H_∞ Type Control of Periodic Stochastic Systems Subject to Multiplicative White Noises: Application to Satellite AOCS Design

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Abstract: The paper presents an H_∞ state feedback type design method for a class of periodic discrete-time stochastic systems subject to multiplicative white noises. It is shown that the gains of the control law for the considered problem may be expressed in terms of the solution of a specific system of linear matrix inequalities with periodic coefficients. The design method is illustrated by an application for the detumbling subsystem of a cubesat in which a linearized model with parametric modeling uncertainties is considered.

1 INTRODUCTION

The design of the satellites AOCS (Attitude and Orbit Control System) is an important direction of research in aerospace applications due to the wide mission profiles and the specific requirements in their different stages. One of the effective approaches to design control laws used in such applications is based on the so-called H_∞ type techniques which became over the last decades a mature design method for a large number of engineering applications (see e.g. (Doyle,1989, Skelton, 1998, Zhou, 1999) for theoretical fundamentals) and for instance, (Simplicio, 2016, La Ballois, 1996, Souza, 2019), for some applications in aerospace engineering. Early results concerning the H_∞ norm minimization referred to the case when the controlled system dynamics is approximated by linear models with constant coefficients. Further, the H_∞ type design methodology was extended to other classes of dynamic systems, including models with time-varying parameters, stochastic systems and nonlinear systems (see e.g. (Dragan, 2010, Zhang, 2017, Coutinho, 2002, Aliyu, 2017)).

In the present paper, an H_∞ type control problem is considered for stochastic discrete-time linear systems with periodic time-varying parameters corrupted with multiplicative white noises. This class of systems extend the results presented for instance in (Lovera, 2000, Lovera, 2002), derived in the absence of the stochastic terms. The interest for stochastic models including multiplicative white noise terms is motivated both for specific applications in which such terms naturally appear (see e.g. (Gershon, 2005, Xing, 2020, Avital, 2023)) but also in the representation of parametric modeling uncertainties (Petersen, 2017). The design approach presented in this paper aims to determine a state feedback control law ensuring an H_∞ performance for systems with time periodic coefficients subject to multiplicative noises. It is shown that the gains of this H_∞ control law depend on the solution of a specific system of coupled linear matrix inequalities (LMIs) with periodic coefficients. The theoretical results are illustrated for the design of the detumbling subsystem of a cubesat.

Throughout the paper, the following notations will be used: $\mathbb{R}$ denotes the set of real numbers, $\mathbb{Z}_+$ is the set of nonnegative integers, $E[\cdot]$ stands for the expectation, $|x|$ represents the Euclidean norm of the vector x and $P(\cdot)$ denotes the probability of an event.

2 PROBLEM FORMULATION

Consider the discrete-time stochastic system

$$\begin{aligned} x(k+1) &= \left(A_0(k)+\textstyle\sum_{\ell=1}^{r} w_\ell(k)A_\ell(k)\right)x(k) \\ &\quad + \left(B_{10}(k)+\textstyle\sum_{\ell=1}^{r} w_\ell(k)B_{1\ell(k)}\right)u_1(k) \\ &\quad + \left(B_{20}(k)+\textstyle\sum_{\ell=1}^{r} w_\ell(k)B_{2\ell(k)}\right)u_2(k) \\ y_1(k) &= C(k)x(k)+D(k)u_2(k); \\ y_2(k) &= x(k),\ k=0,1,\ldots \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the state vector, $u_1 \in \mathbb{R}^{m_1}$ is the exogenous input, $u_2 \in \mathbb{R}^{m_2}$ stands for the

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control variable, $y_1 \in \mathbb{R}^{p_1}$ is the quality output and y_2 denotes the measured output. $w(k) = \begin{bmatrix} w_1(k) & \dots & w_r(k) \end{bmatrix}^T$ are random vectors which components $w_\ell(k), \ell = 1,...,r$ are independent, zero mean variables on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with $\sup_{k\in\mathbb{Z}_+} E[|w(k)|^2] < \infty$ and $E[w(k)w^T(k)] = I_{r\times r}$ for all $k \in \mathbb{Z}_+$. Throughout the paper it will be assumed that all matrices are time-periodic functions with the positive integerperiod N. A motivation for the considered model will be given in Section 4 where an application for the periodic control law design of a cubesat detumbling is presented.

Remark 1. *In the above stochastic system (1) the same noises $w_\ell, \ell = 1,...,r$ were considered in the expressions of the state and the control matrices. In the situation when these matrices are perturbed with different independent multiplicative noises, one may define a vector w obtained by concatenating all noises in a single vector w and setting accordingly the coefficients $A_\ell, B_\ell, \ell = 1,...,r$.*

The problem consists in determining a periodic state feedback gain $F(k), k = 0,1...$ such that the resulting system obtained with $u_2(k) = F(k)x(k)$ is exponentially stable in mean square (ESMS) and it satisfies for a given $\gamma > 0$, the H_∞ type condition

$$\sum_{k=0}^{\infty} E\left[|y_1(k)|^2 - \gamma^2|u_1(k)|^2\right] < 0 \qquad (2)$$

for all $u_1 \in L^2[0,\infty;\mathbb{R}^{m_1}]$, where L^2 denotes the space of all sequences u_1 with $\sum_{k=0}^{\infty}|u_1(k)|^2 < \infty$. It is reminded that a discrete-time stochastic system with time varying coefficients of form $x(k+1) = (A_0(k) + \sum_{\ell=1}^{r} w_\ell(k)A_\ell(k))x(k)$, $k \in \mathbb{Z}_+$ is ESMS if there exist $\alpha \in (0,1)$ and $\beta \geq 1$ such that $E[|x(k)|^2] \leq \beta\alpha^k|x_0|^2$ for all $x_0 \in \mathbb{R}^n$ (see e.g. (Morozan, 1997, El Bouhtouri, 1999)).

3 TIME-PERIODIC H_∞ STATE FEEDBACK CONTROL LAW

The next result which proof may be found in (Morozan, 1999) is a version of the Bounded Real Lemma for discrete-time time-varying systems with multiplicative noises.

Theorem 1. *The following assertions are equivalent*
i) *The system*

$$\begin{array}{rcl} x(k+1) & = & (A_0(k) + \sum_{\ell=1}^{r} w_\ell(k)A_\ell(k))\,x(k) \\ & & (B_0(k) + \sum_{\ell=1}^{r} w_\ell(k)B_\ell(k))\,u(k) \\ y(k) & = & (C_0(k) + \sum_{\ell=1}^{r} w_\ell(k)C_\ell(k))\,x(k) \\ & & +\,(D_0(k) + \sum_{\ell=1}^{r} w_\ell(k)D_\ell(k))\,u(k) \end{array}$$

is ESMS and its associated input-output operator has the norm less than γ;
ii) *The system of linear matrix inequalities*

$$\begin{bmatrix} \mathcal{R}_{11}(k,k+1) & \mathcal{R}_{12}(k,k+1) \\ \mathcal{R}_{12}^T(k,k+1) & \mathcal{R}_{22}(k,k+1) \end{bmatrix} < 0 \qquad (3)$$

where

$$\begin{array}{rcl} \mathcal{R}_{11}(k,k+1) & = & \sum_{\ell=0}^{r}\left[A_\ell^T(k)X(k+1)A_\ell(k)\right. \\ & & \left.+C_\ell^T(k)C_\ell(k)\right] - X(k) \\ \mathcal{R}_{12}(k,k+1) & = & \sum_{\ell=0}^{r}\left[A_\ell^T(k)X(k+1)B_\ell(k)\right. \\ & & \left.+C_\ell^T(k)D_\ell(k)\right] \\ \mathcal{R}_{22}(k,k+1) & = & -\left[\gamma^2 I - \sum_{\ell=1}^{r}\left(D_\ell^T(k)D_\ell(k)\right.\right. \\ & & \left.\left.+B_\ell^T(k)X(k+1)B_\ell(k)\right)\right] \end{array}$$

has a bounded positive definite solution $\{X(k)\}_{k\in\mathbb{Z}_+}$. Moreover, if the coefficients are periodic then the system (3) has a periodic positive definite solution with the same period. $\square$

Based on the above theorem, the state feedback periodic gain solving the H_∞ type control problem is given by the following result.

Theorem 2. *If there exist $Y(k) \in \mathbb{R}^{n\times n}$, $Y(k) > 0$ and $Z(k) \in \mathbb{R}^{n\times m_2}$, $k = 0,1,...,N$ with $Y(N) = Y(0)$ solving the system of linear matrix inequalities*

$$\begin{bmatrix} -Y(k) & 0 & \mathcal{M}(k) & \mathcal{N}(k) \\ 0 & -\gamma^2 I_{m_1} & \mathcal{B} & 0 \\ \mathcal{M}^T(k) & \mathcal{B}^T & -\mathcal{Y}(k+1) & 0 \\ \mathcal{N}^T(k) & 0 & 0 & -I_{p_1} \end{bmatrix} < 0, \quad (4)$$

$k = 0,1,...,N-1$ where

$$\begin{array}{l} \mathcal{M}(k) := \left[\left(Y(k)A_0^T(k) + Z(k)B_{20}^T(k)\right),\dots\right. \\ \qquad \left.\dots,\left(Y(k)A_r^T(k) + Z(k)B_{2r}^T(k)\right)\right] \\ \mathcal{N}(k) := Y(k)C^T(k) + Z(k)D^T(k) \\ \mathcal{B} := \left[B_{10}^T,\dots,B_{1r}^T\right] \\ \mathcal{Y}(k+1) := diag\left(Y(k+1),\dots,Y(k+1)\right), \end{array}$$

then the stabilizing state feedback periodic gains for which the H_∞ type condition (2) is fulfilled are given by $F(k) = Z^T(k)Y^{-1}(k), k = 0,1,...N-1$.
Proof. For $u_2(k) = F(k)x(k)$, the system (1) becomes:

$$\begin{array}{l} x(k+1) = [A_0(k) + B_{20}(k)F(k) \\ \qquad + \sum_{\ell=1}^{r} w_\ell\left(A_\ell(k) + B_{2\ell}(k)F(k)\right)]\,x(k) \\ \qquad + \left(B_{10}(k) + \sum_{\ell=1}^{r} w_\ell B_{1\ell}(k)\right)u_1(k) \\ y(t) = (C(k) + D(k)F(k))\,x(k),\ k = 0,1,\dots \end{array}$$

Using Theorem 1 for the above closed loop system, direct algebraic computations based on Schur complements arguments, give (4) after multiplication of the equivalent inequality to the left and the right with diag$(Y(k), I, ..., I)$, where one denoted $Y(k) := X^{-1}(k)$ and $Z(k) := X^{-1}(k)F^T(k)$. $\square$

4 CUBESAT DETUMBLING TIME-PERIODIC CONTROL LAW

The attitude dynamics of a rigid satellite is expressed by the known angular momentum equation (see e.g. (Wertz, 1978, Wie, 1998))

$$\dot{\omega}(t) \quad = \quad -\omega(t) \times \mathbf{I}\omega(t) + T_c(t) + T_d(t) \qquad (5)$$

where $\omega \in \mathbb{R}^3$ denotes the angular rate expressed in the body frame, $\mathbf{I} \in \mathbb{R}^{3\times 3}$ is the inertia matrix, $T_c \in \mathbb{R}^3$ stands for the control torques and $T_d \in \mathbb{R}^3$ includes the exogenous disturbances torques. The control torque is given by the cross product between the geomagnetic field vector $\mathbf{B}$ and the magnetic dipole moment generated by magnetic coils of the magnetorquer actuator. The external disturbances are generated by the forces acting on a satellite in low-earth orbit as gravity gradient, solar radiation pressure, magnetic torques and air drag (see e.g. (Wisniewski, 1999, Wallado, 2001)).

The attitude kinematics is parametrized as follows (Wertz, 1978, Wie, 1998)

$$\dot{\mathbf{q}}(t) \quad = \quad \tfrac{1}{2}\mathbf{W}(\omega)\mathbf{q}(t), \qquad (6)$$

where $\mathbf{q} \in \mathbb{R}^4$ is the quaternion vector with unit Euclidian norm and the skew matrix $\mathbf{W} \in \mathbb{R}^{4\times 4}$ has the expression

$$\mathbf{W}(\omega) = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}$$

in which ω_x, ω_y and ω_z denote the components of the angular rate ω. An important phase of the satellite mission after its ejection from the launch vehicle is the so-called "detumbling", consisting in reducing its angular rate to a value close to zero. A linear approximation of the dynamics and kinematics equations (5) and (6) around equilibrium conditions for the state $x := [\mathbf{q}^T \ \omega^T]^T$, namely for $\mathbf{q} = [0\,0\,0\,1]^T$ and $\omega = [0\,0\,0]^T$, respectively, leads to the following approximative model with time-varying coefficients

$$\dot{x}(t) = Ax(t) + B(t)u(t) \qquad (7)$$

where one denoted by u the magnetic dipole moment vector generated by the magnetic coils aligned with the principal inertia axes and where (see e.g. Lovera, 2000, Silano, 2005)

$$A = \begin{bmatrix} \frac{\partial\dot{\mathbf{q}}}{\partial\mathbf{q}} & \frac{\partial\dot{\mathbf{q}}}{\partial\omega} \\ \frac{\partial\dot{\omega}}{\partial\mathbf{q}} & \frac{\partial\dot{\omega}}{\partial\omega} \end{bmatrix} = \begin{bmatrix} 0_{3\times 4} & \frac{1}{2}I_{3\times 3} \\ 0_{4\times 4} & 0_{4\times 3} \end{bmatrix}$$

$$B(t) = \begin{bmatrix} 0_{4\times 3} \\ \mathbf{I}^{-1} \end{bmatrix} \mathbf{B}(b(t))$$

where $\frac{\partial\dot{\mathbf{q}}}{\partial\mathbf{q}}$, $\frac{\partial\dot{\mathbf{q}}}{\partial\omega}$, $\frac{\partial\dot{\omega}}{\partial\mathbf{q}}$, $\frac{\partial\dot{\omega}}{\partial\omega}$ represent the derivatives of the right hand sides of equations (6) and (5), respectively, and

$$\mathbf{B}(b(t)) = \begin{bmatrix} 0 & b_z(t) & -b_y(t) \\ -b_z(t) & 0 & b_x(t) \\ b_y(t) & -b_x(t) & 0 \end{bmatrix}$$

in which $b_x(t)$, $b_y(t)$ and $b_z(t)$ are time-periodic components of the geomagnetic field vector. They may be measured on board using magnetometers but they also be determined using the International Geomagnetic Reference Field (IGRF) models based on the latitude, longitude and altitude of the satellite. In Figure 1, the time variation of the components of the Earth magnetic field are presented, corresponding to a polar low Earth orbit (LEO) of a cubesat at 500 km altitude and 87^o longitude.

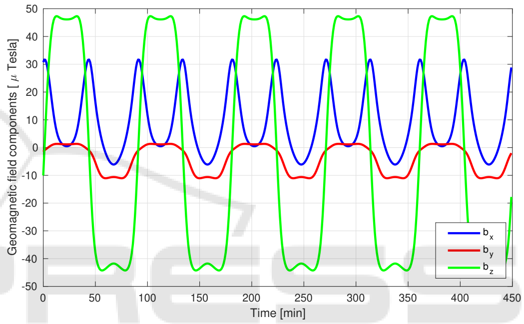

Figure 1: Geomagnetic field components for a polar orbit at 500 km altitude and 87^o longitude.

These components of the magnetic field may be approximated by simplified expressions derived using the Fourier coefficients, for an orbital period $T = 90\,\text{min}$, as follows

$$b_x(t) = 10^{-6}\left(10.7150 + 2.4674 \sin\tfrac{2\pi}{90}t \right.$$
$$\left. +4.1390 \cos\tfrac{2\pi}{90}t - 9.7118 \sin\tfrac{4\pi}{90}t + 11.5496 \cos\tfrac{4\pi}{90}t\right)$$
$$b_y(t) = 10^{-6}\left(-34927 + 5.9779 \sin\tfrac{2\pi}{90}t \right.$$
$$\left. +42.7726 \cos\tfrac{2\pi}{90}t - 1.8465 \sin\tfrac{4\pi}{90}t + 1.0177 \cos\tfrac{4\pi}{90}t\right)$$
$$b_z(t) = 10^{-6}\left(1.2491 + 48.5761 \sin\tfrac{2\pi}{90}t \right.$$
$$\left. +20.594 \cos\tfrac{2\pi}{90}t - 3.7489 \sin\tfrac{4\pi}{90}t + 11.5496 \cos\tfrac{4\pi}{90}t\right).$$

For the application presented in Section 4, a cubesat with the inertia matrix $\mathbf{I} = \text{diag}(0.005, 0.005, 0.002)\text{kg}\,\text{m}^2$ has been considered, for which the time varying control matrix $B(t)$ has the expression

$$B(t) = \begin{bmatrix} 0_{4\times 1} & 0_{4\times 1} & 0_{4\times 1} \\ 0 & 200\,b_z(t) & -200\,b_y(t) \\ -200\,b_z(t) & 0 & 200\,b_x(t) \\ 500\,b_y(t) & -500\,b_x(t) & 0 \end{bmatrix}.$$

One can see that in the above model, the control matrix $B(t)$ is time periodic with the orbital period T. Another aspect taken into account in the considered application is the influence of the parametric modeling uncertainties. One may represent such uncertainties using white multiplicative noise terms. Thus, if for instance, the element $B(5,2)$ which equals $200\,b_z(t)$, has a variation of $\pm 10\%$ around its nominal amplitude $\bar{b}_{52} = 200$ due to modeling uncertainties or to the satellite mass change, then this uncertainty may be represented as $b_{52} = \bar{b}_{52} + \xi_1(t)$ where ξ_1 is a Gaussian white noise. Its variance $\sigma^2_{\xi_1}$ may be determined using the 3σ rule stating that $P\left(|b_{52} - \bar{b}_{52}| \leq 3\sigma_{\xi_1}\right) \geq 0.997$ from which it results that $\sigma_{\xi_1} = 6.67$. Similarly, for $b_{63} = \bar{b}_{63} + \xi_2(t)$ and $b_{71} = \bar{b}_{71} + \xi_3(t)$, where $\bar{b}_{63} = 200$ and $\bar{b}_{71} = 500$, considering the same level of uncertainty $\pm 10\%$, one obtains the following representation of the control matrix $B_u(t)$ with modeling uncertainties

$$B_u(t) = B(t) + \xi_1(t)B_1(t) + \xi_2(t)B_2(t) + \xi_3(t)B_3(t) \quad (8)$$

where

$$B_1(t) = \begin{bmatrix} 0_{4\times 1} & 0_{4\times 1} & 0_{4\times 1} \\ 0 & 200\,b_z(t) & -200\,b_y(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_2(t) = \begin{bmatrix} 0_{4\times 1} & 0_{4\times 1} & 0_{4\times 1} \\ 0 & 0 & 0 \\ -200\,b_z(t) & 0 & 200\,b_x(t) \\ 0 & 0 & 0 \end{bmatrix}.$$

and

$$B_3(t) = \begin{bmatrix} 0_{4\times 1} & 0_{4\times 1} & 0_{4\times 1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 500\,b_y(t) & -500\,b_x(t) & 0 \end{bmatrix},$$

and where the noises have the standard deviations $\sigma_{\xi_1} = \sigma_{\xi_2} = 6.67$ and $\sigma_{\xi_3} = 11.67$, respectively. For this application, the matrices $B_{2\ell}, \ell = 0,\ldots,3$ of the generalized discrete-time model (1) were obtained by discretization of the above matrices A, $B(t)$ and $B_\ell(t), \ell = 1,2,3$ with a sampling period $T_s = 15\,\text{min}$. Taking into account the average magnitude of the disturbances torques (see e.g. (Wisnievski, 1999, Wallado, 2001)), the matrix coefficients of $u_1(k)$ in (1) where considered $10^{-2}B_{2\ell}, \ell = 0,\ldots,3$. As concerns the quality output, one defined

$$y_1(k) = \begin{bmatrix} W_1\,\omega(k) \\ W_2 u_2(k) \end{bmatrix}$$

with the positive scalar weights W_1 and W_2. Solving the system of LMIs (4) one obtained for $W_1 = 100, W_2 = 1$ and for an attenuation level $\gamma = 0.5$, the following time-varying gains for an orbital period $T = 90\,\text{min}$:

$$F(1) = \begin{bmatrix} 0_{1\times 4} & -0.0001 & 0.00301 & -0.0011 \\ 0_{1\times 4} & -0.0225 & 0.0001 & 0.0776 \\ 0_{1\times 4} & 0.0006 & -0.0415 & 0 \end{bmatrix}$$

$$F(2) = \begin{bmatrix} 0_{1\times 4} & 0 & 0.0467 & -0.0021 \\ 0_{1\times 4} & -0.0467 & 0.0000 & 0.0017 \\ 0_{1\times 4} & 0.0008 & -0.0007 & 0 \end{bmatrix}$$

$$F(3) = \begin{bmatrix} 0_{1\times 4} & -0.0009 & 0.0438 & -0.0031 \\ 0_{1\times 4} & -0.0345 & 0.0009 & 0.0308 \\ 0_{1\times 4} & 0.0016 & -0.0157 & 0 \end{bmatrix}$$

$$F(4) = \begin{bmatrix} 0_{1\times 4} & -0.0059 & -0.0403 & 0.0117 \\ 0_{1\times 4} & 0.0213 & 0.0059 & 0.0410 \\ 0_{1\times 4} & -0.0100 & -0.0352 & 0 \end{bmatrix}$$

$$F(5) = \begin{bmatrix} 0_{1\times 4} & -0.0022 & -0.0505 & 0.0299 \\ 0_{1\times 4} & 0.0484 & 0.0022 & -0.0188 \\ 0_{1\times 4} & -0.0108 & 0.068 & 0 \end{bmatrix}$$

$$F(6) = \begin{bmatrix} 0_{1\times 4} & -0.0080 & -0.0435 & 0.0159 \\ 0_{1\times 4} & 0.0302 & 0.0079 & 0.0336 \\ 0_{1\times 4} & -0.0113 & -0.0240 & 0 \end{bmatrix},$$

corresponding to the moments of time $\{0;15;30;45;60;75\}$ min. With the above time-periodic gains, the following time responses of the angular rates and of coil's magnetic dipoles illustrated in Figure 2, have been obtained. These time-responses indicate the cubesat stabilization with a reduced control effort together with disturbances attenuation. The time response performance may be adjusted by changing the ratio between the weights W_1 and W_2 in the quality outputs according with the AOCS requirements.

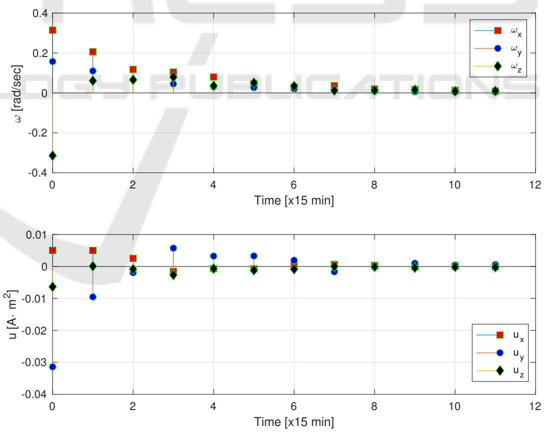

Figure 2: Angular velocities and dipoles moments at the sampling instants for two orbits.

5 CONCLUDING REMARKS

The problem of stable feedback H_∞-control of linear discrete-time systems with periodic coefficients and corrupted with state dependent noise has been analyzed and illustrated for a small satellite using only magnetic coils as actuators. The main result states that the time-varying optimal state feedback gains are periodic functions expressed in terms of the solution of a specific system of linear matrix inequalities. The theoretical result is used for a detumbling application of a cubesat subject to parametric modeling uncertainties, aiming to stabilize it with a low control effort. Further developments will be devoted to synthesis of time-periodic optimal control laws for satellite formation.

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