

Balancing Act: Navigating the Privacy-Utility Spectrum in Principal Component Analysis

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Abstract: A lot of research in federated learning is ongoing ever since it was proposed. Federated learning allows collaborative learning among distributed clients without sharing their raw data to a central aggregator (if it is present) or to other clients in a peer to peer architecture. However, each client participating in the federation shares their model information learned from their data with other clients participating in the FL process, or with the central aggregator. This sharing of information, however, makes this approach vulnerable to various attacks, including data reconstruction attacks. Our research specifically focuses on Principal Component Analysis (PCA), as it is a widely used dimensionality technique. For performing PCA in a federated setting, distributed clients share local eigenvectors computed from their respective data with the aggregator, which then combines and returns global eigenvectors. Previous studies on attacks against PCA have demonstrated that revealing eigenvectors can lead to membership inference and, when coupled with knowledge of data distribution, result in data reconstruction attacks. Consequently, our objective in this work is to augment privacy in eigenvectors while sustaining their utility. To obtain protected eigenvectors, we use *k*-anonymity, and generative networks. Through our experimentation, we did a complete privacy, and utility analysis of original and protected eigenvectors. For utility analysis, we apply HIERARCHICAL CLUSTERING, RANDOM FOREST regressor, and RANDOM FOREST classifier on the protected, and original eigenvectors. We got interesting results, when we applied HIERARCHICAL CLUSTERING on the original, and protected datasets, and eigenvectors. The height at which the clusters are merged declined from 250 to 150 for original, and synthetic version of CALIFORNIA-HOUSING data, respectively. For the *k*-anonymous version of CALIFORNIA-HOUSING data, the height lies between 150, and 250. To evaluate the privacy risks of the federated PCA system, we act as an attacker, and conduct a data reconstruction attack.


1 INTRODUCTION

The demand for Artificial Intelligence (AI) tools that align with legal regulations such as GDPR (Voigt and Von dem Bussche, 2017) and individual privacy preferences has become crucial. Federated Learning (FL), introduced by McMahan *et al.* (McMahan *et al.*, 2017), addresses this need by enabling collaborative model learning among distributed clients without transmitting raw data. Despite its initial portrayal as a privacy-preserving solution, it is now acknowledged that FL is susceptible to various attacks on data, models, and communication links (Zhu *et al.*,

2019). Therefore, developing FL frameworks, which also preserves privacy is our main goal. FL was initially proposed for deep learning models. But, now it has been applied to many classical machine learning algorithms as well. Our study focuses on FL algorithms (Hartebrodt and Röttger, 2022) that perform PCA (Principal Component Analysis). To perform data analysis of high dimensional data, we need dimensionality reduction techniques, and PCA is one of the most popular dimensionality reduction techniques. Through our research, we want to show that FL-PCA algorithms, particularly those that share information like local eigenvectors computed from each distributed client's data, lack privacy, as sharing eigenvectors can reveal the members of training data, as shown in the paper (Zari *et al.*, 2022),

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which showed a Membership Inference attack (MIA) against PCA. When this knowledge is combined with the knowledge of data distribution, the attacker can estimate the original data of clients participating in the FL process, as shown in the paper (Kwatra and Torra, 2023), which showed a data reconstruction attack against PCA. Hence, our objective is to introduce protection in the eigenvectors and evaluate the utility and privacy of protected eigenvectors compared to the original eigenvectors. Our proposed privacy-preserving approach can also be applied to real-life scenarios. For example, consider there are hospitals located at different locations, which have high-dimensional data of RNA sequences, and their aim is to identify genes associated with certain conditions or diseases. Hence, hospitals can apply privacy protected FL-PCA algorithms to facilitate the collaboration while preserving privacy, and also saving the computation resources by removing the overhead of collecting all the data at one location. Finally, we list the contributions of this paper as follows.

- Introduction of privacy measures in the the computation of eigenvectors by computing them on private data. For creating private data, we use k -anonymity, and synthetic data generated by Conditional Tabular Generative Adversarial Network (CTGAN).
- Evaluation of the privacy of the system by acting as an intruder/attacker, who has some background knowledge, such as knowledge of some top eigenvectors, and the knowledge of data distribution, and then conduct a data reconstruction attack. We compare the case when the attacker is aware of some top private (k -anonymous, and synthetic) eigenvectors with the case when the attacker is aware of some top original eigenvectors.
- Evaluation of utility through RANDOM FOREST, and dendrogram analysis on both original and protected (k -anonymous, and synthetic) eigenvectors.

The subsequent sections of this paper are organized as follows: Section 2 reviews essential concepts, including PCA, FL-PCA algorithms, membership inference attack, and data reconstruction attack. Section 3 elaborates on our contributions, where we provide a comprehensive analysis of utility and privacy using a RANDOM FOREST, HIERARCHICAL CLUSTERING, and data reconstruction attack. This analysis is relevant and adaptable to centralized and Federated Learning (FL) scenarios. Section 4 outlines the datasets and attack settings. Section 5 presents and discusses the results, and Section 6 concludes the paper with insights into future directions.

2 BACKGROUND AND RELATED WORK

In this section, we explain all the relevant background theories needed to understand our proposed analysis.

2.1 Principal Component Analysis

Given a set $\mathcal{D} = \{x_n \in R^d : n = 1 : N\}$, where N is the number of samples, and x_n is a sample in R^d , PCA aims to determine a p dimensional subspace that approximates each sample x_n (Abdi and Williams, 2010) in a way that the maximum variance of the data is retained. The formulation of PCA is as follows:

$$\min_{\pi_p} E = \frac{1}{N} \sum_{n=1}^N E_n = \frac{1}{N} \sum_{n=1}^N \frac{1}{N} \|x_n - \pi_p x_n\|_2^2 \quad (1)$$

In the above expression, π_p represents the projection matrix onto the p -dimensional subspace. E is the reconstruction error or the mean squared error, representing the average squared distance between the original sample x_n and its projection $\pi_p x_n$. The solution to this optimization problem (1) can be obtained through the Singular Value Decomposition (SVD) of a sample covariance matrix Σ_{cov} of the standardized data matrix \mathcal{D} . The SVD of Σ_{cov} is given by $\sum_{i=1}^d \lambda_i v_i v_i^T$, where $\lambda_1 \geq \lambda_2 \dots \lambda_d$ are the eigenvalues, and $v_1, v_2 \dots v_d$ are the corresponding eigenvectors of Σ_{cov} , respectively. Let V_p denote the matrix whose columns are the top p eigenvectors. The optimal projection matrix is then defined as $\pi_p = V_p V_p^T$, and it provides the solution to the PCA optimization problem in (1). This projection matrix π_p allows for the representation of the data in a lower-dimensional subspace while minimizing the reconstruction error.

2.2 Federated PCA Algorithms

There are many existing algorithms to perform PCA in a federated setting. In FL-PCA algorithms, the clients compute the reduced subspace from its data, and sends it to the aggregator. The aggregator does its job by aggregating those reduced subspaces, and sends the aggregated subspace back to the clients. In this section, we discuss some of the FL-PCA algorithms in detail, with the aim of analysing their privacy breaches, and further improving their privacy protection while maintaining their utility. PCA is an unsupervised machine learning method. There is also a supervised version of FL-PCA in this recent work (Briguglio et al., 2023). Nevertheless, in this paper, our focus is on unsupervised FL-PCA algorithms. In (Hartebrodt and Röttger, 2022) Hartebrodt *et al.* present a plethora of state-of-the-art approaches for

FL-PCA. They analyze both iterative and single-shot approaches for horizontally partitioned data.

Regarding the first approach, Federated Subspace Iteration (FSI) (Pathak and Raj, 2011) is one of the state-of-the-art methods. FSI uses an exact approach for privacy-preserving computation of eigenvector matrices. Clients and aggregator exchange and compute local and global matrices iteratively. With a large number of iterations, the complete covariance matrices can be recovered, which is a privacy breach, as discussed in Section 2.3.

In terms of single-shot approaches, they require the computation of local subspaces at the client side, and the server aggregates the local subspaces received from the distributed clients, and sends the global subspace back to the clients, such as P-COV, AP-COV, AP-STACK (Liang et al., 2014). In AP-COV and AP-STACK, the clients perform SVD on its data and sends the local eigenvectors to the aggregator. In AP-COV, the aggregator aggregates the local eigenvectors by doing element wise addition, while in AP-STACK, the aggregator aggregates the local eigenvectors by stacking the local eigenvectors vertically. In the study by Hartebrodt *et al.*, it was empirically proven that both methodologies exhibit comparable performance. For this reason, for our investigation, we will concentrate on AP-COV. This choice is also influenced by the presence of the parameter k , which constrains the sharing of local eigenvectors. In a broader context, sharing eigenvectors poses risks of revealing membership information and data, as discussed in Section 2.3.

2.3 Privacy Attacks Against PCA

This Section briefly present two of the most popular state-of-the-art privacy attacks against PCA-based approaches.

Membership Inference Attack(MIA) was first published in 2017 (Shokri et al., 2017). It is a privacy attack against Machine Learning models, with the objective of determining the membership of a record to the original training dataset. In the case of MIA for PCA-based approaches, Zari *et al.*(Zari et al., 2022) defined a variant of MIA in which it is assumed that the adversary intercepts certain principal components (eigenvectors) from PCA-transformed data, which may contain sensitive information. The adversary employs these intercepted eigenvectors to calculate the reconstruction error for a given target sample, representing the disparity between the original and projected samples. The main insight is that samples from the training set will show a lower reconstruction error compared to those outside the training set. This highlights the importance of protecting

eigenvectors in privacy-preserving scenarios to mitigate such membership inference risks.

Data Reconstruction Attack Recently, Kwatra *et al.* (Kwatra and Torra, 2023) empirically proved that in the context of PCA-based approaches, also a data reconstruction attack is possible. The reconstruction attack tries to approximate as closely as possible the original data and to perform it requires the knowledge of leaked eigenvectors. This attack generates synthetic data, exploiting a Conditional Tabular Generative Adversarial Networks (CTGAN) (Xu et al., 2019) and obtaining: $\hat{X} = X_{\text{anonymized or synthesized}}VV^T$. At this point, the efficacy of the attack is assessed by computing the reconstruction accuracy, which quantifies the proximity between the estimated data and the original data. Kwatra *et al.* showed the reconstruction attack, and did not mention about the utility of protected eigenvectors. Hence, in this work we also do the utility analysis of the protected eigenvectors, and aim to provide a complete picture for the utility and privacy. In this work, we experiment with k -anonymous (Samarati, 2001; Samarati and Sweeney, 1998; Sweeney, 2002) eigenvectors, where we compute eigenvectors from k -anonymous data using Mondrian.

3 PRIVACY-PRESERVING COMPUTATION OF PCA

In this paper we consider the FL setting similar to the one outlined in AP-COV algorithm, where each client participating to the federation, sends to the aggregator the local eigenvectors and the server will aggregate the local contribution to compute an approximation of the hypothetical global covariance matrix. Given the privacy issues discussed above, we propose a preliminary study which analyzes the effect of applying a mitigation strategy on the client data before the eigenvector computation. Our methodology, illustrated in Figure 1, involves the use of Mondrian k -anonymity transformation (LeFevre et al., 2006) and a synthetic data generation by CTGAN at the client level.

3.1 Threat Model and Attack Methodology

In our attack setting, the client generates anonymous or synthetic data, one at a time. We denote the k -anonymous data as $\mathcal{D}_{\text{anonymized}}$, and the synthetic data as \mathcal{D}_{syn} . $\mathcal{D}_{\text{anonymized}}$ can be created by choosing the value of parameter k according to the privacy requirements of client, and the synthetic data can be created

using different percentages (10%, 30%, 50%, 70%, 100%) of samples from the original data \mathcal{D} , using CTGAN. The client then computes the eigenvectors E_p of the synthetic data or the anonymized data, and sends these to a trusted party, which is an aggregator in FL. We assume that the attacker \mathcal{A} intercepts some or all of the eigenvectors computed by eavesdropping on the communication channel. To do a successful data reconstruction attack, the attacker needs two things, eigenvectors, and the knowledge about the distribution of the data. For the distribution of the data, we assume that the attacker has access either to the synthetic data, which is created using some percentage of samples from the original data or to the k -anonymous data, which is a noisy version of the original data. Both, k -anonymous, and synthetic data have reduced re-identification risks, as compared to the original data, which means they are not personal data anymore. So, GDPR does not apply. Hence, the attacker can utilize them to conduct the attack.

In Table 1, we list the possible combinations of the assumptions for the attacker to conduct a data reconstruction attack. E.g., the attacker may have access to the original eigenvectors, and the synthetic data created using 10% of the samples from the original data, where those 10% samples can be selected either using random sampling or stratified sampling.

After conducting the attack, it is needed to measure the efficacy of the attack to quantify the privacy breach. We evaluate the success of our attack as follows.

Definition 1. Suppose R is the reconstructed data, which is the estimator for the original data O . Let δ be a parameter for reconstruction error, which quantifies the acceptable deviation. The reconstruction accuracy, $R.A.$ is defined as follows:

$$R.A. = \frac{\#\{\hat{R}_{i,j} : |O_{i,j} - R_{i,j}|, i = 1, \dots, n, j = 1, \dots, d \leq \delta\}}{n \times d} \quad (2)$$

where $\#$ means count, and n is the number of records. Hence, $R.A.$ expresses the percentage of reconstructed entries for which the relative errors are within δ .

4 DATA AND EXPERIMENTAL SETTINGS

We conducted experiments on CALIFORNIA-HOUSING, and COD-RNA datasets. The CALIFORNIA-HOUSING dataset has 20,640 records, and the COD-RNA dataset has 59,535 records. Both

Table 1: Cases for the privacy evaluation concerning data reconstruction attack. Here, O.D. stands for Original Distribution. S.S. stands for Stratified Sampling, and R.S. stands for Random Sampling.

Eigenvectors (EVs)	Data Distribution Information
Original EVs	Synthetic data from complete O.D. Synthetic data from 10% O.D. via R.S. Synthetic data from 10% O.D. via S.S.
Synthetic EVs	Synthetic data from the complete O.D. Synthetic from 10% O.D. via R.S. Synthetic from 10% of O.D. via S.S.
k -anonymous EVs	Used all the k -anonymous data Synthetic data from complete k -anonymous data Synthetic data from 10% k -anonymous data (via R.S. and S.S.)

datasets have 9 features. The CALIFORNIA-HOUSING dataset is for a regression task, where the goal is to estimate the housing prices, based on features such as income, housing occupancy, and geographical location attributes across various districts in California. These features contain sensitive information. Hence, privacy incorporation is important for data analysing. The COD-RNA dataset is for a classification task. For the utility analysis of CALIFORNIA-HOUSING dataset, we use R^2 , also known as Coefficient of Determination. R^2 determines the proportion of variability in the dependent variable that can be explained by the independent variable(s) included in the model. Mathematically, R^2 is expressed as follows.

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (3)$$

Both, CALIFORNIA-HOUSING and COD-RNA are numerical datasets. Therefore, as part of the pre-processing, we implement standardization using the `scikit-learn` library in Python. In our evaluation, we utilize a 10-fold cross validation, and report mean \pm standard deviation for R^2 in Table 2.

5 RESULTS AND DISCUSSION

We explain our main experimental findings as follows.

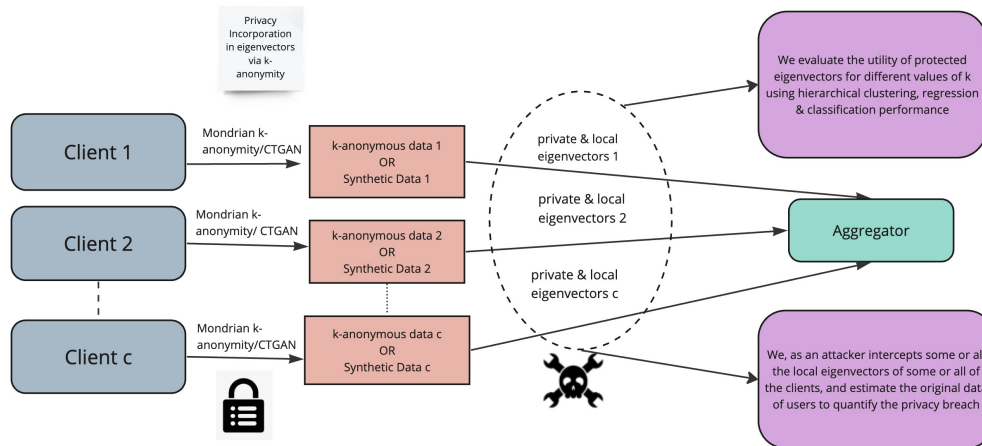
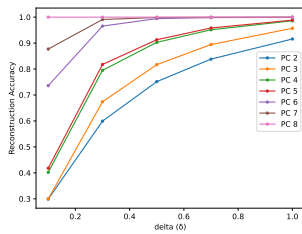
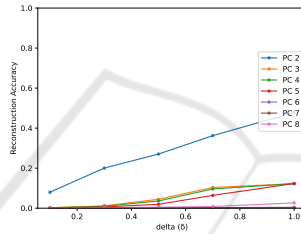


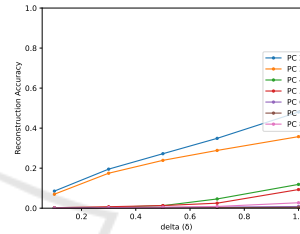
Figure 1: Our Methodology.



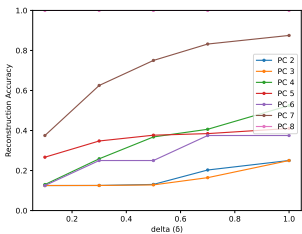
(a) R.A. in general PCA where we reach close to the original data on increasing no. of PCs.



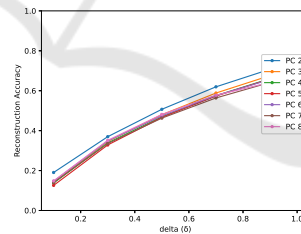
(b) R.A. b/w original and reconstructed data when 20-anonymous eigenvectors and 20-anonymous data is known.



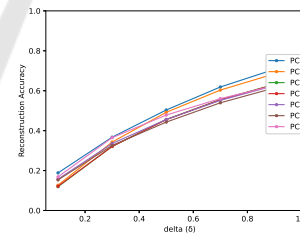
(c) R.A. b/w original and reconstructed data when 10-anonymous eigenvectors and 10% data is randomly drawn from 10-anonymous data is known.



(d) R.A. b/w anonymous and reconstructed data when eigenvectors computed from the original data, and synthetic data generated using 10% samples from the original data via random sampling.



(e) R.A. b/w original and reconstructed data when eigenvectors computed from the original data, and synthetic data generated using 10% samples from the original data via stratified sampling.



(f) R.A. b/w original and reconstructed data when eigenvectors computed from the original data, and synthetic data generated using 10% samples from the original data via random sampling.

Figure 2: Reconstruction Accuracy (R.A.) for California housing dataset with varied assumptions by the attacker.

- From the results in Table 2, we found out that as we increase the number of principal components, utility improves, and as the value of k increases upto 20, the utility is almost constant, which shows that the utility of data can be pre-

served while enhancing data privacy. This is because the machine learning models aim to avoid overfitting.

- We show reconstruction attack results in the Figure 2, which shows that the reconstructed

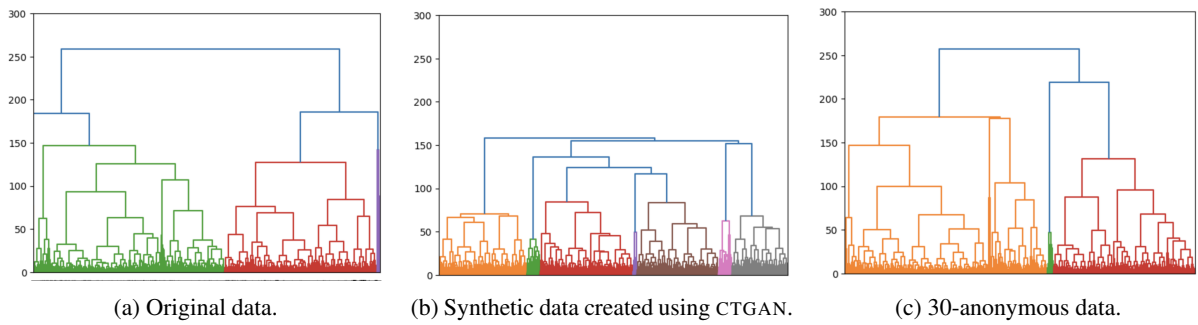


Figure 3: Dendrograms showing the HIERARCHICAL CLUSTERING for the CALIFORNIA-HOUSING data.

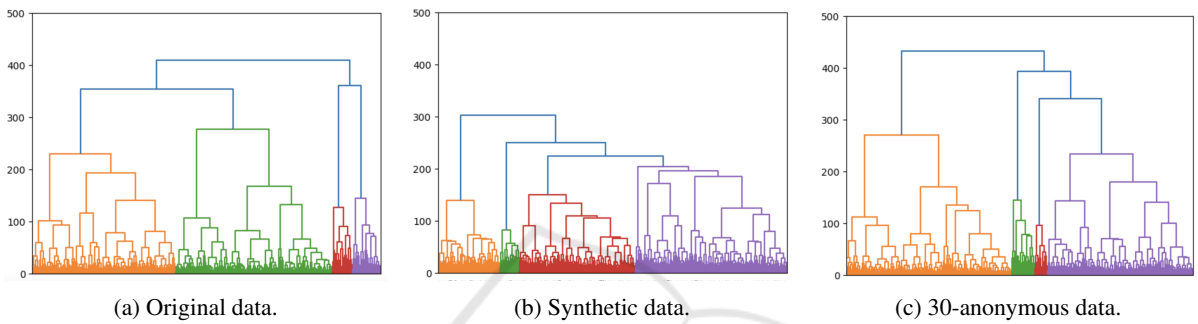


Figure 4: Dendrograms showing the HIERARCHICAL CLUSTERING for the cod-rna data.

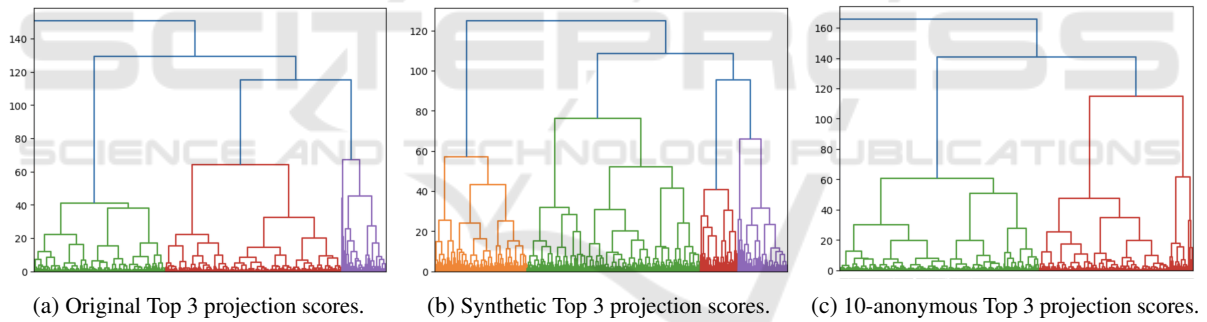


Figure 5: HIERARCHICAL CLUSTERING for California-housing's Top 3 projection scores.

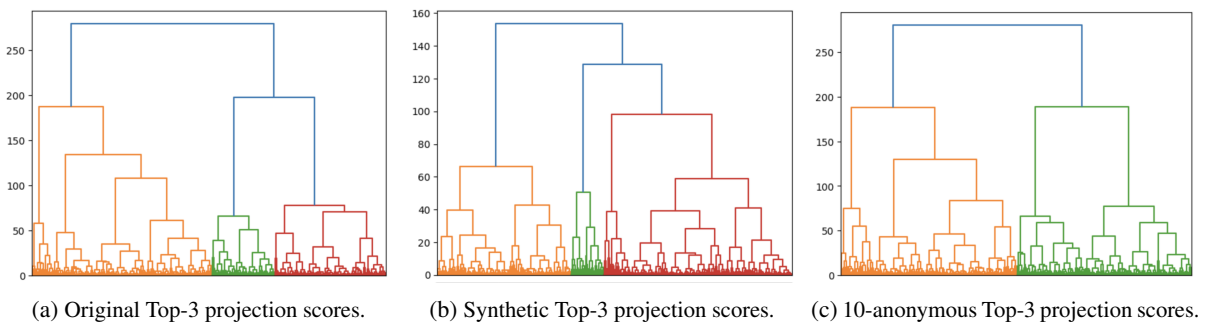


Figure 6: Dendrograms showing results of HIERARCHICAL CLUSTERING for cod-rna's Top 3 projection scores.

Table 2: Utility analysis via regression task on CALIFORNIA-HOUSING dataset. O-PCA refers to *Original* PCA, S-PCA is *Synthetic* PCA and A-PCA is *Anonymized* PCA with different values for k .

PCA	# PCs	R^2
<i>Baseline</i>	<i>all</i>	0.781 ± 0.019
O-PCA	3	0.148 ± 0.034
S-PCA	3	0.134 ± 0.030
A-PCA ($k=5$)	3	0.135 ± 0.030
A-PCA ($k=10$)	3	0.147 ± 0.035
A-PCA ($k=15$)	3	0.134 ± 0.030
A-PCA ($k=20$)	3	0.134 ± 0.030
O-PCA	4	0.455 ± 0.038
S-PCA	4	0.445 ± 0.034
A-PCA ($k=5$)	4	0.444 ± 0.034
A-PCA ($k=10$)	4	0.454 ± 0.038
A-PCA ($k=15$)	4	0.445 ± 0.034
A-PCA ($k=20$)	4	0.445 ± 0.035
O-PCA	5	0.631 ± 0.034
S-PCA	5	0.624 ± 0.029
A-PCA ($k=5$)	5	0.624 ± 0.029
A-PCA ($k=10$)	5	0.629 ± 0.035
A-PCA ($k=15$)	5	0.624 ± 0.029
A-PCA ($k=20$)	5	0.623 ± 0.029
O-PCA	6	0.697 ± 0.029
S-PCA	6	0.689 ± 0.003
A-PCA ($k=5$)	6	0.690 ± 0.032
A-PCA ($k=10$)	6	0.696 ± 0.030
A-PCA ($k=15$)	6	0.689 ± 0.032
A-PCA ($k=20$)	6	0.689 ± 0.032

dataset is farthest from the original dataset in the case when eigenvectors are computed on the k -anonymous data in comparison with the case when eigenvectors are computed on the synthetic dataset. Using the anonymous eigenvectors, we can reach closer to the anonymous data, but not to the original data, which means that anonymous eigenvectors provide protection concerning the reconstruction attack. Hence, we observe that the efficacy of attacker in inferring the data of users in CALIFORNIA-HOUSING dataset declines, if we incorporate privacy protection mechanism in our data before the data analysis.

- We show the dendrograms obtained after employing HIERARCHICAL CLUSTERING on the CALIFORNIA-HOUSING, and COD-RNA datasets in Figure 3. The dendrograms of the original data, and the anonymous data are quite similar for both the datasets. For the synthetic data, the dendrograms look quite different, in the sense that the Y-axis in figures, which shows the height at which the clusters are merged is declined in synthetic data, which shows that the clustering information

is somewhat lost in the synthetic datasets. In k -anonymous datasets, as the value of k increases, the clusters become compact.

- We show the dendrograms for the original and protected projection scores in Figures 5, and 6. We got the similar trends as we got when we applied HIERARCHICAL CLUSTERING on the original, anonymous, and the synthetic datasets. The reason for the different clustering results for the synthetic datasets/eigenvectors from the original datasets/eigenvectors is that the synthetic data generation algorithm, which is CTGAN in our case reproduce data points within a fixed range, which leads to loss of information concerning the actual number of clusters.
- We found out that datasets, and eigenvectors protected using k -anonymity produces more accurate clustering results in comparison with the synthetic datasets created using CTGAN. For synthetic datasets, the size of clusters becomes compact in comparison with the original, and the k -anonymized dataset. Hence, generative algorithms can be utilised, if we intend to protect the outliers in the data. But, it should be avoided if we want better clustering results, especially for critical applications.
- In conclusion, eigenvectors computed from k -anonymous data provide better privacy-utility tradeoff in comparison with the eigenvectors computed from synthetic data, and eigenvectors with no privacy, in our attack, and utility analysis setup.

6 CONCLUSION AND FUTURE DIRECTIONS

This paper focuses on FL-PCA algorithms, in which each client participating in the FL framework shares the information of eigenvectors with the central aggregator, which can leak the data of clients. Hence, we explore Privacy Preserving Principal Component Analysis (PP-PCA). We propose that each client creates a protected database using k -anonymity, and generative networks, one at a time. This protected database is basically k -anonymous, and synthetic database, respectively. On the protected database, each client computes private eigenvectors. We evaluate the utility and privacy of private eigenvectors against their original counterparts. We employed regression, classification and HIERARCHICAL CLUSTERING for utility assessment, demonstrating strong performance of anonymized data and eigenvectors. Privacy evaluation involves a data reconstruction at-

tack on PCA, showcasing the attacker’s success in reconstructing the original databases. Our results reveal that anonymized eigenvectors maintain good utility compared to the original ones. Differential Privacy (DP) is also utilised in FL-PCA (Grammenos et al., 2020). In future, we will also consider DP to protect eigenvectors. As, in this work, we quantify individual privacy leakage arising from sharing of local eigenvectors, which were derived from the data of individual clients. So, our future investigation will extend to privacy leakage post-aggregation. In the case, when global eigenvectors are compromised, there is a potential risk of the attacker to deduce the records of specific individuals, particularly those who are influencing the aggregation step predominantly. Hence, our future research aims to focus on the performance of PP-PCA when aggregated or global eigenvectors are compromised in a FL scenario.

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