Decision of Production Combinations Based on Cournot Model and Linear Programming

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Keywords: Cournot Model, Linear Programming, Market Competition, Strategic Decisions.

Abstract: This paper explores production strategies for micro electric vehicle manufacturers in small cities using a revised Cournot model and linear programming. Focusing on cities like Liuzhou, Nanyang, and Shangqiu, where market dynamics favour oligopolies, the traditional Cournot model is adapted to emphasize cost-based strategies. By integrating unit costs as variables representing service levels, the model examines competition among manufacturers providing different service levels within the same market. A payoff matrix and linear programming determine optimal strategies for maximizing profits while maintaining market stability. The study highlights the importance of strategic decisions in cost control, brand positioning, and supply chain management. Results indicate that both firms should adopt mixed strategies to enhance competitiveness and profitability. The paper suggests future research to refine the model for more complex market scenarios, including additional competitors and dynamic market conditions. This research provides insights into market competition strategies and lays the groundwork for more nuanced economic and mathematical analyses.

1 INTRODUCTION

In recent years, the surge in popularity of micro electric vehicles in smaller cities has presented a unique set of market dynamics, particularly evident in cases such as Liuzhou, Nanyang and Shangqiu, which usually emerges in small cities. From 9.7% in 2017 to 53.3% in 2022, Liuzhou's new energy vehicle market penetration rate has been steadily increasing (Yunjing, 2023). Where a monopolistic trend is emerging. In many of these cities, oligopoly and even monopoly market is observed.

This phenomenon has sparked interest among researchers aiming to understand the implications of such market structures. On the on hand, for companies, which strategies they should adopt to achieve their goals, like maximize their profits and put the competitors at a disadvantage, need to be clarified. On the other hand, for the market, whether a stable state, or technically, a Nash equilibrium exist is essential to define whether this market is efficient.

The Cournot model, a classic economic framework for analyzing oligopoly markets, becomes highly relevant in this context. The Cournot duopoly model, or Duopoly model, is another name for the Cournot model. An early example of an oligopoly is the Cournot model. The French economist Cournot first put up the idea in 1838 (Yan, Da and Pei, 2013). The first use of the Nash equilibrium is the Cournot model, which is frequently utilized as a jumping off point for oligopoly theory study (Tang, 1997). According to the Cournot model, there are just two sellers of a product in the market, and they don't cooperate with one another. Instead, they anticipate each other's actions and decide on the best output to maximize profits (Du, 2005). The Cournot model's conclusions can be applied with ease to situations involving three or more oligopolistic manufacturers (Zhang and Xiao, 2003). Studies such as those by prominent scholars have applied it to similar markets, emphasizing the strategic behaviors of oligopolies.

Given the niche nature of micro electric vehicle markets in smaller cities, the sales volume tends to be relatively stable due to the limited market size. Despite the homogeneity in product characteristics, manufacturers can still strategize by varying the level of services provided with their products. For example, in the case of similar models, companies can achieve product differentiation by providing customization service, such as customized auto parts. Shortening

Li, H., Li, Y. and Wu, X.
Decision of Production Combinations Based on Cournot Model and Linear Programming.
DOI: 10.5220/0013001000004601

In *Proceedings of the 1st International Conference on Innovations in Applied Mathematics, Physics and Astronomy (IAMPA 2024)*, pages 119-125
ISBN: 978-989-758-722-1

delivery time can also enhance the competitiveness of enterprises. However, higher service level requires higher cost. These costs will be shared into the cost of every car. Hence, the service level is quantified in terms of cost. It suggests a significant shift in market modeling, moving from quantity-based variables to cost-based ones. In the traditional Cournot model, two producers have identical items with linear demand curves, and one decides how to maximize profit by varying the volume of sales dependent on the other's actions (Yuan et al., 2003). Thus, the traditional Cournot model with sales volume as variable is no longer applicable. A research based on revised Cournot model is both necessary and timely, aiming to delve deeper into how these cost-based strategies affect market dynamics and competition among few but powerful players in small yet pivotal market segments.

Actually, the researchers have noticed these cases and made some attempts. A famous model named the Bertrand model was proposed by French economist Joseph Bertrand in 1883. Unlike the Cournot model, the Bertrand model describes price competition rather than quantity competition (Tremblay and Tremblay, 2019). In this model, it is assumed that there are two or more firms selling identical or non-differentiated products and that they compete on price. When setting prices, firms must consider their competitors' reactions and anticipate their potential pricing strategies. Consumers will buy from the firm offering the lowest price. In equilibrium, each firm's price equals its marginal cost. If a firm's price is above its marginal cost, it will lose market share.

There are also researchers combines the Cournot model with Bertrand model. They believe that firms can decide to compete in both cost and quantity. It appeared in early studies as “mixed oligopoly", a model with a “mixed equilibrium”, or a “mixed strategy setting" (Bylka and Komar, 1976 & Singh and Vives, 1084). In some later studies, it is referred as “Cournot–Bertrand Model” discussed a duopoly where competitors can adjust their output or price (Sato, 1996 & Correa-Lopez, 2007). However, these models do not fit the market discussed by this paper very well, either. On the one hand, customers’ choices are not always the products with the lowest price. Customers have various preference based on the quality and the brand of the products. On the other hand, to maintain the brand image, there exists a satisfied threshold of customers a firm should keep. Therefore, a revised model is needed (Maggi, 1996).

The objective of this paper is to build a revised model based on Cournot model model to calculate the best strategies for companies in a market segment and analyze the Nash equilibrium of it. In this model, Unit costs of the products are regarded as variables, which is able to measure the service level of similar products. To achieve this goal, a payoff matrix of a firm in this market is derived first based on the revised model. Then, a linear programming model is built, considering the satisfaction constraints. To solve this model, duality theory is considered. The dual problem not only ensures the primal problem can be solved, but at the same time derives a Nash equilibrium. Finally, examples which are closed to reality, related data and python algorithms will be used to assess the validity and reliability of the models. The model can help provide valuable advice and insights for companies to make informed decisions and estimate the final balance of a certain market segment.

2 METHODOLOGY

2.1 Data Source

There are many factors in the market model that are difficult to analyze with available data. It is difficult to do regression analysis to obtain continuous quantified factors. Thus, to facilitate the establishment of mathematical models and calculations, this paper quantifies the factors involved in this model based on the basic rule of the market, instead of using certain direct data. To ensure the authority and accuracy of the data, current prices and strategies in the market are also considered as reference (Ma et al., 2018). This paper especially refers to the markets of Liuzhou and Nanyang, and get the picture in Figure1 after investigation, the top two brands, which are the blue and orange parts, occupy the majority of the market share in both Liuzhou and Nanyang.

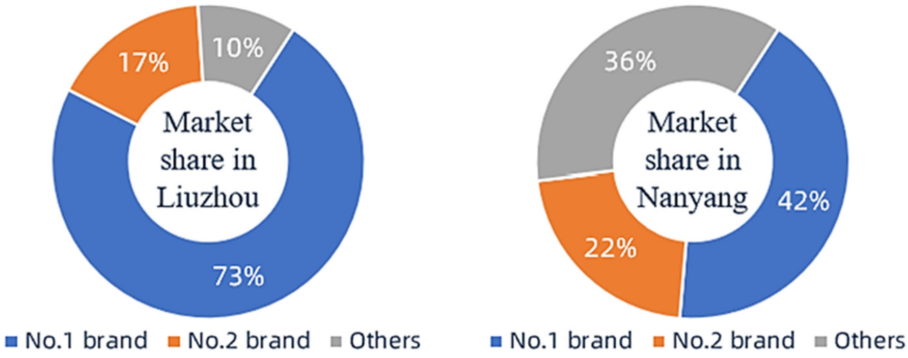

Figure 1: Market Shares in Liuzhou and Nanyang.

2.2 Variable Introduction

Firstly, In the oligopolistic market, similar but different products are selling. The differences between these products are not indicated in the product itself, but in the related service level. For instance, the products with higher level of customization, quick response and excellent after-sales service will be the high-end product in this market. In this model, there are two firms. Firm 1 focuses on high-end products while Firm 2 focuses on the low-end ones. The firm's investment in service levels is spread over each product. Therefore, the service level of the product is measured by the unit product cost.

Secondly, due to the different positioning of the firms, customers have certain expectations for the strategies adopted by the firm. Part of that expectation is not adjustable. The non-adjustable part represents the bottom line of the company's image maintenance, which the company must meet. For example, once a company with a high-end positioning sells a product with a very low positioning, old customers who pursue high quality will be disappointed and stop buying the company's products. The other part of expectations is elastic. Although the company will face the loss of certain expectations that are not met, the company can make up for the loss with the profit resulting from this adjustment. In addition, the budgets of the firms are different since the sizes of them are different.

2.3 Method Introduction

To make the description clear, the example of Firm 1 producing Product 1 is considered. Since the objective of a firm is to maximize their profit, a system of functions needs to be built. The researchers considered different parameters for the firm and the product, such as the firm's production budget, the product's unit price, and the market expectation.

For Firm 1, the budget b that it has for production is a constant. If it only produces Product 1, the relation between the cost of Product 1 and the quantity of them can be represented as below. In the equation (1), c_1 is the cost of Product 1. The quantity of Product 1 is q_1.

$$b = c_1 q_1 \tag{1}$$

The price of Product 1 (P_1) is affected by its unit cost c_1 and its expected production q_0. Specifically, if the firm produced more products than it is expected, the price of the product will drop, and vice versa. The researchers consider these effects to be linear, respectively controlled by two coefficients λ_1 and k_1.

$$P_1 = \lambda_1 c_1 - k_1(q_1 - q_0) \tag{2}$$

The profit of the firm by selling only Product 1(π_1) can be calculated as below.

$$\pi_1 = P_1 q_1 - c_1 q_1 \tag{3}$$

Then, by solving the simultaneous equations, the final expression of π_1 is obtained.

$$\pi_1 = b\lambda_1 - \frac{k_1 b^2}{{c_1}^2} + \frac{k_1 q_0 b}{c_1} - b \tag{4}$$

The profit function is of unit cost. Thus, the researchers set the derivative of the function equal to zero to find the optimal unit cost that maximizes profit. This represents the optimal pure strategy for Product 1. The whole process is shown in Figure 2.

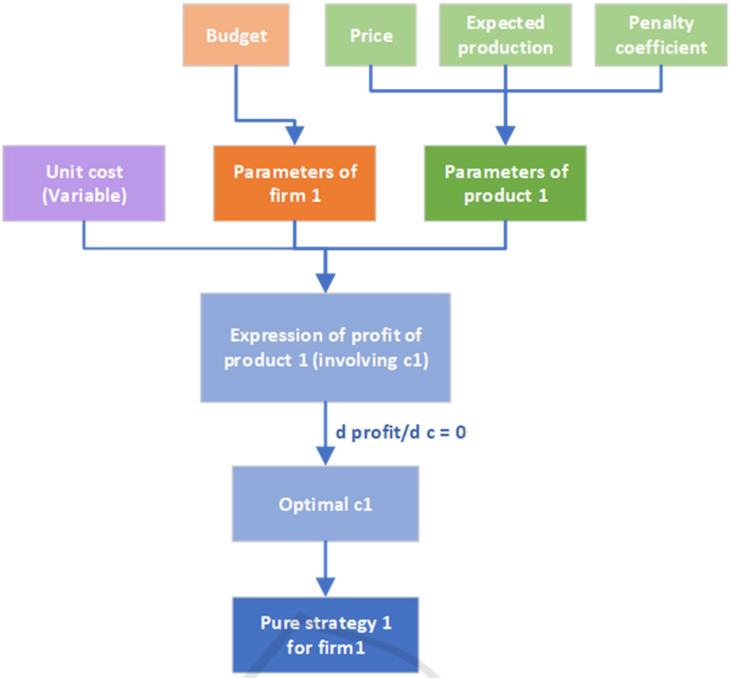

Figure 2: Pure Strategy Model.

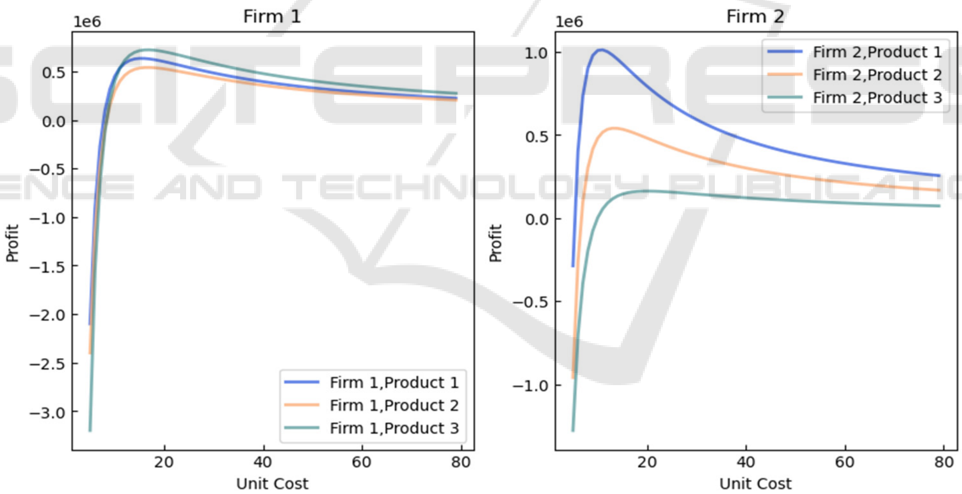

Figure 3. Relationship Between Unit Cost and Profit.

3 RESULTS AND DISCUSSION

3.1 Preliminary Work

Similarly, other pure strategies can be calculated. In the model, Firm 1 is high-end and Firm 2 is low-end. The differences between them are reflected in the parameter. For instance, the budgets of the firms are different since the volume of them are different. The customers' expectation of different products is different since the firm's image determines their market. By changing the parameters, the pure strategies for Firm 2 can be found. How the profits of companies producing different products vary with changes in cost are shown by Figure 3. For a certain product, the cost that corresponds to the highest profit point is the best pure strategy.

Recognizing that a company wouldn't produce just one type of product, the researchers mixed these pure strategies in proportion to form mixed strategies.

The proportions of the mixed strategies are shown in Table 1. The first row indicates that in strategy 1, 20% of the budget is allocated to producing Product 1, 40% to Product 2, and the remaining 40% to Product 3. Similar for the rest.

It is important to point out that the unit cost of Product 1 is the lowest, Product 2's is middle and Product 3's is the highest. This means that the quality of the products is higher with the increase of their index. Thus, mixed strategy 1 with emphasis on production of Product 2 and Product 3 is a high-end strategy.

Table 1: Mixed Strategy.

	Product 1	Product 2	Product 3
Strategy 1	0.2	0.4	0.4
Strategy 2	0.3	0.5	0.2
Strategy 3	0.5	0.3	0.2

As mentioned earlier, a firm's goal is to maximize profits while putting their competitors at a disadvantage. The payoff is defined as their profit minus the competitor's profit.

$$Payoff\ of\ Firm\ 1 = \pi_1 - \pi_2 \tag{5}$$

With the previously derived profit expression, the payoff matrix of Firm 1 is obtained.

Table 2: Payoff Matrix.

	Strategy 1	Strategy 2	Strategy 3
Strategy 1	1484540	254760	-689600
Strategy 2	1214890	-14890	-959250
Strategy 3	1401590	171810	-772500

Using the data above, the researchers came up with the following linear programming model (Table 2). A_1 is the payoff matrix of Firm 1. By pre-multiplying transpose of Firm 1's strategy vector x and post-multiplying Firm 2's strategy vector y, the payoff of Firm 1(u) under certain strategies are determined.

For Firm 2, not knowing which strategy Firm 1 will take, it also in the face of determine a strategy-choosing problem. For a given x, Firm 2 will take the strategy that minimizes u to put Firm 1 at disadvantage. Thus, the objective of Firm 2 is to minimize u by choosing y.

$$Minimize\ u = x^T A_1 y \tag{6}$$

In addition, y is under several constraints: Summation of the proportions of the strategies equals 1. The strategy Firm 2 take should meet the satisfaction constraint. The proportions of the strategies should be greater than or equal to 0.

The satisfaction matrix of Firm 2 is S_2. The satisfied threshold of customers for Firm 2 is c_2. Then the constrains of the linear programming model can be written as, subject to:

$$\sum_{i=1}^{3} y_i = 1 \tag{7}$$

$$S_2 y \geq c_2 \tag{8}$$

$$y \geq 0 \tag{9}$$

For Firm 1, it will choose the strategy that will maximize its payoff. With the model above, x can be estimated. However, u is not a linear function. To solve the problem, linear programming duality is used.

The dual problem is obtained by using linear programming duality as below:

$$Maximize\ v = x_0 + c_2 x_0' \tag{10}$$

Subject to,

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & S_{2\_1} & -A_{1\_1,1} & -A_{1\_2,1} & -A_{1\_3,1} \\ 1 & S_{2\_2} & -A_{1\_1,2} & -A_{1\_2,2} & -A_{1\_3,2} \\ 1 & S_{2\_3} & -A_{1\_1,3} & -A_{1\_2,3} & -A_{1\_3,3} \\ 0 & 0 & S_{1\_1} & S_{1\_2} & S_{1\_3} \end{bmatrix} \begin{bmatrix} x_0 \\ x_0' \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{matrix} = \\ \geq \\ \geq \\ \geq \\ \geq \end{matrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ c_2 \end{bmatrix} \tag{11}$$

$$x_0 \leq 0 \tag{12}$$

$$x_{1,2,3} \geq 0 \tag{13}$$

In the dual problem, Firm 1 maximizes its own payoff while meeting its own market satisfaction criteria, represented by the satisfaction matrix S_1. S_{i_j} represents the jth element in S_i. $A_{1\_i,j}$ represents the element in A_1 that is in the ith row and the jth column. Now, the problem can be solved by calling linprog in Python.

3.2 Sensitivity Analysis

In this analysis, researchers performed a sensitivity analysis on a Cournot competition model implemented using linear programming. The primary goal was to understand how small changes in key parameters affect the model's outcomes. Specifically, researchers increased and decreased certain parameters by 10% to observe the impact on the model's results.

After adjusting the parameters and rerunning the model, the results showed minimal changes in the outcomes. Below are the comparative results for the original parameters, +10% adjustment, and -10% adjustment. The Cournot competition model used here is a linear programming model, which means it optimizes a linear objective function subject to linear constraints. Linear models tend to be more stable and less sensitive to small parameter changes compared to non-linear models. Adjusting parameters by 10% might not be significant enough to push the system into a different region of the feasible solution space. In other words, the feasible region defined by the constraints may not change much with these small adjustments, resulting in similar optimal solutions.

The Nash equilibrium in the Cournot model represents a stable state where firms have optimized their strategies given the strategies of their competitors. This equilibrium tends to be robust to small perturbations in parameters, meaning that small changes in costs or budget do not significantly alter the strategic interactions and outcomes.

3.3 Payoff Matrix and Mixed Strategy

Solving the problem with the payoff matrix and mixed strategy, the final strategies are shown in Table 3. As a result, Firm 1 will choose mixed strategy 2, and Firm 2 will choose mixed strategy 1.

3.4 Competitive Strategies Analysis

In today's fiercely competitive business environment, companies employ well-crafted competitive strategies to enhance their market position and profitability. These strategies not only impact a company's immediate benefits but also shape its long-term development. For example, Firm 1 employs a Cost Leadership and Price War Strategy, which has its advantages and disadvantages. The advantages include economies of scale, where scaling production spreads fixed costs over more units, reducing the cost per unit and gaining a competitive edge in price-sensitive markets, and rapid market penetration, where lower price points attract a large customer base quickly, effectively expanding market share. However, the risks involve profit margin compression, as long-term low pricing can lead to sustained decreases in profit margins, especially when raw material or production costs rise, and brand value dilution, where continuous price wars may degrade the brand's perceived value, making it difficult to raise prices or expand into higher-end product lines later.

On the other hand, Firm 2's Brand Positioning and Customer Loyalty Strategy also comes with its own set of advantages and risks. The advantages include the ability to charge a premium for products due to strong brand influence, achieving higher profit margins, and enhanced customer retention, where increased customer satisfaction and emotional connection lead to repeat purchases and new customer referrals. However, the risks include high costs associated with maintaining a brand image and improving customer service, involving significant marketing and advertising expenses, and poor market adaptability, where an overemphasis on a specific brand position may limit the company's ability to adapt to market changes.

Lastly, the importance of supply chain management cannot be overlooked. Advantages include cost efficiency, where optimized supply chain operations reduce the cost of acquiring raw materials and enhance production efficiency, and market responsiveness, where a flexible and efficient supply chain allows for quick adaptation to market demand changes, reducing inventory backlog and increasing customer satisfaction. However, risks include supply chain disruptions, where dependence on single or key suppliers includes risks of production halts and logistical delays, and raw material price volatility, where uncertainty in raw material costs can lead to budget overruns.

Table 3: Final Strategies.

	x_0	x_0'	Mixed Strategy 1	Mixed Strategy 2	Mixed Strategy 3
Firm1	1.22×10^6	0	0	1	0
Firm2	1.95×10^6	-3.15×10^5	1	0	0

4 CONCLUSION

This research provides a comprehensive examination of strategic approaches within micro electric vehicle markets, utilizing a modified Cournot model alongside linear programming to assess competition dynamics in small cities. By integrating service levels and costs into the Cournot framework, researchers reveal that firms can enhance their competitive edge and profitability through strategic diversification. The study confirms that both high-end and low-end market strategies can coexist successfully by balancing service quality and cost-effectiveness. This paper's findings also suggest that mixed strategies, blending different levels of service and pricing, allow firms to optimize their market presence and financial performance effectively. Looking ahead, further refinement of this model is recommended to include a broader range of competitors and dynamic market conditions, which would provide deeper insights into the complexities of market competition and strategy optimization. This research not only underscores the utility of advanced economic and mathematical tools in market analysis but also sets the stage for future studies aimed at evolving these methodologies for more comprehensive market scenarios.

AUTHORS CONTRIBUTION

All the authors contributed equally and their names were listed in alphabetical order.

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