Identification of TITO Systems Using Modified Decentralized Relay Feedback

M. Hofreiter[a]
Department of Instrumentation and Control Engineering, Czech Technical University in Prague, Faculty of Mechanical Engineering, Prague, Czech Republic

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Abstract: The paper is devoted to the identification of systems with two inputs and two outputs (TITO systems) from one short, decentralized relay feedback experiment. The proposed modifications help to excite the process so that all parameters of the model describing the process can be estimated. The proposed procedure can be used to estimate the parameters of linear models without the need to achieve a steady-state output cycle. The proposed modification of relay feedback identification is demonstrated on a simulated TITO process.

1 INTRODUCTION

A relay feedback experiment for process identification is often used in control design. It was originally used for process identification by Rotač et al. (1961) and lately also for tuning mainly proportional-integral-derivative (PID) controllers, e.g. Åström & Hägglund (1984); Bi et al. (1997); Luyben (1987). A review of methods using relay feedback identification for single-input single-output (SISO) systems can be found in publications, e.g. Dharmalingam & Thangavelu (2019); Liu et al. (2013); Liu & Gao (2011); Ruderman (2019); Yu (2006).

Although there are many publications devoted to relay feedback identification, most of them consider only SISO systems. However, in many industrial processes, we often encounter systems with multiple inputs and multiple outputs, i.e. MIMO systems. The methods proposed for relay feedback identification of SISO systems can be used for the identification of MIMO systems in the case of negligible cross-couplings. Some methods are also dedicated to designing a proper decoupler to limit cross-couplings, e.g. Padhy & Majhi, (2006). If cross-couplings in MIMO systems are not negligible, then according to Wang et al. (1997) sequential tuning or decentralized relay feedback can be used. Sequential tuning is used, for example, in Shen & Yu (1994). In the case of decentralized relay feedback all loops are closed with relay feedback simultaneously, see Fig. 1 for the two-input two-output (TITO) process P. In this case, all cross-couplings influence the process. Published methods using the relay feedback experiment to identify TITO systems mostly assume low-order linear models, steady-state oscillations with the same fundamental period, and require multiple experiments, see e.g. Chidambaram & Sathe (2014); Wang et al. (1997); Bajarangbali & Majhi (2012); Hofreiter (2022). The mentioned limitations are solved in Berner et al. (2017a); Berner et al. (2017b) by changing the relay parameters during the relay feedback test and the identification is solved as an optimization problem.

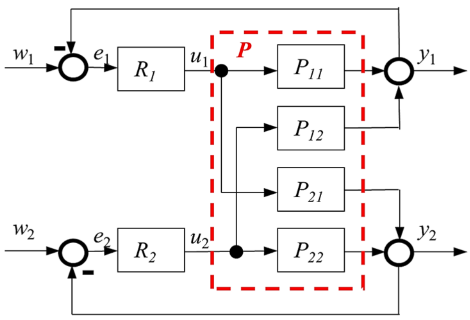

Figure 1: Block diagram of the decentralized relay control of the TITO process.

a https://orcid.org/0000-0001-9373-2988

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In this paper, we restrict ourselves to TITO systems and our goal is to use a decentralized relay test to identify the process. The aim of this paper is not to design TITO systems control, but to obtain a matrix of transfer functions that can be used for control design.

This paper is organized as follows. After an introductory section presenting the issues addressed, Section II explains the frequency method for identification of TITO strongly coupled systems using a relay-based closed-loop test. Here one recommends inserting an integrator before the relay to improve the estimation of the TITO system's static sensitivities. Section III demonstrates the proposed method on a TITO strongly coupled process where second-order time-delayed models model all interactions. The basic properties of this method are summarized in the Conclusions section.

2 RELAY IDENTIFICATION FOR TITO SYSTEMS

Let us consider a time-invariant strongly coupled TITO process, which is controlled by two biased relays R_1, R_2 with hysteresis, see Figure 1, where y_1, y_2 denote the controlled variables, w_1, w_2 are the desired variables, e_1, e_2 are the control errors and u_1 and u_2 are the manipulated variables. The static characteristics of the relays are depicted in Figure 2. The TITO process is described by the following frequency transfer function matrix (j is the imaginary unit).

$$\boldsymbol{P}(j\omega) = \begin{bmatrix} P_{11}(j\omega) & P_{12}(j\omega) \\ P_{21}(j\omega) & P_{22}(j\omega) \end{bmatrix} \qquad (1)$$

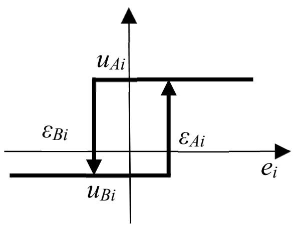

Figure 2: Steady-state characteristics of a biased relay with hysteresis (i=1,2).

In the frequency domain, it holds for the TITO process

$$Y_i = P_{i1} \cdot U_1 + P_{i2} \cdot U_2;\ i = 1,2 \qquad (2)$$

where Y_i, U_i are the Fourier images of y_i, u_i and P_{i1}, P_{i2} are the frequency transfer functions of the TITO process. We choose the model structure M_{i1}, M_{i2}, i=1,2 for describing the frequency transfer functions P_{i1}, P_{i2}, i=1,2 of the TITO process. The unknown parameters of the model transfer functions M_{i1}, M_{i2}, i=1,2 can be obtained by minimizing the criterion reflecting the errors of equations (2) for individual frequencies where P_{i1}, P_{i2}, i=1,2 are substituted by M_{i1}, M_{i2}, i=1,2. The relationship (2) implies that we can identify the process subsystems related to one output separately from the subsystem related to the other output. Therefore, the model parameters can be obtained for i=1,2 by the criterion

$$Cr_i(\theta_{i1},\theta_{i2}) = \sum_{k=1}^{n_f} \left| \left(Y_i(\omega_k) - \sum_{l=1}^{2} M_{il}(\theta_{il},\omega_k) \cdot U_l(\omega_k) \right) \right|^2 \qquad (3)$$

where θ_{i1}, θ_{i2}; i=1,2 are the estimated model parameters, n_f is the number of frequencies, and ω is the frequency. The model parameters θ_{i1}, θ_{i2}; i=1,2 are estimated by iterative calculation using

$$\theta_{i1},\theta_{i2} = \arg\min_{\theta_{i,1},\theta_{i2}} Cr_i(\theta_{i,1},\theta_{i2}); i = 1,2 \qquad (4)$$

This optimization problem was solved in the Matlab environment using the *tfest* command in System Identification Toolbox (Release 2024a). The algorithm of the *tfest* command is described in Ozdemir & Gumussoy (2017).

Asymmetric relays help to excite the process and thus improve parameter estimation. A significant improvement in parameter estimates can be achieved by inserting an integration term into one input (see Fig. 3) or both inputs. This will excite the process at low frequencies, which significantly improves the estimates of steady-state gains.

3 EXAMPLE

The following description of the TITO process was taken from Wang et al. (1997). Consider the transfer function matrix $\mathbf{P}(s)$ of a TITO process

$$\boldsymbol{P}(s) = \begin{bmatrix} \frac{0.5}{(0.1s+1)^2(0.2s+1)^2} & \frac{-1}{(0.1s+1)(0.2s+1)^2} \\ \frac{1}{(0.1s+1)(0.2s+1)^2} & \frac{2.4}{(0.1s+1)(0.2s+1)^2(0.5s+1)} \end{bmatrix} \qquad (5)$$

(s is the complex variable in the Laplace transform).

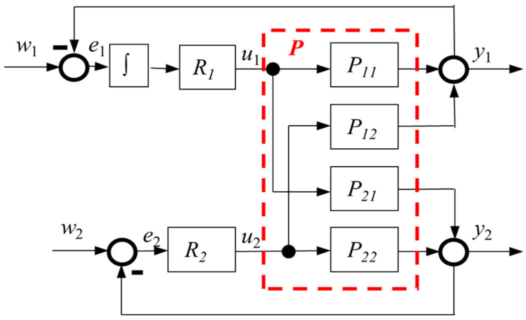

Figure 3: Block diagram of the decentralized modified relay test of the TITO system.

The block diagram used to identify the TITO system is shown in Fig. 1. The relays parameters in the given example were chosen as follows.

$$u_{A1} = 1.5,\ u_{B1} = -0.5,\ \varepsilon_{A1} = 0.1,\ \varepsilon_{B1} = -0.1 \quad (6)$$

$$u_{A2} = 1.4,\ u_{B2} = -0.4,\ \varepsilon_{A2} = 0.1,\ \varepsilon_{B2} = -0.1 \quad (7)$$

The relay outputs u_1, u_2 and the process outputs y_1, y_2 obtained from the relay feedback test to identify the TITO process are shown in Fig. 4. Input and output data in the range of 5 s were used to estimate the parameters of models M_{i1}, M_{i2}, i=1,2. Inputs and outputs were recorded with a sampling period of 0.01 s. A fast Fourier transform was used to convert the inputs and outputs from the time domain to the frequency domain, see Fig. 5. Second-order time-delayed (SOTD) models (8) were used for the model fitting because this model is very versatile and can describe most non-oscillating or oscillating proportional systems with or without a transport delay.

$$M_{mn}(s) = \frac{K_{mn} \cdot e^{-s \cdot \tau_{mn}}}{a_{2mn}s^2 + a_{1mn}s + 1},\ m=1,2;\ n=1,2 \quad (8)$$

Therefore, it is necessary to estimate 16 parameters (K_{mn}, τ_{mn}, a_{2mn}, a_{1mn}, m=1,2; n=1,2) using relay feedback identification.

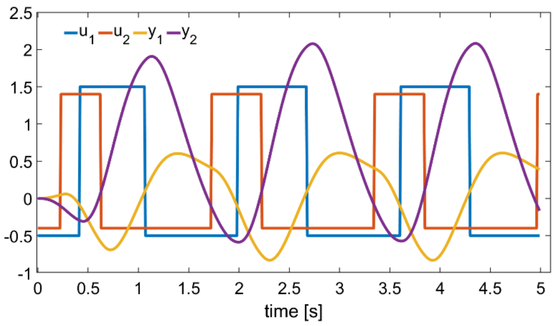

Figure 4: Input and output data used to estimate model parameters.

Model M of the TITO process was obtained using the procedure described in Section 2 in the form

$$\boldsymbol{M}(s) = \begin{bmatrix} \frac{6.817 \cdot exp(-0.08s)}{s^2+7.146s+15.8} & \frac{-12.98 \cdot exp(-0.03s)}{s^2+5.293s+15.46} \\ \frac{19.69 \cdot exp(-0.02s)}{s^2+6.988s+11.68} & \frac{7.085 \cdot exp(-0.13s)}{s^2+2.966s+10.03} \end{bmatrix} \quad (9)$$

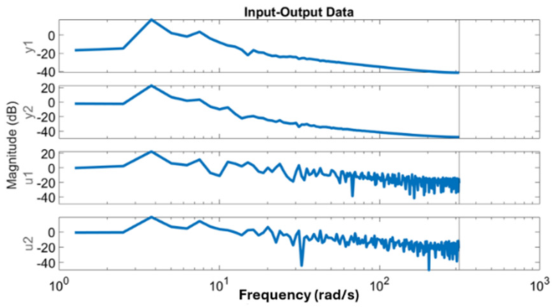

Figure 5: Magnitude spectrum of signals y_1, y_2, u_1 and u_2.

Fig. 6 to 9 show the Nyquist frequency responses of process **P** and model **M**. Further comparison is made using unit step responses shown in Fig. 10 to13. It is clear from the frequency and step responses that, especially at lower frequencies, there is little agreement between the frequency characteristics of the process and the model, which is a consequence of the small excitation of the process at lower frequencies. At the same time, it can be stated that with the mentioned procedure, it was possible to estimate all 16 parameters from the transition process by means of one short experiment using decentralized relay control.

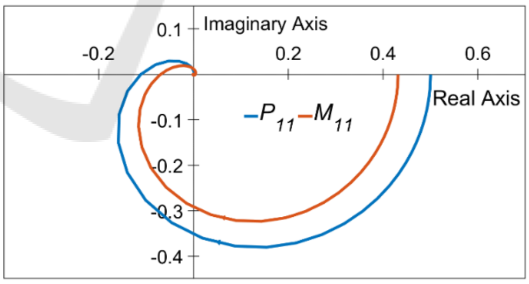

Figure 6: Nyquist frequency responses of P_{11} and M_{11}.

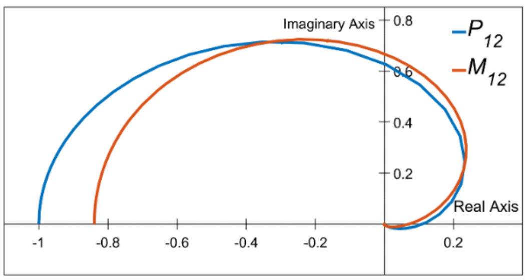

Figure 7: Nyquist frequency responses of P_{12} and M_{12}.

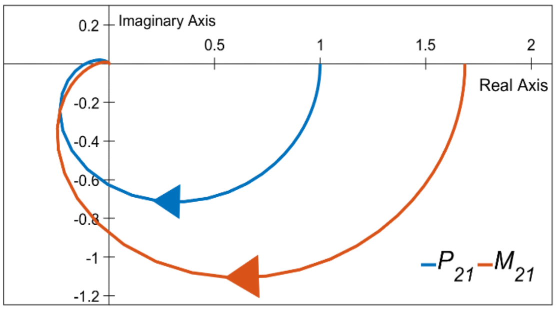

Figure 8: Nyquist frequency responses of P_{21} and M_{21}.

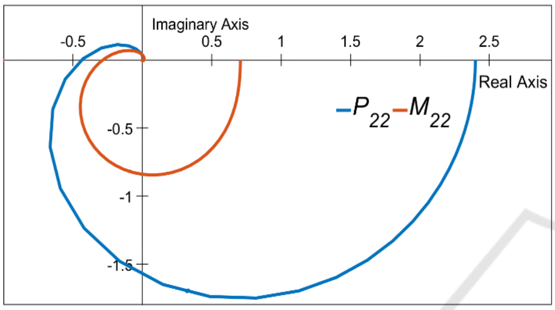

Figure 9: Nyquist frequency responses of P_{22} and M_{22}.

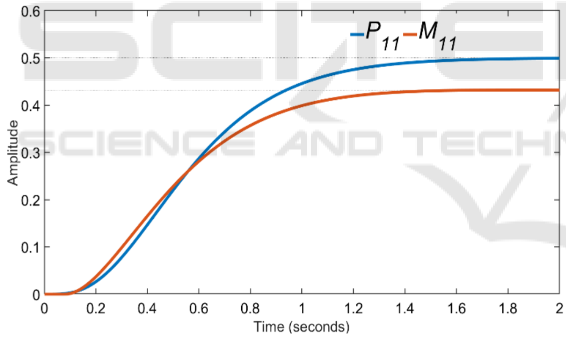

Figure 10: Unit step responses of P_{11} and M_{11}.

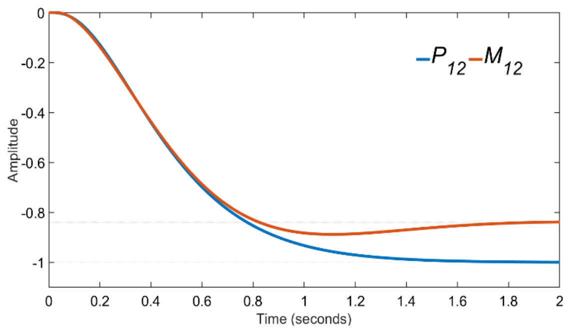

Figure 11. Unit step responses of P_{12} and M_{12}.

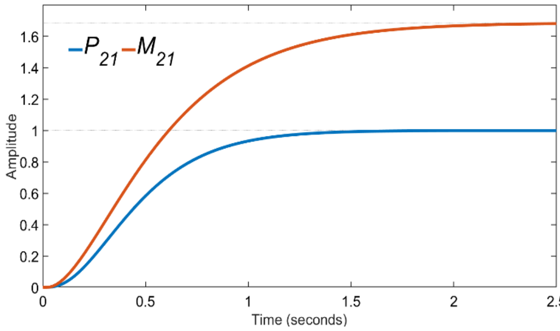

Figure 12. Unit step responses of P_{21} and M_{21}.

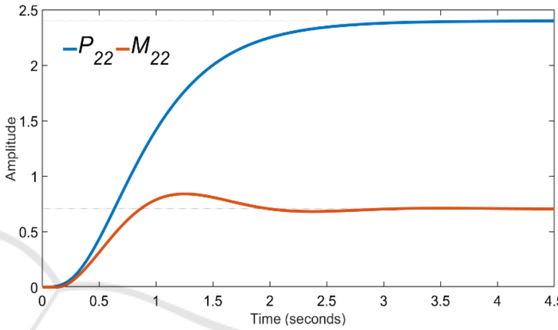

Figure 13: Unit step responses of P_{22} and M_{22}.

The estimation of the model parameters in the above example can be improved by inserting an integration term on one of the inputs (see Fig. 3) or on both inputs. The relay outputs u_1, u_2 and the process outputs y_1, y_2 obtained from the relay feedback test by inserting an integration term into the input u_1 to identify the TITO process are shown in Fig. 14. This will cause a greater excitation of the process at lower frequencies (see Fig. 15) and improve the estimation of the model parameters. Model **M** of the TITO process obtained using the modified decentralized relay feedback is in the form

$$\boldsymbol{M}(s) = \begin{bmatrix} \frac{167.9\cdot exp(-0.13s)}{s^2+150.6s+346.3} & \frac{-14.35\cdot exp(-0.02s)}{s^2+5.97s+14.11} \\ \frac{13.58\cdot exp(-0.01s)}{s^2+6.044s+12.5} & \frac{10.56\cdot exp(-0.13s)}{s^2+3.279s+4.877} \end{bmatrix} \quad (10)$$

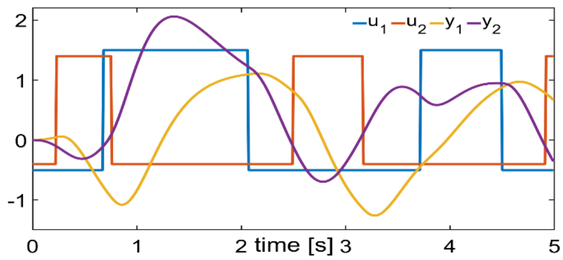

Figure 14: Input and output data used to estimate model parameters.

The Nyquist frequency responses of process **P** and model **M** obtained by the modified relay feedback test are depicted in Fig. 16 to 19. Unit step responses of process **P** and model **M** obtained by the modified relay feedback test are depicted in Fig. 20 to 23.

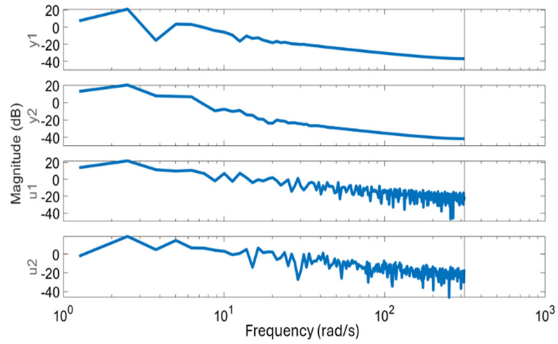

Figure 15: Magnitude spectrum of signals y_1, y_2, u_1 and u_2.

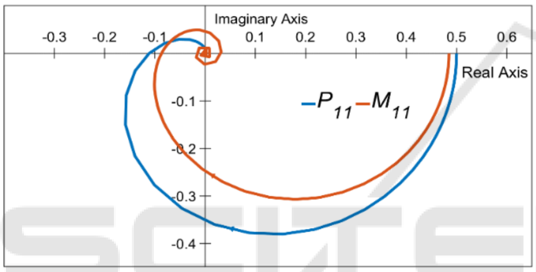

Figure 16: Nyquist frequency responses of P_{11} and M_{11} obtained by the modified decentralized relay feedback.

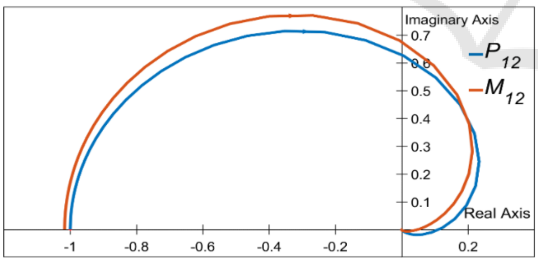

Figure 17: Nyquist frequency responses of P_{12} and M_{12} obtained by the modified decentralized relay feedback.

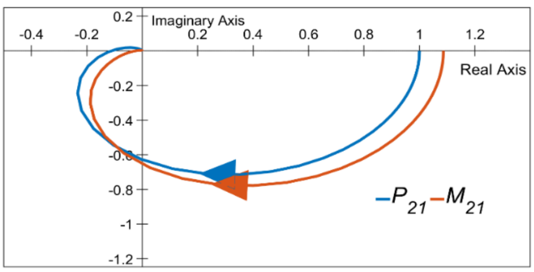

Figure 18. Nyquist frequency responses of P_{21} and M_{21} obtained by the modified decentralized relay feedback.

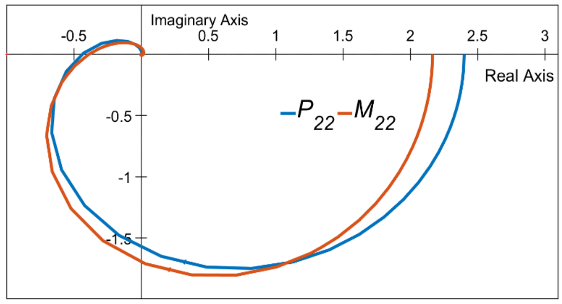

Figure 19: Nyquist frequency responses of P_{22} and M_{22} obtained by the modified decentralized relay feedback.

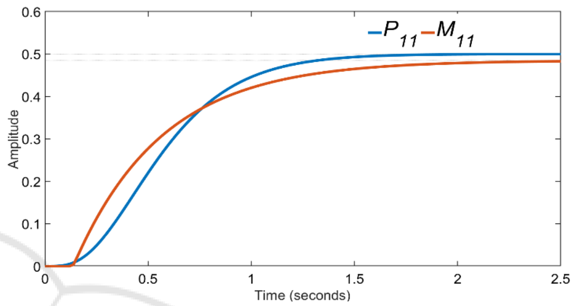

Figure 20: Unit step responses of P_{11} and model M_{11} obtained by the modified relay feedback test.

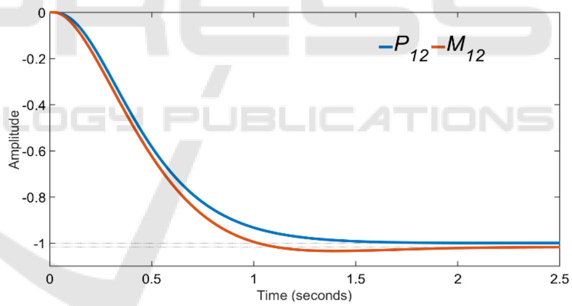

Figure 21: Unit step responses of P_{12} and model M_{12} obtained by the modified relay feedback test.

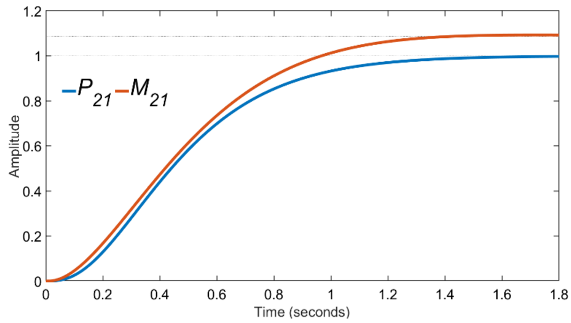

Figure 22: Unit step responses of P_{21} and model M_{21} obtained by the modified relay feedback test.

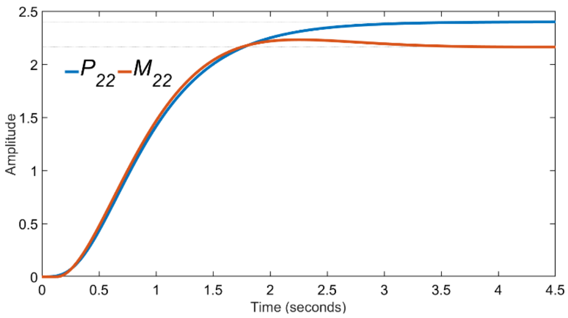

Figure 23: Unit step responses of P_{22} and model M_{22} obtained by the modified relay feedback test.

The percentage fit between the TITO process (5) and the estimated TITO models (9) and (10) is shown in Table 1. The fit between P_{ij} and M_{ij} is calculated using the relationship

$$Fit_{ij} = 100 \cdot \left(1 - \frac{\|P_{ij} - M_{ij}\|}{\|P_{ij} - \bar{P}_{ij}\|}\right), i,j = 1,2$$

where P_{ij} and M_{ij} are vectors where vector elements are 128 linearly spaced frequency responses of process $P_{ij}(j\omega)$ and $M_{ij}(j\omega)$ up to frequency 314 rad/s. $\bar{P}_{ij}$ denotes the mean value of elements of P_{ij}, $\omega \in (0,314)$ rad·s^{-1}, $\|x\|$ is Euclidean norm of a vector x.

Table 1: The fit between the TITO process (5) and the estimated TITO models (9) and (10).

	Model(9)	Model(10)
Fit_{11}	81.67 %	77.00 %
Fit_{12}	82.24 %	93.13 %
Fit_{21}	19.06 %	89.04 %
Fit_{22}	14.20 %	86.76 %
mean	49.29 %	86.48 %

The steady-state gains of the TITO process (5) and the estimated TITO models (9) and (10) are shown in Table 2.

Table 2: The steady-state gains of the TITO process (5) and the estimated TITO models (9) and (10).

Process(5)	Model(9)	Model(10)
$P_{11}(0)$=0.5	$M_{11}(0)$=0.43	$M_{11}(0)$=0.48
$P_{12}(0)$=-1	$M_{12}(0)$=-0.84	$M_{12}(0)$=-1.02
$P_{21}(0)$=1	$M_{21}(0)$=1.68	$M_{21}(0)$=1.08
$P_{22}(0)$=2.4	$M_{22}(0)$=0.71	$M_{22}(0)$=2.17

4 CONCLUSIONS

Most methods using relay feedback identification for estimating the model parameters of MIMO systems rely on experimental determination of the steady-state oscillation period. For cross-coupled MIMO systems, these identification methods require the process to be able to achieve steady-state oscillations under relay control, which requires a significantly longer experimental measurement period compared to relay feedback identification of SISO systems. The procedure presented in this paper reduces the experimental measurement time compared to commonly used relay identification methods because it does not require achieving stable oscillations to determine the fundamental period. The proposed modification using the integration term and frequency characteristics of the TITO system then allows better estimation of the static gains of the individual transfer functions. The procedure is demonstrated using the TITO process and shows that TITO processes can be identified from a non-stationary time course with one decentralized relay test. Further research will be devoted to the sensitivity of the relay feedback method to disturbances, the identification of more complex MIMO systems, and the control design of real TITO systems using models obtained through the decentralized relay test.

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