Non Linear Homogenization of Laminate Magnetic Material by Computing Equivalent Magnetic Reluctivity

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Keywords: Laminate Material, Magnetic Reluctivity, Homogenization Technique, Finite Element Method, Non Linear Material, Inverse Problem, Optimization.

Abstract: In the present study we present a numerical modeling of a laminate magnetic material using an homogenization technique which exploit an inverse problem resolution. The laminate magnetic material is consisting of an alternate of magnetic and insulates layers. The equivalent magnetic reluctivity is computed for the homogenized domain by considering a non linear behavior of the magnetic layers. A finite element method is used to solve the 2D non-linear electromagnetic partial differential equation. An optimization problem is constructed and solved with the association of the 2D finite element resolution and a conjugate bi-gradient algorithm. The computation of the equivalent magnetic reluctivity is then performed for different excitation field value according to B-H curve. The comparison of the obtained B-H curves of the laminate and the homogenized domains to the theoretical B-H curve (experimental data) show a good agreement of laminate results.

1 INTRODUCTION

Generally used material in electrical, mechanical, structures are: metallic, polymers, ceramic and composites. The physical characteristics of a composite material primarily of the laminates are the result of a combination of the properties of matrix, reinforcement and additives. The materials nature are strongly heterogeneous and anisotropic (Trichet , 2000). The quality of simulation results is directly related to the precise knowledge of the physical properties of composite material.

The goal of homogenization of heterogeneous material is to reduce the complexity involved by the wall geometry of the solving domain and the non linearity of the physical properties with anisotropy. It's then more suitable and convenient, as proposed by several researchers the use of methods of The algorithm of optimization based on the method homogenisation (Meunier, 2010; Bensaid, 2006; Charmoille, 2008; Waki, 2005). To determine the equivalency of electromagnetic characteristics of an homogeneous material replacing the heterogeneous one (Charmoille, 2008; Waki, 2005; Ren, X, 2016; Achkar, 2021).

The current study is focused on the computation of the anhysteretic curve and the equivalent magnetic reluctivity of homogeneous material equivalent to laminate one. For this goal , the method of inverse problem which is coupled with finite elements is exploited (Szeliga, 2004; Martin, 2015; Gavazzoni, 2022). The algorithm of optimization is based on the method of the gradient which uses a function cost defined as a difference between the stratified and homogeneous magnetic fields. The homogenization of electromagnetic characteristics of heterogeneous material are then computed and compared to theoretical results (Feliachi, 1991).

2 ELECTROMAGNETIC EQUATION

The modeling of the electromagnetic problems is based on the Maxwell's equations. The

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Yousfi, G. and Mohellebi, H.
Non Linear Homogenization of Laminate Magnetic Material by Computing Equivalent Magnetic Reluctivity.
DOI: 10.5220/0012795200003758

In *Proceedings of the 14th International Conference on Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH 2024)*, pages 348-353
ISBN: 978-989-758-708-5; ISSN: 2184-2841

electromagnetic equation, by considering magnetic vector potential unknown $\vec{A}$, is then deduced from Maxwell equations as following:

$$\vec{\nabla} \wedge (\upsilon(B)\vec{\nabla} \wedge \vec{A}) + j\omega\sigma\vec{A} = \mu \cdot \vec{J}_s \qquad (1)$$

$\nu(B)$ is the magnetic reluctivity which depends on magnetic flux density $[H/m]^{-1}$, σ is the electric conductivity [S/m], ω is the angular frequency [rd/s] and $\vec{J}_s$ is the current density [A/m^2].

In case of (x,y) Cartesian coordinates, equation (1) could be written according to each solving sub-domain "i" as fellow:

$$\frac{\partial}{\partial x}\left(\nu_i(B)\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_i(B)\frac{\partial A}{\partial y}\right)-j\omega\sigma_i A=-J_{si} \qquad (2)$$

In case of ferromagnetic layers $J_{si} = 0$ and for insulator layers $J_{si} = 0$ and $\sigma_i = 0$.

3 FINITE ELEMENTS FORMULATION

The finite element formulation of the electromagnetic problem defined by non linear equation (2) consists on the substitution of the partial differential equation by an integral formulation of the problem. According to that, we choice the projective formulation based on Galerkin method which has the advantages, with some considerations, of providing a system whose mass matrix become symmetrical. Thus, finite element formulation of Equ. (2) is:

$$\iint_\Omega\left(\upsilon\left(\frac{\partial\alpha_i}{\partial x}\frac{\partial A_z}{\partial x}+\frac{\partial\alpha_i}{\partial y}\frac{\partial A_z}{\partial y}\right)+j\sigma\omega\alpha_i A_z-\alpha_i J_{sz}\right)dxdy=0 \qquad (3)$$

The final matrix system to be solve is given by:

$$[[K(\nu)]+j\omega[M]][\overline{A}]=[S] \qquad (4)$$

$$\overline{A} = A_r + jA_{im} \qquad (5)$$

$\overline{A}$ is a complex unknown which represents the magnetic vector potential of real (A_r) and imaginary (A_{im}) components respectively.

$$K_{ij} = \iint_\Omega \upsilon(B)\left(\frac{\partial\alpha_i}{\partial x}\frac{\partial\alpha_j}{\partial x}+\frac{\partial\alpha_i}{\partial y}\frac{\partial\alpha_j}{\partial y}\right)dxdy. \qquad (6)$$

$$M_{ij} = \iint_\Omega \sigma\,\alpha_i\,\alpha_j\,dxdy \qquad (7)$$

$$S_i = \iint_\Omega \alpha_i\,J_{sz}\,dxdy \qquad (8)$$

K_{ij} are coefficients of non linear mass matrix.

M_{ij} are coefficients of harmonic matrix.

S_i are coefficients of source vector.

α_i and α_j are projection and shape function at nodes "i" and j" respectively.

4 MAGNETIC LAYERS MODELING

The magnetic layers used in the electromagnetic arrangement are a ferromagnetic type by considering nonlinear hypothesis. A non linear model of the magnetic reluctivity which governs the magnetic behavior of ferromagnetic layers, according to magnetic flux density variation, is given by expression below(Feliachi,1984; 1991)

$$\upsilon(B)=\nu_0\left(\nu_i+\left(\nu_f-\nu_i\right)\frac{B^{2\eta}}{B^{2\eta}+\tau}\right) \qquad (9)$$

B is the magnetic flux density modulus, ν_0 is the magnetic reluctivity of vacuum, ν_i and ν_f are initial and final magnetic reluctivity respectively. When η and τ are parameters of magnetic reluctivity model which could be deduced from experimental curve (data). The values of the different parameters of magnetic reluctivity model are summarized in table 1.

The theoretical curve representing anhysteretic curve obtained when using non linear magnetic reluctivity model with the associated parameters presented in Table 1 is plotted and shown in Fig.1.

Table 1: Parameters of magnetic reluctivity model.

Parameters	ν_i	ν_f	η	τ (USI)
Values	0.005	1	5.419	1339

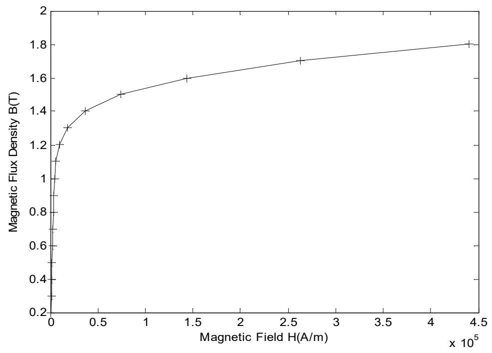

Figure 1: Anhysteretic curve B-H.

5 OPTIMIZATION PROBLEM

The method of inverse problem is applied in the current study of the homogenization problem (fig. 3). The goal consists to search for the behavior of electromagnetic characteristics of the homogenized solving domain with the determination of the optimal value of the magnetic reluctivity by taking account of non linearity. This is performed with the use of the proposed objective function J above:

$$J = \frac{1}{2}\left(H_{stra} - H_{\text{hom}}\right)^2 \tag{10}$$

H_{stra} is a magnetic field of laminated material, H_{hom} is a magnetic field of homogenized material. The flowchart of the optimization procedure in shown in Fig.2.

The treatment of the inverse problem is carried out while being based on the method of the gradient with the algorithm described by (Szeliga, 2004):

$$u^{k+1} = u^k - \alpha^k \vec{\nabla} J\left(u^k\right) \tag{11}$$

α^k : optimal step; u^k: search point at k iteration; $\vec{\nabla} J\left(u^k\right)$: gradient of objective function.

The computation of the gradient of the objective function is obtained using the proposed formula:

$$\vec{\nabla} J\left(u^k\right) = \frac{\partial J}{\partial u^k} \cdot \frac{\partial u^k}{\partial \nu} \tag{12}$$

$$\frac{\partial u^k}{\partial \nu} = \frac{u^{k+1} - u^k}{\nu^{k+1} - \nu^k} \tag{13}$$

The optimal step was calculated with the method of Quasi-Newton, whose algorithm is written as follows:

$$x^{k+1} = x^k - \alpha^k s^k \vec{\nabla} f\left(x^k\right) \tag{14}$$

S^k: symmetric matrix; α^k: optimal step given by linear minimization

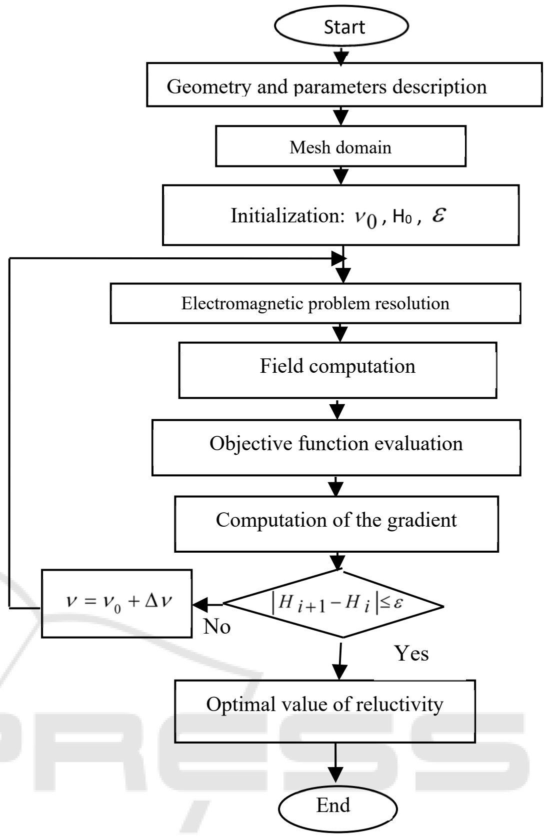

Figure 2: Flowchart of the optimization problem.

6 APPLICATIONS AND RESULTS

The electromagnetic device considered in the current work is consisting of an alternates arrangement of ferromagnetic and insulator layers. The complex algebraic system (4) is solved using finite element code developed under Matlab PDETOOL package.

6.1 Study of the Laminate Material

6.1.1 Geometric and Physical Characteristics

The laminated armature is made of two kinds of materials: ferromagnetic and insulator. Ten (10) layers are ferromagnetic ones with 0.25 mm width and nine (09) layers are insulators with 0.1 mm width, the height of the stratified material is about 8 mm (Fig. 3).

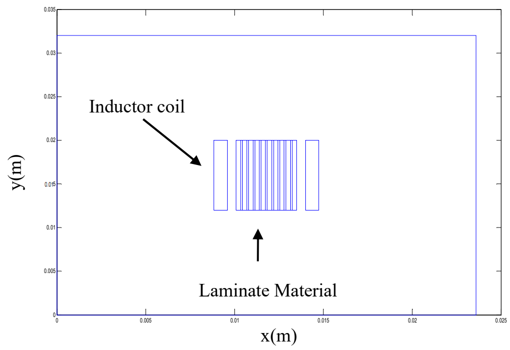

Figure 3: Laminate solving domain.

The physical properties of the studied device with stratified material are given in the following Table 2:

Table 2: Physical parameters.

	Electric Conductivity	Magnetic Reluctivity
Magnetic material	$4\ 10^6\ [S.m]$	$\nu(B)\ [m/H]$
Insulator	0	$\nu_0 = 7.95\,10^5$

6.1.2 Mesh Domain

The mesh of the resolution domain with stratified material is shown in Fig. 4.

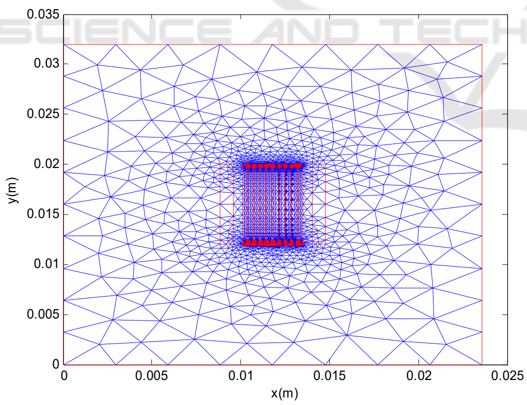

Figure 4: Meshed solving domain.

The mesh domain consists of 10694 nodes and 21314 triangles.

6.1.3 Results and Discussion

After resolution of the non linear problem, the results obtained are shown in figures 5, 6, 7 and 8. In Figure 5 and 6 are given the behaviors of the magnetic vector potential distribution and the magnetic anhysteretic curve compared to experimental one. It shows a good agreement between results. In Fig. 7 is given the magnetic reluctivity behavior with magnetic flux density of laminate material.

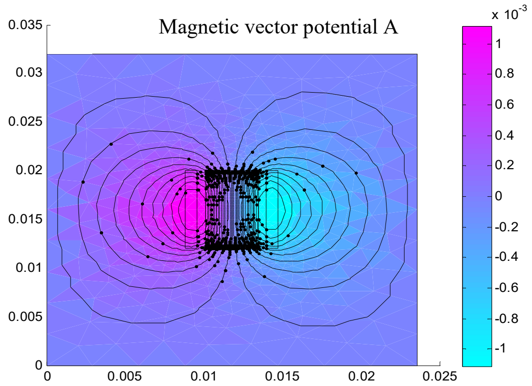

Figure 5: Magnetic vector potential distribution (Laminate domain).

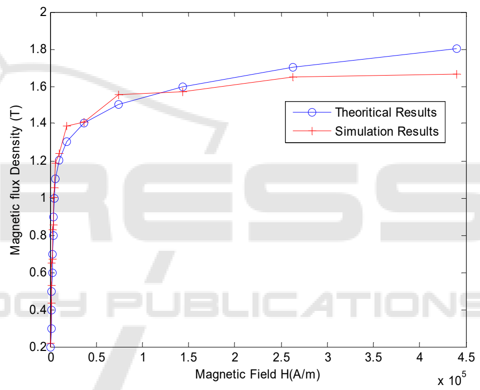

Figure 6: Anhysteretic magnetic curves.

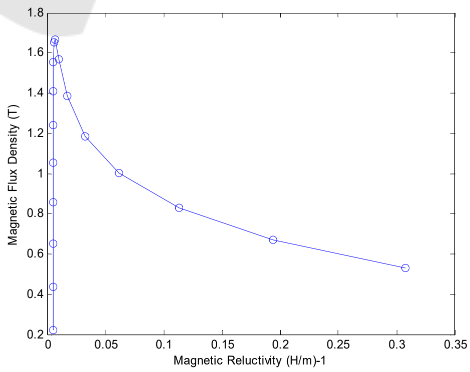

Figure 7: Magnetic reluctivity variation of laminate.

6.2 Study of the Homogenized Structure

The homogenized structure is shown in Fig. 8 where the load region is consisting of a single domain replacing the laminate one as shown in the previous Fig.3.

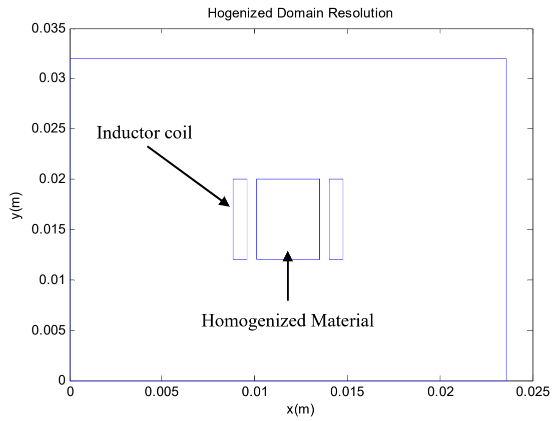

Figure 8: Homogenized solving domain.

6.2.1 Mesh Domain

The mesh of homogenized domain is presented in Fig.9.

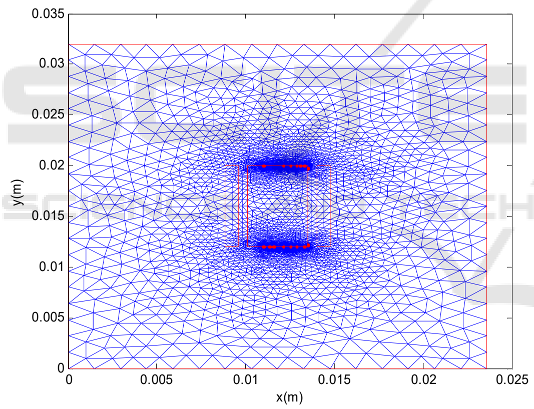

Figure 9: Mesh of homogenized domain.

6.2.2 Results and Discussion

The complex algebraic system is solved using finite element code developed under Matlab PDETOOL package and coupled to the optimization conjugate gradient algorithm as illustrated in flowchart (Fig.2).

After resolution of the optimization problem, the results obtained are shown in figures 10, 11 and 12.

The distribution of the magnetic vector potential shown in Fig. 10 is reproduced correctly in the homogenized material and seems to be identical to the distribution obtained in the case of laminate material.

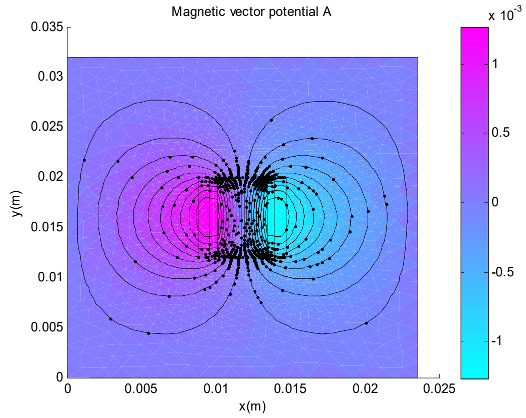

Figure 10: Magnetic vector potential distribution (Homogenized domain).

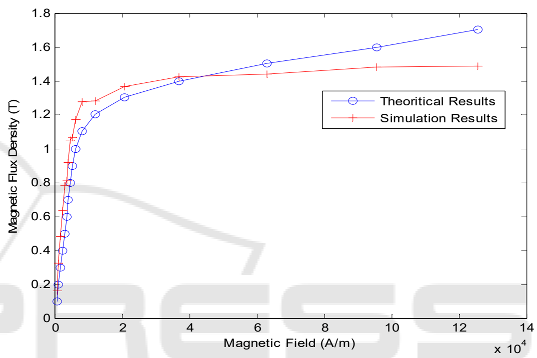

Figure 11: Anhysteretic magnetic curves.

In Fig. 11, the anhysteretic curve obtained in the homogenized material is compared to theoretical anhysteretic curve (experimental data) where the results seem to be comparable with some difference for high excitation field.

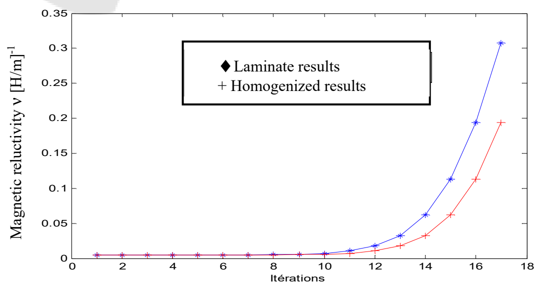

Figure 12: Magnetic reluctivity behavior curves.

The results are acceptable and permit to conclude to the validity of the optimization process used. Thee magnetic reluctivity behavior with iterations shown for both laminate and homogeneous material in Fig. 12 have a similar variation with a difference which appears when the excitation magnetic field becomes higher. The difference shown could be investigated by performing an experimental setup.

7 CONCLUSION

This works presents the characterization of laminated material in terms of physical properties. The finite elements method is used and associated to inverse algorithm problem based on gradient search method of optimum value. The magnetic reluctivity is calculated considering the objective function depending on the square difference between magnetic fields of laminated and homogenized materials. Results seems interesting to be able to apply the model to the identification of the electric conductivity of laminated material used in electrical machinery which are subjected to eddy current and saturation effects.

The difference recorded on the magnetic anhysteretic curves and the magnetic reluctivity behavior could be investigated by performing an experimental setup. This brings us to conclude that the used algorithm supplies reproducible results were from his robustness.

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