Empirical Analysis of Convertible Bond Pricing and Arbitrage Based on Black-Scholes Model

Simo Li

Sino-French Institute, Renmin University of China, Beijing, 100872, China

Abstract: A convertible bond is a special financial instrument. In China, the size of the convertible bond market continues to grow. However, due to the dual characteristics of stocks and bonds, there are complications in pricing. There are currently a variety of convertible bond pricing methods, including the Black-Scholes model, binary tree model, etc. This article uses the Black-Scholes model and takes the convertible bond Chongda zhuan 2 as an example to research the pricing of convertible bonds, calculate the bond value and option value of this convertible bond, and analyzes the investment opportunities of convertible bonds by contrasting the variation between the theoretical value and the real price. The research on convertible bond pricing in academic circles is biased towards theory. The efficiency of the convertible bond market can be improved and investment possibilities in the convertible bond market can be explored with the aid of data-based research on pricing.

1 INTRODUCTION

A convertible bond is a special financial instrument that has the dual characteristics of stocks and bonds. Convertible bondholders are entitled to convert their bonds, at a price set at the time of issuance, into shares. Holders of convertible bonds can also choose to hold the bond and receive interest and principal, or sell the bond in the market (Wenshi 2022).

According to the terms of the agreement, under certain circumstances, When the price of the underlying stock falls below a specific threshold, the holder of the convertible bond has the option to call the bond back to the issuer, and the issuer has the option to redeem the bond when the price of the underlying stock rises above a specific threshold (Zhao 2022).

Therefore, convertible bonds can be viewed as a hybrid of stocks and bonds. The bond value and the conversion value make up the two components of a convertible bond's value. Alternatively, a convertible bond can be viewed as a combination of a regular bond and a corresponding call option on a stock (Junbo 2021). Convertible bond pricing theory, which is based on the Black-Scholes option pricing model, advanced quickly after the model's introduction in the 1970s.

The launch of convertible bonds is of great significance to listed companies. Convertible bonds create a low-cost, long-term, and stable financing channel. At the same time, listed companies can improve their financial structure through convertible bond financing (Zaiqiao 2023).

China's convertible bond market emerged relatively late. In China, the first convertible bond appeared in the early 1990s. As the Chinese capital market has grown, the convertible bond market has attracted more and more attention from institutions and individuals, and the scale of financing has grown rapidly. As of the end of 2021, the number of convertible bonds issued in the Chinese market was close to 700 (Leying 2022).

However, compared to the stock market, the convertible bond market in China developed later and has not gotten as much attention. The convertible bond market is inefficient, and it is easy for convertible bond prices to deviate from the intrinsic reasonable value. Therefore, research on convertible bond pricing will help adjust prices in the convertible bond market and enhance its effectiveness (Yu 2021).

This article will introduce in detail how to use the B-S model to value convertible bonds and verify the feasibility of the investment method based on the B-S model based on real historical data.

2 CALCULATING THE VALUE OF CONVERTIBLE BONDS

Convertible bond pricing can be divided into two parts: bond value and option value.

For the calculation of bond value:

Assume that the value of the convertible bond is B, the bond interest is I, n is the term of the convertible bond, F is the maturity redemption price of the convertible bond, and the discount rate d is the interest rate of ordinary bonds in the same industry and with the same credit rating. The following is the calculation of the value of the convertible bond bond:

$$B = \sum_{t=1}^{T} \frac{I}{(1+d)^t} + \frac{F}{(1+d)^t} \tag{1}$$

For the calculation of option value:

Assume model parameters: S is the underlying stock price, X is the option exercise price, σ is the stock price volatility, and T is the relative remaining period of the option (T=remaining period/365). C is the option value of the convertible bond. According to the B-S model, the following is the calculation of the convertible bond option value:

$$C = S * N(d_1) - X * e^{-rT} * N(d_2) \tag{2}$$

$$d_1 = \left[\ln(S/X) + (r + \sigma^2/2)T\right]/(\sigma\sqrt{T}) \tag{3}$$

$$d_2 = d_1 - \sigma * \sqrt{T} \tag{4}$$

(N(d) is the cumulative possibility distribution function of normally distributed variables)

For convertible bonds, X is the conversion price. Each unit of convertible bonds corresponds to A units of options. A is called the conversion ratio, A=100/conversion price.

Convertible bond value = bond value + option value ∗ conversion ratio

3 ANALYSIS OF CONVERTIBLE BOND ARBITRAGE STRATEGY

3.1 Convertible Bond Arbitrage Strategies

Long strategy: When convertible bonds are undervalued, buy convertible bonds and choose to convert them to make profits if there is a profit during the convertible period (Jinhua & Yujuan 2023).

Overnight spread arbitrage: When the underlying stock of a convertible bond is allowed to short-sell through securities lending if the conversion price of the convertible bond is underestimated relative to the stock price, the underlying stock can be obtained through securities lending and sold, and a corresponding amount of convertible shares can be purchased. Then the bonds are converted into shares and the stocks borrowed are returned the next day (Zefeng 2023).

Other strategies: Arbitrage based on different delta volatilities of financial products related to convertible bonds.

3.2 Arbitrage Strategy Based on the Black-Scholes Model

The Black-Scholes model-based arbitrage strategy is long. Determine the convertible bond's theoretical value using the Black-Scholes model. It is a sign that a convertible bond is undervalued when its theoretical price is higher than its actual market price. It is believed that the convertible bond has a large room for growth and can be purchased to build a position. Conversely, convertible bonds can be sold to reduce positions if the theoretical price is less than the market price, which indicates that the bonds are overvalued by the market.

4 EMPIRICAL RESEARCH ON ARBITRAGE STRATEGY-TAKING CONVERTIBLE BOND CHONGDA ZHUAN 2 AS AN EXAMPLE

To verify the effectiveness of the B-S model pricing, starting from 2021/1/04, the data on the 4th of each

month for 6 consecutive months will be calculated (postponed if the market is closed).

4.1 Calculation of Bond Value

In the bond value calculation part, Chongda Technology, the issuer of Chongda zhuan 2, is an electronic product manufacturing company. Select the average interest rate of general corporate bonds found by companies with the same credit rating in the same industry as the discount rate (4.3%).

According to the Chongda zhuan 2 issuance information, the interest rates of the convertible bond within 6 years are 0.30%, 0.60%, 1.00%, 1.50%, 1.80%, and 2.00% respectively. The redemption price at maturity is 110(This means B=110 not 100).

As shown in Table 1, the bond value of the convertible bond is calculated.

Table 1: Bond value of Chongda Zhuan 2.

Chongda Zhuan 2	Remaining period	Bond value
2021/1/4	5.68	91.13
2021/2/4	5.59	91.46
2021/3/4	5.52	91.75
2021/4/6	5.42	92.09
2021/5/6	5.34	92.42
2021/6/4	5.26	92.72

4.2 Calculation of Option Value

In the option value calculation part, the 3-year Chinese government bond interest rate (3%) for the same period is chosen to be the risk-free interest rate. According to historical information of the wind database, the annualized volatility rate of Chongda Technology (002815) is 30.09%. The corresponding conversion price from 2021/1/4 to 2021/5/6 is 19.54, and the corresponding conversion price on June 4, 2021 is 19.29.

As shown in Table 2, the option value is calculated.

Table 2: Option value of Chongda Zhuan 2.

Chongda Zhuan 2	Remaining period	Stock price	Conversion price	Option value
2021/1/4	5.68	13.55	19.54	2.89
2021/2/4	5.59	11.09	19.54	1.68
2021/3/4	5.52	11.84	19.54	1.97
2021/4/6	5.42	11.53	19.54	1.8
2021/5/6	5.34	11.10	19.54	1.59
2021/6/4	5.26	11.40	19.38	1.71

4.4 Convertible Bond Value Calculation

As convertible bonds' conversion prices change, it is necessary to consider the conversion ratios corresponding to different conversion prices (conversion ratio = 100/conversion price).

As shown in Table 3, the theoretical value of convertible bonds is calculated.

Table 3. The theoretical value of the convertible bond

Chongda Zhuan 2	Remaining period	Stock price	Conversion price	Bond value	Option value	Conversion ratio	The theoretical value of the convertible bond	Actual price of convertible bond
2021/1/4	5.68	13.55	19.54	91.13	2.89	5.118	105.916	106.5
2021/2/4	5.59	11.09	19.54	91.46	1.68	5.118	100.054	94.4
2021/3/4	5.52	11.84	19.54	91.75	1.97	5.118	101.828	98
2021/4/6	5.42	11.53	19.54	92.09	1.8	5.118	101.306	99.9
2021/5/6	5.34	11.10	19.54	92.42	1.59	5.118	100.553	99.9
2021/6/4	5.26	11.40	19.38	92.72	1.71	5.160	101.548	102.3

4.3 Verification of Arbitrage Strategy

Since the actual price of Chongda Zhuan 2 on 2021/1/4 is higher than the theoretical value, for the convenience of calculation, it is assumed that on January 4, 2021, 100 units of Chongda Zhuan 2 are held, totaling 10,650 yuan. Due to the high actual price Based on the theoretical value, it is believed that the price is overvalued by the market and the bonds should be sold. On February 4, 2021, the

actual price was lower than the theoretical value, and the price was greatly underestimated. The bonds should be bought again, a total of 112.82 units. During the period from 2021/3/4 to 2021/5/6, because the theoretical value is higher than the actual price, continue to hold the bonds. On June 4, 2021, the actual price was higher than the theoretical value. So sold the bonds and received a total of 11,541.49 yuan. Then from 2021/1/4 to 2021/6/4, the total income from the above operations is 8.37% ((11541.49-10650)/10650=8.37%).

Observing the above data, the pricing of convertible bonds based on the B-S model is generally consistent with the actual price, and the actual difference is small. There is a large difference between the theoretical value and the actual price on 2021/2/4 (As marked in Figure 1), and there is an obvious arbitrage opportunity. According to the K-line chart, the actual price is close to a local low, and rebounded on the second trading day, heading towards theoretical value.

Figure 1: K-line of Chongda Zhuan 2 (Wind database 2024).

4.5 Limitations Analysis

In the empirical analysis based on historical data, no transaction costs were considered. Therefore, the actual rate of return drops when transaction costs are considered. Furthermore, a tiny discrepancy between the real price and the theoretical value does not always indicate an opportunity for arbitrage because the Black-Scholes model cannot adequately capture the inherent value of convertible bonds (Zhenghang 2020).

The market price of convertible bonds is influenced by numerous factors, including corporate development status, market supply and demand, alternative financial products, market sentiment, and other factors that are difficult to quantify (Wind database 2024). Therefore, pricing and arbitrage strategies based on the Black-Scholes model cannot perfectly describe the price and changing trends of convertible bonds. Therefore, specific issues need to be analyzed in detail. For convertible bonds that have a high probability of triggering a forced redemption clause, it is also necessary to consider the impact of the forced redemption clause on the price of convertible bonds.

5 CONCLUSION

Convertible bond values are separated into two categories: option values and pure bond values.

Thus, convertible bonds can be priced and arbitraged using the Black-Scholes model. The primary determining elements in the BS model-based convertible bond pricing model are the stock price, stock price volatility, conversion price, remaining term, risk-free interest rate, etc. This article verifies that this model has a reference value for investors to find convertible bond investment targets by taking the convertible bond Chongda zhuan 2 as an example. Although the theoretical value calculated by this model cannot perfectly correspond to the actual value, it is of reference significance for predicting the price change trend of convertible bonds.

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