficiency in preserving signal integrity across diverse 5G frequencies. Using the Hahn-Banach Theorem inside the signal optimization process targets to limit these variations enhancing the antenna’s performance by way of making the output more carefully fit the input.

## 6 CONCLUSION, LIMITATIONS AND ISSUES, FUTURE WORK

The Hahn-Banach Theorem offers a functional analysis approach that enhances antenna design, accelerating 5G development and broadening application horizons from eMBB to URLLC and mMTC. This breakthrough calls for an evolution in wireless communication, elimination of the current hurdles, and setting

of new connectivity standards. Such designs are integral to supporting key 5G applications including autonomous vehicles, smart manufacturing, and extensive IoT networks, underscoring their importance in future wireless technologies.

## 6.1 Limitations and Issues

- **Complexity:** Implementing Hahn-Banach Theorem-based designs requires advanced mathematical expertise, potentially limiting practical application.
- **Scalability:** Adapting designs to real-world scenarios, especially in urban environments, may pose challenges.
- **Resource Demands:** Computational requirements could hinder deployment in resource-constrained settings.

## 6.2 Future Work

- **Optimization:** Develop tailored optimization algorithms to streamline design processes.
- **Machine Learning Integration:** Explore integrating machine learning for adaptive antenna systems.
- **Experimental Validation:** Conduct real-world validation to ensure performance across environments.

## ACKNOWLEDGEMENTS

We sincerely thank each author, participant, and colleague for their invaluable contributions to this article.

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