A Modified Preference-Based Hypervolume Indicator for Interactive Evolutionary Multiobjective Optimization Methods

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Keywords: Multiple Objective Optimization, Interactive Methods, Performance Indicators, Region of Interest, Evolutionary Algorithms, Preference-Based Hypervolume.

Abstract: Various interactive evolutionary multiobjective optimization methods have been proposed in the literature for problems with multiple, conflicting objective functions. In these methods, a decision maker, who is a domain expert, iteratively provides preference information to guide the solution process while gaining insight into the problem. To compare interactive evolutionary multiobjective optimization methods, a preference-based hypervolume indicator (PHI) has been proposed to quantify the performance of the methods. PHI was the first indicator designed based on some desirable properties of indicators for interactive evolutionary multiobjective optimization methods. However, it has some shortcomings, such as excluding some potentially interesting solutions and being limited to consider a reference point as a type of preference information. In this paper, a modified indicator called PHI$^+$ is proposed to address the mentioned drawbacks. PHI$^+$ modifies the region of interest in PHI. While PHI is directed at methods where a decision maker provides preference information in the form of a reference point, PHI$^+$ is applicable for methods that utilize desirable ranges of objective function values as preference information. Therefore, PHI$^+$ is the first indicator that can handle preference information provided as desirable ranges when evaluating interactive methods. Experimental results show that PHI$^+$ can also better distinguish differences in the performance of interactive evolutionary multiobjective optimization methods.

1 INTRODUCTION

Many real-world problems involve multiple (conflicting) objective functions, and these problems are called multiobjective optimization problems (MOPs) (Sawaragi et al., 1985). For MOPs, it is typically impossible to find a solution where all objective functions can attain their optimal values. Instead, there are many compromise solutions, called Pareto optimal solutions (Sawaragi et al., 1985), representing different trade-offs between the objective functions.

When multiple Pareto optimal solutions exist, a decision maker (DM), a person with domain expertise of the problem being solved, is usually introduced to express preference information and determine the most preferred solution. If preferences of a DM are considered, multiobjective optimization methods can be divided into three categories (Miettinen, 1999): *a priori*, *a posteriori*, and *interactive* methods.

In *a priori* methods, a DM provides preference information before optimization. In contrast, *a posteriori* methods generate a representative set of Pareto optimal solutions to be considered. In interactive methods, a DM iteratively provides preference information to guide the solution process, and focus only on those Pareto optimal solutions that are of interest to the DM. Therefore, these methods can save computational resources and put less cognitive load on the DM at a time.

There are many interactive multiobjective optimization methods (Miettinen et al., 2008, 2016) that can be used to solve MOPs. Among them, evolutionary multiobjective optimization methods (Branke et al., 2008) are population-based methods that can be used to solve problems that have, e.g., non-differentiable and discontinuous objective functions. For compactness, in the following, we use the term

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Liang, M., Shavazipour, B., Saini, B., Emmerich, M. and Miettinen, K.
A Modified Preference-Based Hypervolume Indicator for Interactive Evolutionary Multiobjective Optimization Methods.
DOI: 10.5220/0012934600003837

In *Proceedings of the 16th International Joint Conference on Computational Intelligence (IJCCI 2024)*, pages 214-221
ISBN: 978-989-758-721-4; ISSN: 2184-3236

"methods" to refer to evolutionary multiobjective optimization methods since we focus on them.

Although many interactive methods have been published, it is difficult to quantify their performance due to the lack of appropriate quality indicators (Afsar et al., 2021). Aghaei Pour et al. (2022) proposed the desirable properties of indicators for interactive methods and the first indicator designed based on these desirable properties was proposed in (Aghaei Pour et al., 2024). It is called a preference-based hypervolume indicator (PHI) and it mainly calculates the hypervolume (HV) (Zitzler and Thiele, 1998) of solutions within a region of interest (ROI) defined using the preference information provided by a DM. This is assumed to be in the form of a reference point representing desirable objective function values.

We observed that certain solutions of potential interest to a DM are excluded from the definition of a ROI in Aghaei Pour et al. (2024), indicating that this definition does not adequately highlight solutions that reflect the preference information. This can influence the comparison. Furthermore, PHI can only be used to compare methods that involve a reference point.

In this paper, we modify the definition of the ROI in PHI, and call the resulting modified PHI as PHI$^+$. This enables us to more accurately understand the performance of the methods being evaluated. Importantly, PHI$^+$ is the first indicator that can be used to compare interactive methods with desirable ranges as the type of preference information. Unlike PHI$^+$, PHI requires setting a point of poor values to calculate HV values. Moreover, PHI values are limited to [0, 2], while PHI$^+$ values range from 0 to infinity. Thus, PHI$^+$ values allow us to distinguish more clearly differences in the performance of the methods.

2 BACKGROUND

A MOP (Sawaragi et al., 1985) can be expressed as:

$$\begin{aligned} &\text{minimize } \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_k(\boldsymbol{x})) \\ &\text{subject } \boldsymbol{x} \in S, \end{aligned} \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ is a decision variable vector in a feasible region S of the decision space $\mathbb{R}^n$. There are k objective functions $f_1, f_2, \ldots, f_k$ which map any feasible $\boldsymbol{x}$ to an objective vector $\boldsymbol{f}(\boldsymbol{x})$ in the so-called objective space $\mathbb{R}^k$. Additionally, while all objective functions are minimized in (1), objective functions to be maximized can be handled by multiplication by -1.

A solution $\boldsymbol{x}^1$ is said to dominate another solution $\boldsymbol{x}^2$ if $\boldsymbol{f}_i(\boldsymbol{x}^1) \leq \boldsymbol{f}_i(\boldsymbol{x}^2)$ for $i = 1, 2, \ldots, k$, and $\boldsymbol{f}_i(\boldsymbol{x}^1) < \boldsymbol{f}_i(\boldsymbol{x}^2)$ for at least one $i = 1, 2, \ldots, k$. A feasible solution is Pareto optimal, if it is not dominated in S. The image of a set of all Pareto optimal solutions is called a Pareto front (PF) in $\mathbb{R}^k$.

An ideal point $\boldsymbol{z}^* \in \mathbb{R}^k$ is a vector consisting of the lowest values for each objective found in a PF. On the other hand, a nadir point $\boldsymbol{z}^{nad} \in \mathbb{R}^k$ consists of the highest values found in a PF. It is usually approximated since the PF is not known Miettinen (1999).

Interactive methods ask a DM to iteratively provide preference information to guide the solution process. When observing solution processes with interactive methods, one can often notice two phases (Miettinen et al., 2008). The goal of a so-called learning phase is to allow a DM to explore different solutions and improve their understanding of problems until an ROI can be identified. The goal of the next phase, called a decision phase, is to fine-tune the search in the ROI to find a solution that satisfies the DM.

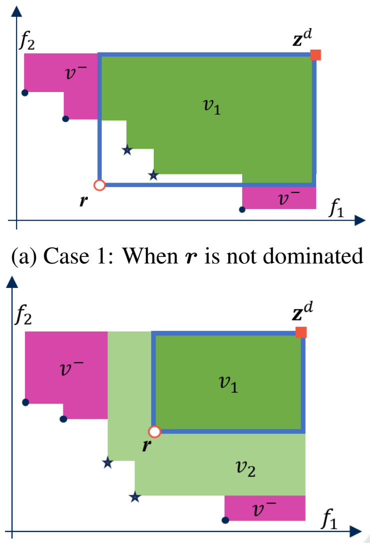
A common way to express preferences is to use reference points (Lárraga and Miettinen, 2022; Wierzbicki, 1980). A reference point $\boldsymbol{r} = (r_1, r_2, \ldots, r_k)$ is a vector in the objective space that represents desirable values for each objective function. Tanabe and Li (pear) identify three most commonly used definitions of ROIs in methods using reference points. They are ROIs based on a closest point, ROIs based on an achievement scalarizing function (ASF) (Wierzbicki, 1980) and ROIs based on the Pareto dominance relation.

Besides a reference point, other types of preference information can be used (Luque et al., 2011). An example is a desirable range of objective function values, where a DM provides information defining a range. The desirable ranges constitute an ROI (Hakanen et al., 2016; Manuel et al., 2022) that includes all Pareto optimal solutions that lie within the desirable ranges.

With an ROI based on the Pareto dominance relation, Aghaei Pour et al. (2024) proposed PHI to evaluate the performance of interactive methods. PHI uses an HV, and besides incorporating the concept of Pareto dominance, it also tries to capture the efficiency of utilizing computational resources by penalizing solutions that fall outside an ROI, i.e., solutions that do not reflect the reference point.

In Aghaei Pour et al. (2024), a so-called dystopian point $\boldsymbol{z}^d \in \mathbb{R}^k$ is defined as $z_i^d = z_i^{nad} + \varepsilon$, $i = 1, 2, \ldots, k$, with a very small constant $\varepsilon > 0$. It is used in the calculation of the HV.

We denote a set of Pareto optimal solutions generated by an interactive method as P. If P does not include any point that dominates the reference point $\boldsymbol{r}$, the ROI includes all solutions of P that are dominated by $\boldsymbol{r}$, and the set composed of these solutions is called P_1. Alternatively, if $\boldsymbol{r}$ is dominated by at least one solution in P, the ROI includes all solutions that dominate $\boldsymbol{r}$, and the set composed of these solutions is called P_2. In Figure 1, black dots represent solutions outside the ROI, black stars represent P_1 or P_2.



(a) Case 1: When $\boldsymbol{r}$ is not dominated

(b) Case 2: When $\boldsymbol{r}$ is dominated

Figure 1: Visualization of components of PHI for a bi-objective problem.

To define PHI, according to the definition of HV and $\boldsymbol{z}^d$, so-called positive parts and negative parts are defined as the green and pink areas in Figure 1, respectively. Formally, for a solution set P, the so-called negative contribution v^- can be expressed as:

$$v^- = HV(P \cup \{\boldsymbol{r}\}, \boldsymbol{z}^d) - HV(P_2 \cup \{\boldsymbol{r}\}, \boldsymbol{z}^d) \tag{2}$$

and the so-called positive contribution v^+ constituting of v_1 and v_2 can be defined as:

$$v_1 = \begin{cases} HV(P, \boldsymbol{z}^d) - v^-, & \text{if } P_2 = \varnothing \\ HV(\boldsymbol{r}, \boldsymbol{z}^d), & \text{otherwise,} \end{cases} \tag{3}$$

$$v_2 = \begin{cases} 0, & \text{if } P_2 = \varnothing \\ HV(P_2, \boldsymbol{z}^d) - v_1, & \text{otherwise,} \end{cases} \tag{4}$$

$$v^+ = v_1 + v_2 = \begin{cases} HV(P, \boldsymbol{z}^d) - v^-, & \text{if } P_2 = \varnothing \\ HV(P_2, \boldsymbol{z}^d), & \text{otherwise.} \end{cases} \tag{5}$$

Based on the negative and positive contributions, PHI can be expressed as:

$$\begin{aligned} PHI(P, \boldsymbol{r}, \boldsymbol{z}^d) &= \frac{v_1}{HV(\boldsymbol{r}, \boldsymbol{z}^d)} + \frac{v_2}{HV(P, \boldsymbol{z}^d)} \\ &= \begin{cases} \frac{v_1}{HV(\boldsymbol{r}, \boldsymbol{z}^d)}, & \text{if } P_2 = \varnothing \\ 1 + \frac{v_2}{HV(P, \boldsymbol{z}^d)}, & \text{otherwise.} \end{cases} \end{aligned} \tag{6}$$

Compared with indicators designed for *a priori* methods, PHI can provide more information about the method being evaluated. If at least one Pareto optimal solution dominates $\boldsymbol{r}$, then $\boldsymbol{r}$ is considered attainable. On the contrary, if $\boldsymbol{r}$ is not dominated, then it is considered unattainable. When the PHI value is in [0,1], we know that $\boldsymbol{r}$ is unattainable, and when the PHI value is greater than 1, we know that $\boldsymbol{r}$ is attainable. From (6), it can be seen that the higher the value of PHI, the better the performance of the method that generated P. The maximum value of PHI is 2.

3 NEW INDICATOR PHI$^+$

Before introducing the proposed indicator PHI$^+$, we modify ROI in the definition of PHI. In this way, we avoid some of the shortcomings of PHI.

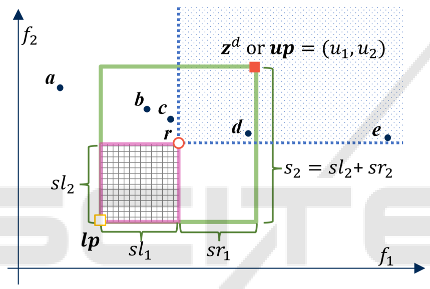
3.1 Modified Region of Interest

As stated in Section 2, the ROI of PHI is determined based only on the points that either dominate the reference point $\boldsymbol{r}$ or are dominated by it, but it falls short in considering points that are incomparable to the reference point. However, one can argue that incomparable solutions that are close to the reference point may also be of interest to the DM and should therefore have a positive contribution to the indicator value. For example, solutions b and c in Figure 2 (a), and solution b in Figure 2 (b) will not be included in the ROI (blue dot zone) as defined in PHI. However, a DM applying an interactive method is expected to learn and update their preferences. These close-by solutions may provide information of interest to the DM and should not be punished using a negative contribution.

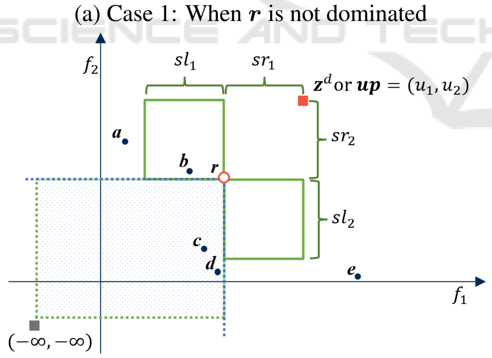
Moreover, if there are some solutions that are far away from $\boldsymbol{r}$ and dominated by $\boldsymbol{r}$, they will be included in the ROI, although the indicator for sets containing such solutions should indicate that these sets are not adequately addressing the preferences of the DM and that these solutions are not close to $\boldsymbol{r}$. Therefore, the definition of ROI that takes these situations into account may provide more insight into the performance of interactive methods applied by a real DM.

Taking into account other ROIs mentioned in Section 2, the ROI based on an ASF cannot be easily used for PHI because parameters required to calculate the value of ASF are not considered within the method to calculate PHI. To consider such an ROI in PHI, many additional parameters need to be defined which are difficult for DM to provide, so we do not use it in this paper. On the other hand, the ROI based on the closest point considers the solution closest to $\boldsymbol{r}$, that is, the solution that best satisfies the preferences. It is included in the ROI. This means that there is at least one solution in the ROI, even though all solutions may be far away from $\boldsymbol{r}$, and the DM may not be interested in all solutions. Moreover, if distances from other solutions to the closest point are all greater than the radius of this ROI, there is no guarantee that this ROI contains the majority of solutions that satisfy the DM's preferences. Another weakness of this ROI is that it is indifferent to Pareto dominance, and solutions dominating the reference point are ignored.

Since the current ROI definitions in the literature are insufficient for filtering solutions that accurately depict the performance of methods, they cannot be used in the context of PHI. Therefore, we introduce a modified ROI to overcome the mentioned limitations, and propose a new indicator PHI$^+$ as a variant of PHI utilizing the modified ROI.



(a) Case 1: When $\boldsymbol{r}$ is not dominated



(b) Case 2: When $\boldsymbol{r}$ is dominated

Figure 2: The modified ROI.

As illustrated in Figure 2, a DM may express the desirable range by providing a reference point and acceptable positive and negative deviations of it. The side length on the left side (or bottom side) of $\boldsymbol{r}$ and the side length on the right side (or top side) of $\boldsymbol{r}$ can be set separately. They can be referred to as sl_i and sr_i on each objective function, $i = 1, 2, \ldots, k$. In this paper, we use the same deviation on each objective function. Then $\boldsymbol{up}$ and $\boldsymbol{lp}$ are the upper right corner and lower left corner of the desirable range, respectively.

Once we have desirable ranges, we can modify the definition of a ROI as shown in Figure 2. When $\boldsymbol{r}$ is not dominated by any solution in P, as shown in Figure 2 (a), solutions in the green zone but not in the pink grid zone are included in the modified ROI. When $\boldsymbol{r}$ is dominated, as shown in Figure 2 (b), solutions in the green zone are included in the modified ROI. The area of the green zone on the lower left is infinite, which represents the zone where solutions dominating $\boldsymbol{r}$ are located.

The modified ROI contains all solutions that satisfy the DM's preferences. It also includes solutions that best satisfy the preferences.

3.2 Modified Preference-Based Hypervolume Indicator

Based on the modified ROI, we propose a modified PHI, and call it PHI$^+$, to address some shortcomings of PHI. If we use the modified ROI for PHI described in Section 2 and set a dystopian point $\boldsymbol{z}^d$ as mentioned in Section 2, when $\boldsymbol{r}$ is not attainable, the PHI value can be greater than 1. For example, in Figure 3, the HV value of solutions in a modified ROI is obviously greater than the HV value of $\boldsymbol{r}$, so the PHI value is greater than 1. In such cases, it is difficult to distinguish whether $\boldsymbol{r}$ is attainable through PHI values.

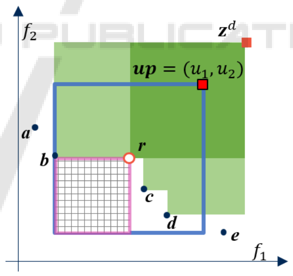


Figure 3: An example of using a modified ROI and setting a $\boldsymbol{z}^d$ that is different from $\boldsymbol{up}$ in PHI.

Moreover, when $\boldsymbol{r}$ is attainable, as in (6), the denominator in PHI depends on the method under evaluation, and the PHI value remains the same when the numerator and the denominator decrease or increase at the same rate. Since denominators are different in the PHI for different methods, we cannot directly know the differences in performance of the compared methods through the PHI values. Furthermore, when the PHI value reaches its maximum value, that is, when $\boldsymbol{r}$ is dominated and $\boldsymbol{r}$ is the same as $\boldsymbol{z}^d$, the PHI value will remain at 2 even if the HV value of solutions in the ROI changes.

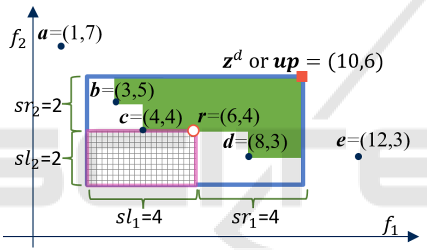
Therefore, to make the indicator work well with the modified ROI and to enable it to reflect more accurately the performance of the methods being evaluated, we propose a new indicator PHI$^+$ that is a variant of PHI. In PHI$^+$, $\boldsymbol{up}$ is used as a dystopian point $\boldsymbol{z}^d$ to calculate HV values, so unlike PHI, we do not need to set a point $\boldsymbol{z}^d$ separately. We refer to the set consisting of solutions in the modified ROI as P^+, and the HV value of solutions in P^+ as mv^+ defined as:

$$mv^+ = HV(P^+, \boldsymbol{up}). \tag{7}$$

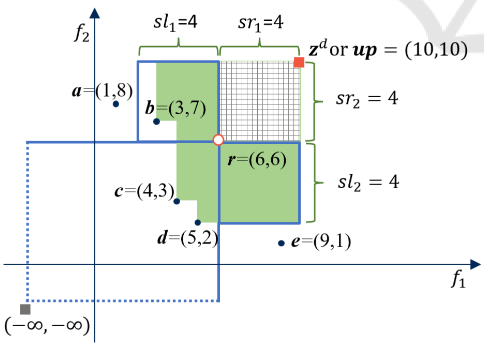
Based on mv^+, we define PHI$^+$ as:

$$PHI^+(P, \boldsymbol{r}, \boldsymbol{z}^d) = \begin{cases} \frac{mv^+}{HV(\boldsymbol{lp}, \boldsymbol{up}) - sl_1 \times sl_2 \times \cdots \times sl_k}, & \text{if } P_2 = \varnothing \\ \frac{mv^+}{HV(\boldsymbol{r}, \boldsymbol{up})}, & \text{otherwise.} \end{cases} \tag{8}$$

To allow PHI$^+$ to communicate similar information as the original PHI, that is, when $\boldsymbol{r}$ is attainable, the PHI$^+$ value exceeds 1, and when $\boldsymbol{r}$ is not attainable, the PHI$^+$ value falls below 1, we set different denominators depending on whether $\boldsymbol{r}$ is attainable or not.



(a) Case 1: When $\boldsymbol{r}$ is not dominated by any solution



(b) Case 2: When $\boldsymbol{r}$ is dominated by some solution

Figure 4: An example of the calculation of PHI$^+$.

When $\boldsymbol{r}$ is unattainable, no solution dominates it, for example, in the gray grid area in Figure 4 (a). To remove this area from the calculation and to make the indicator value more accurate, we subtract $sl_1 \times sl_2 \times \cdots \times sl_k$ from HV$(\boldsymbol{lp}, \boldsymbol{up})$. We calculate the PHI$^+$ value by dividing the HV value of the solution in the modified ROI by the HV value of $\boldsymbol{lp}$ excluding the region where no solutions exist.

In this case, when the diversity and convergence of solutions within the modified ROI is better, the PHI$^+$ value is higher, and the maximum PHI$^+$ value is 1. For example, in Figure 4 (a), using $\boldsymbol{up} = (10, 6)^T$ as $\boldsymbol{z}^d$ for the calculation of HV, the HV value of solutions in the modified ROI is 15, namely mv^+ is 15, the HV value of $\boldsymbol{lp}$ (green area) minus the gray grid region area is 24, so the PHI$^+$ value is 0.625.

When $\boldsymbol{r}$ is attainable, the PHI$^+$ value is calculated by dividing the HV value of the solution in the modified ROI by the HV value of $\boldsymbol{r}$. In this case, the better the diversity and convergence of the solutions within the modified ROI, the higher the PHI$^+$ value, and the value is higher than 1. For example, in Figure 4 (b), using $\boldsymbol{up} = (10, 10)$ as $\boldsymbol{z}^d$ for the calculation of HV, the HV value of solutions in the modified ROI is 50, namely mv^+ is 50, the HV value of $\boldsymbol{r}$ is 16, so the PHI$^+$ value is 3.125.

Unlike the original PHI, PHI$^+$ evaluates solutions that are close to $\boldsymbol{r}$ and incomparable to $\boldsymbol{r}$, the denominator is the same when $\boldsymbol{r}$ is attainable or $\boldsymbol{r}$ is unattainable. If all solutions are far from $\boldsymbol{r}$, resulting in no solutions within the modified ROI, PHI$^+$ values will not inaccurately represent the performance of methods. Thus, the PHI$^+$ values allow us to understand the performance differences between interactive methods more accurately and clearly. Furthermore, PHI can only be used for interactive methods with a reference point, while PHI$^+$ can be used for methods with desirable ranges as preference information. For this, a DM can provide a reference point and deviations of it.

4 NUMERICAL EXAMPLE

In this section, we illustrate the functionality of PHI$^+$ in assessing interactive methods, and compare it against PHI. We apply the Interactive RVEA method (Hakanen et al., 2016), referred to as IRVEA, which uses RVEA (Cheng et al., 2016) as the underlying evolutionary algorithm. We chose this method because the source code is openly available in the DESDEO framework (Misitano et al., 2021). Due to the page limitation, supplementary materials, and the source code of PHI$^+$ are available at https://optgroup.it.jyu.fi/material.php or https://doi.org/10.5281/zenodo.13587354.

4.1 Test Problem

We use the problem RE2-3-2, which has 2 objective functions and 3 decision variables, from the REal world problem (RE) suite (Tanabe and Ishibuchi,

2020) as a test problem, because it is built on real-world problems. We assume 6 iterations to be taken. As in Aghaei Pour et al. (2024), we consider two phases so that the first 4 iterations are in the learning phase, and the last 2 iterations in the decision phase.

For a fair comparison, we set the same number of generations in each iteration. We set the number of generations as 1 since we use it only to visualize how PHI$^+$ works and to avoid all solutions converging to one location. To make it more likely to obtain a feasible solution, we set the population size to 100. Since the ideal and nadir points of problems are provided in Tanabe and Ishibuchi (2020), we normalize function values before calculating the PHI$^+$ value to improve its accuracy. The dystopian point for PHI is set to the corresponding nadir point.

4.2 Optimization Process of RE2-3-2

To visually compare the differences between the way PHI and PHI$^+$ work, we show how they are calculated on the RE2-3-2 in Figure 5. In the visualization, the ROI of PHI is represented by the area surrounded by blue lines, the solutions that dominate $\boldsymbol{r}$ are in the square surrounded by blue solid lines (Figure 5 (a)), and the solutions dominated by $\boldsymbol{r}$ are in the square surrounded by blue dashed lines. The modified ROI of PHI$^+$ is indicated by the area enclosed by the blue line, excluding the gray grid area (Figure 5 (c)).

Naturally, the preferences depend on the solutions of the previous iteration. For compactness, we list them as follows:

a) Iteration 1: The DM sets desirable ranges, we derive $\boldsymbol{r}$ = (25, 12.5) and the deviations on objective functions are 25 and 12.5 from there. The DM express the initial preference to understand what types of solutions may exist.

b) Iteration 2: The DM wants to learn more about functions, and sets desirable ranges from which we derive $\boldsymbol{r}$ = (160, 10) and the deviations on objective functions are 25 and 12.5. Based on the solutions found in Iteration 1, the DM sets the preference information to see more solutions in the ROI.

c) Iteration 3: The DM sets new desirable ranges that correspond to $\boldsymbol{r}$ = (20, 100) and the deviations on objective functions are 25 and 12.5, to study more the first objective function. The DM is interested in learning about different parts of the PF. Therefore, based on the solutions seen in Iteration 2, they provide new preference information to see more solutions in the new ROI.

d) Iteration 4: The DM sets desirable ranges leading to $\boldsymbol{r}$ = (10, 5) the deviations on objective functions are 10 and 5, expecting to see lower values on objective functions. Same as Iteration 3, the DM is interested in seeing more solutions in a different part of the PF.

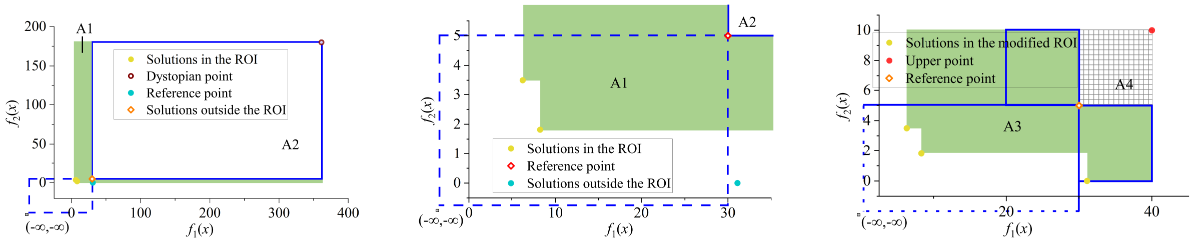
e) Iteration 5: The DM provides desirable ranges to see if the solutions obtained meet their expectations, and we get $\boldsymbol{r}$ = (30, 5) and the deviations on objective functions are 10 and 5. In the previous iteration, the DM has found the region that they are most interested in. They set the new preference information to focus more in the same (and nearby) region and find more solutions of interest.

f) Iteration 6: The DM sets desirable ranges corresponding to $\boldsymbol{r}$ = (5.9, 0) and the deviations on objective functions are 10 and 5, expecting to see lower values on objective functions. The DM found that their preferences were pessimistic and easily achieved in Iteration 5. They provide new preference information based on the knowledge gained to find more desirable solutions.

We have no room for studying all iterations, but to demonstrate differences between PHI and PHI$^+$ in the decision phase of IRVEA solving RE2-3-2, we observe their performance in Iterations 5 and 6. In the 5th iteration, when $\boldsymbol{r}$ is attainable, as shown in Figure 5 (a), the PHI value is $(1 + A1/(A1 + A2))$, that is, one plus the volume of the green area divided by the sum of volumes of white and green areas. As shown in Figure 5 (c), the PHI$^+$ value is $(A3 + A4)/A4$, that is, the sum of volumes of green and gray grid areas divided by the volume of the gray grid area.

In Figure 5 (a), since $\boldsymbol{z}^d$ is far away from $\boldsymbol{r}$, all solutions appear to be close to $\boldsymbol{r}$. To clearly show the distance between them, we zoom in a part of Figure 5 (a) and show the resulting image in Figure 5 (b). As shown in Figure 5 (b) and 5 (c), there is a solution that is close to $\boldsymbol{r}$ and incomparable to $\boldsymbol{r}$. It may provide information of interest to the DM. This solution is in the modified ROI for PHI$^+$ but not in the ROI for PHI, and it is evaluated in PHI$^+$ but not evaluated in PHI. Therefore, it can be seen that PHI$^+$ can more accurately represent the performance of the method.

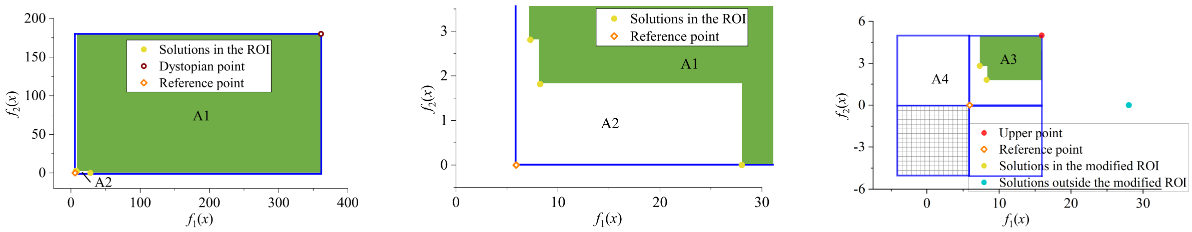
We show the PHI and PHI$^+$ values of the six iterations with IRVEA in Table 1. In the 5th iteration in Table 1, the PHI value is 1.083, and the PHI$^+$ value is 5.779. When $\boldsymbol{r}$ is attainable, the PHI value is limited to [1,2], the PHI$^+$ value ranges from 1 to infinity. As all the calculations were done after normalization of objective function values, the difference in the attained PHI and PHI$^+$ values is due to the denominator used in their calculations. PHI$^+$ has a smaller denominator, emphasising performance differences due to small changes in the solution sets. Therefore, the PHI$^+$ can more clearly measure the differences in the performance between the methods than the PHI.



(a) Calculation of PHI in iteration 5

(b) Calculation of PHI in iteration 5 (Partially enlarged)

(c) Calculation of PHI$^+$ in iteration 5



(d) Calculation of PHI in iteration 6

(e) Calculation of PHI in iteration 6 (Partially enlarged)

(f) Calculation of PHI$^+$ in iteration 6

Figure 5: Pareto optimal solutions obtained by IRVEA on RE2-3-2.

In the 6th iteration, when $\boldsymbol{r}$ is unattainable, as shown in Figure 5 (d), the PHI value is $(A1/(A1+A2))$, that is, the volume of the green area divided by the sum of volumes of the white and green areas. As shown in Figure 5 (f), the PHI$^+$ value is $A3/(A3+A4)$, that is, the volume of the green area divided by the sum of volumes of the white and green areas. To clearly show the distance between solutions, we zoom in a part of Figure 5 (d) and show the resulting image in Figure 5 (e). In Figures 5 (e) and 5 (f), there is a solution that is far away from $\boldsymbol{r}$ and dominated by $\boldsymbol{r}$, so the DM may not be interested in it. And it is not in the modified ROI of PHI$^+$ but it is in the ROI of PHI. Thus, it is not evaluated in PHI$^+$ but evaluated in PHI. Therefore, PHI$^+$ can more accurately represent the performance of methods.

Table 1: The PHI and PHI$^+$ values of IRVEA on RE2-3-2.

Iteration	**1**	**2**	**3**
PHI	1.093990076	1.461817041	1.55402991
PHI$^+$	3.104009333	12.48452761	13.512463
Iteration	**4**	**5**	**6**
PHI	1.02790275	1.082717159	0.995462552
PHI$^+$	2.182101576	5.779339391	0.176329054

In the 6th iteration in Table 1, the PHI value is 0.995 and the PHI$^+$ value is 0.176. The large difference is because, when $\boldsymbol{r}$ is not dominated, the ROI of PHI is much larger than the ROI of PHI$^+$. As this is the final iteration of the solution process, the DM is confident about the solutions they wish to obtain, and thus have a good intuition about the bounds of their ROI. PHI$^+$ more accurately captures this information, due to the more strictly bounded nature of its ROI, compared to PHI. Therefore, PHI$^+$ is more sensitive to changes in the solution set when $\boldsymbol{r}$ is unattainable as well, leading to a more accurate measurement of performance differences between different methods.

5 CONCLUSIONS

In this paper, we discussed the limitations of the only quality indicator, PHI, developed for evaluating the performance of interactive evolutionary multiobjective optimization methods based on their desirable properties. Furthermore, we proposed a new indicator PHI$^+$ by modifying the ROI that is an element of the indicator, and the indicator formulation to address the limitations of PHI. Compared to PHI, PHI$^+$ can better evaluate the performance of interactive methods, and it can be used to compare methods with desirable ranges as preference information.

In addition to the method of defining the modified ROI in Section 3.1, there is another way to determine the modified ROI only when the deviations on each objective function are the same. The preference information provided by a DM can be a reference point and the upper bounds of acceptable objective function values. The modified ROI includes this range and the extension range that includes solutions better than solutions in this range.

Overall, PHI$^+$ follows all the desirable properties possessed by PHI, and PHI$^+$ is more sensitive to small differences in the solution sets found by different methods than PHI. Furthermore, we can directly understand the differences between the compared methods through PHI$^+$ values. The experimental results show that PHI$^+$ can more clearly reflect the changes in the solutions within the ROI.

Although PHI$^+$ has more advantages than PHI, there is still room for further improvement. An aspect to explore further is the situation where multiple methods do not obtain a solution in the modified ROI. In this case, it is not easy to distinguish these methods because PHI$^+$ values for them are all zero. Therefore, our next step is to evaluate solutions outside the modified ROI so that PHI$^+$ can work properly even when there is no solution in the modified ROI.

Additionally, to provide information in the PHI$^+$ values about the attainability of the desirable ranges, we had to accept some discontinuity in the PHI$^+$ values near 1. From our observations, resolving this discontinuity without harming other valuable properties is challenging. Therefore, this is also our future work.

ACKNOWLEDGEMENTS

This research has received part of the funding from the European Union – NextGenerationEU instrument and was therefore partly funded by the Research Council of Finland, grant number 352784, partly by grant number 355346 of the same Council and is related to the thematic research area Decision Analytics utilizing Causal Models and Multiobjective Optimization (jyu.fi/demo) of the University of Jyvaskyla.

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