



Local Motion Planning for Overtaking Maneuvers in a Rural Road Environment

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Abstract: This paper introduces an application of local motion planning designed explicitly for overtaking maneuvers in a rural road environment. The approach integrates multiple driving strategies for enhanced passenger comfort, including the fastest path and minimum jerk trajectory. A robust trajectory planner technique is developed using the Frenet frame, effectively considering real traffic situations, curves, and moving obstacles. Comprehensive analyses are performed on vehicle dynamics, individual cost function components, and planning and tracing times to assess the performance and computational efficiency of the proposed methods. The simulation results highlight the approach's strengths in maintaining dynamic feasibility, ensuring safety, and enhancing passenger comfort while identifying areas for potential improvements, such as computational overhead in complex scenarios.

1 INTRODUCTION

Autonomous vehicle systems must reliably and accurately plan trajectories in real-time within dynamic and changing environments, considering obstacles and moving goals. Achieving this requires a comprehensive, hierarchical software system that integrates multiple layers (Paden et al., 2016), each essential for ensuring the vehicle's safe and efficient navigation.

As the first step in the motion planning process, observing the vehicle's environment and representing it as a map is necessary (Bresson et al., 2017). Subsequently, the vehicle must determine a global route that avoids static obstacles while considering various constraints, such as the vehicle's kinematic limitations and the road's geometry. While traversing the global path, the vehicle has to make decisions at specific points and then plan its local trajectories (Schwartz et al., 2018). In the process of doing so, the vehicle must generate and periodically update a local


path that follows the global route as a reference, considering real-time constraints, the vehicle's dynamics, and dynamically moving objects. The term "local" indicates that this path is short-term, and the planning occurs in the vehicle's local coordinate system. The vehicle must follow the planned local path using control algorithms that determine longitudinal and lateral control inputs. All of this must be performed in real-time, accounting for uncertainties and the vehicle's continuously changing environment.


This paper presents a method for local trajectory planning in a Frenet frame to implement a lane-keeping and overtaking maneuver.


1.1 Related Work


The problem of trajectory planning and control for autonomous vehicles, particularly in overtaking scenarios, has been addressed through various innovative approaches. These efforts balance multiple objectives, such as safety, smoothness, and efficiency, while considering practical constraints and dynamic environments.

Several methodologies illustrate these concepts.

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For instance, (You et al., 2015) proposed a cooperative vehicle infrastructure system for improving lane change maneuvers through real-time adjustments based on vehicle dynamics. Similarly, (Palatti et al., 2021) developed a risk assessment and decision-making framework using a finite state machine to enhance overtaking safety. During motion planning for overtaking situations, the authors used a graph-based method in (Hegedűs et al., 2020) that reduces complexity by clustering and predicts the movement of surrounding vehicles with density functions, ensuring a safe and comfortable trajectory.

Model Predictive Control (MPC) has been extensively applied to autonomous vehicle overtaking. Studies by (Li et al., 2023) and (Batkovic et al., 2022) demonstrate MPC's real-time optimization of trajectories, accounting for dynamic road conditions. Additionally, (Dixit et al., 2020) integrated potential fields and reachability sets with MPC for high-speed overtaking on highways.

Distributed motion planning techniques also show promise. (Wu et al., 2021) and (Kala and Warwick, 2014) emphasize decentralizing decision-making to improve traffic safety and system responsiveness. (Xie et al., 2022) extended this approach with the Artificial Potential Field method for multi-vehicle environments.

Reinforcement learning (RL) is another advanced methodology. A continuous reinforcement learning method was developed to determine the trajectory of the double lane change maneuver (Fehér et al., 2020). The real-time solution was compared with the performance of human drivers. In (Lelekó and Németh, 2024), the authors present a control framework that combines a robust H_∞ controller and an RL agent to ensure the safe movement of autonomous vehicles. (Kulathunga, 2022) and (Wang et al., 2023) highlighted RL's effectiveness in improving decision-making and trajectory planning, with significant success in the Frenet coordinate system. Similarly, (Huang et al., 2023) proposed a multiobjective optimization algorithm within the Frenet frame to enhance driving comfort and safety.

Virtual target-based algorithms are also notable. (Chae and Yi, 2020) developed a method incorporating human driving behavior for improved driver acceptance and safety. (Ghumman et al., 2008) proposed a rendezvous guidance-based trajectory generation approach for real-time safety and comfort.

Dynamic trajectory planning within the Frenet coordinate system remains crucial. (Wang et al., 2019) and (Paden et al., 2016) explored comprehensive surveys and hierarchical urban and highway driving frameworks, focusing on safety and consistency.

(Moghadam and Elkaim, 2021) further developed a hierarchical framework combining long-term and short-term trajectory optimization.

For specific scenarios involving frequent acceleration and deceleration, (Zhang et al., 2019) presented an optimal trajectory generation method considering centripetal acceleration constraints, beneficial for curvy roads.

1.2 Contributions of the Paper

As a contribution, we propose a method to plan an optimal local trajectory for lane-keeping and overtaking maneuvers in a rural road environment. In each planning step, the planner generates several alternative trajectories in the feasible range and assigns a cost to each of them. The optimization objective can be multiple and is achieved by weighting the cost function. The method is designed to be easily integrated into a hierarchical approach to vehicle decision control structure. In order to demonstrate and test the method, a local planning solution was integrated into a self-developed simulation environment.

Section 2 offers a detailed formulation of the problem and essential topological information pertinent to the task. In Section 3, we introduce the Hierarchical Control Structure utilized in the construction of our system. Following this, Section 4 outlines the local trajectory generation process within the Frenet Frame, conducted in the Simulation Environment described in Section 5. Finally, Section 6 discusses the results of the successful maneuver, highlighting the cost function-based driving style management and the decision-making and control strategies employed for maneuver handling.

2 LOCAL MOTION PLANNING AND FRENET FRAME

Local trajectory planning focuses on generating a feasible and safe trajectory for a vehicle over a short time horizon, typically in response to dynamic changes in the environment. This includes continuously updating the planned trajectory based on new sensor data and the vehicle's current state. Unlike route planning, which provides a static sequence of points describing a plan, trajectory planning also includes additional velocity profiles.

Trajectory planning in dynamic environments is inherently complex and is considered PSPACE-hard (Paden et al., 2016). This complexity increases in dynamic settings, where previously manageable problems become intractable. As exact algorithms for

non-trivial trajectory planning in autonomous driving are unavailable, numerical methods are often employed. These methods address trajectory planning by utilizing variational methods within the time domain or by transforming the problem into a path planning challenge within a configuration space that incorporates a time dimension. The path planning solution accommodates differential constraints and is then transformed back into a trajectory.

The Frenet-Serret (FS) frame (Werling et al., 2010) is a powerful mathematical tool also used in vehicle dynamics and path planning to describe vehicle motion along a planned trajectory. This frame defines a moving reference frame with tangential and normal vectors at a certain point on a curve, called the center line. This center line can be an ideal path on a free road, the center of the road, or a path planned by an algorithm. In this paper, the reference line will be the center of the right-hand lane. As shown in Figure 1, the s -axis runs parallel to the lane line, and the d -axis is perpendicular to the reference line. This simplification makes it easier to plan within the proposed frame as the local coordinate system moves with the vehicle, and constraints such as lane boundaries and obstacles are easier to handle.

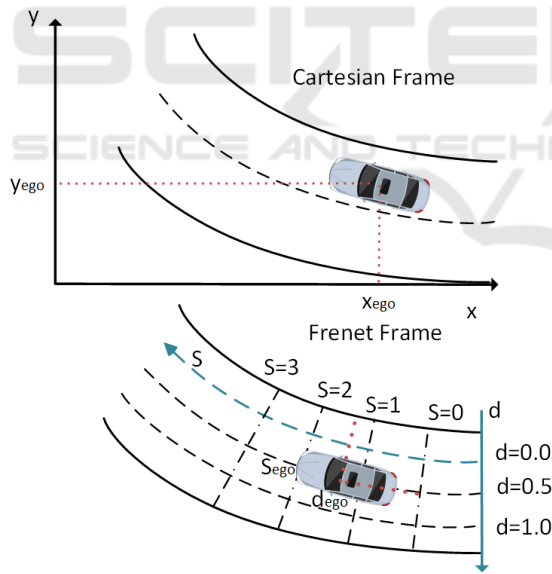


Figure 1: Frenet Frame Coordinate System.

Given the initial and final states of the vehicle along the d -axis $(d_s, \dot{d}_s, \ddot{d}_s, d_i, \dot{d}_i, \ddot{d}_i)$, six equations can be formed to determine the polynomial coefficients, representing the d -axis trajectory with a quintic polynomial. For the s -axis, using its start and end states $(s_s, \dot{s}_s, \ddot{s}_s, s_i, \dot{s}_i, \ddot{s}_i)$, a quartic polynomial represents the trajectory. The planning period t ranges from 0 to T , creating a time series $[0, \Delta t, \dots, T]$, which is ap-

plied to both axes' trajectory equations to calculate each trajectory point's coordinates. This will be further discussed in Section 4.

3 HIERARCHICAL CONTROL APPROACH

Several advanced methodologies for controlling highly automated vehicles have been developed recently. One prevalent approach is the hierarchical structure (Paden et al., 2016), which consists of four levels, each with a specific role in the vehicle's operation, as illustrated in Fig. 2

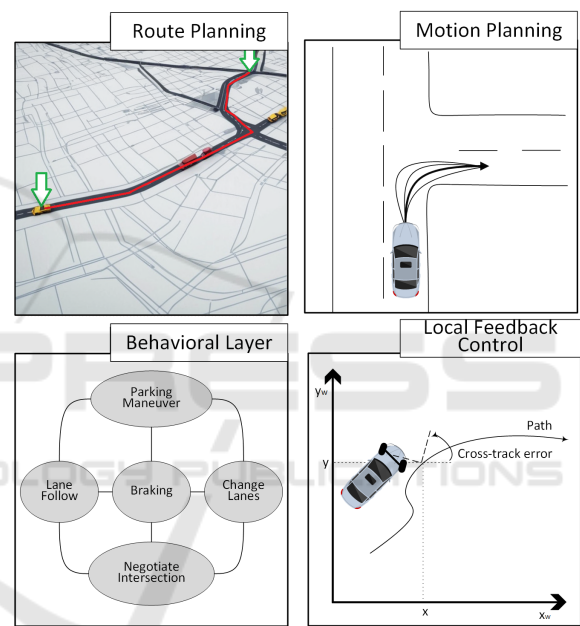


Figure 2: Hierarchical Control Structure.

At the first level, autonomous vehicles plan a global route within a given road network to reach their destination from the current position. The road network is represented as a directed graph, where weighted edges determine the cost of traversing the network. These weights can represent various factors such as distance, travel time, traffic density, and road conditions. Standard algorithms used for global route planning include Dijkstra's algorithm and the A* search algorithm.

Once a global route plan is selected, the vehicle must navigate the route and interact with other road users according to traffic conditions and rules. The behavioral layer is responsible for selecting the appropriate driving behavior at any given time, considering the actions of other road users, road conditions, and infrastructure signals. This decision-making pro-

cess can be automated using finite state machines and probabilistic planning formalisms, such as Markov Decision Processes (MDPs) and their generalizations.

After the behavioral layer decides on the driving behavior, the local motion planning layer defines the vehicle's trajectory. This trajectory must be dynamically feasible, ensuring passenger comfort and avoiding obstacles detected by onboard sensors. Numerical approximation methods are commonly used to solve the motion planning problem. The solution discussed in this article focuses on this layer.

At the fourth level, the planned path and trajectory are executed. The feedback controller calculates the corresponding actuator inputs for driving, steering, and braking to execute the planned movement and reduce tracking errors. This layer relies on well-established algorithms that emphasize robustness and stability.

4 LOCAL TRAJECTORY PLANNING IN FRENET FRAME

Local trajectory planning and tracking are conducted within a simulation environment that adheres to real-world traffic restrictions. The global path is defined as the centerline of the right lane of the road, serving as our reference route within the Frenet frame, as previously discussed.

In the Frenet frame coordinate system, the lateral displacement d is zero when the vehicle is on the reference path. The value of d increases as the vehicle moves towards the left edge of the road and decreases (becoming negative) as it approaches the right edge. The longitudinal position s represents the vehicle's position along the local trajectory, reset to zero at each planning interval.

The vehicle is initialized on this global trajectory at the start, with a planning interval of $T = 3$ seconds. This planning interval allows the vehicle to plan ahead for 41.6 meters at a target speed of $\frac{50}{3.6}$ m/s, which is deemed safe for dynamic obstacle detection and avoidance.

4.1 Planning Process

During the planning process of the local trajectory (as shown in Fig. 4a), several possible route alternatives are planned. To ensure thorough exploration of the available space, the alternatives are generated at every 0.1-meter point along the full width of the road. These trajectories are described with different polynomials in the lateral and longitudinal directions. The polynomials give the deviation from the global path.

Firstly, the lateral characteristics are determined by quintic polynomial fitting, where the 1st, 2nd, 3rd, and 4th derivatives describe the lateral characteristics of the trajectory at each time step. The quintic polynomial is expressed as:

$$d(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5 \quad (1)$$

where $d(t)$ represents the lateral position at time t . The coefficients a_0, a_1, a_2, a_3, a_4 , and a_5 are determined based on boundary conditions such as initial and final positions, velocities, and accelerations.

The first derivative represents the rate of change of the lateral position with respect to time, indicating how fast the vehicle is moving laterally at any given moment. The second calculates the rate of change of lateral velocity, providing information about the lateral forces acting on the vehicle. The third derivative represents the rate of change of lateral acceleration, which is important for determining ride comfort.

The longitudinal characteristics are defined at each time step using a quartic polynomial given by:

$$s(t) = b_0 + b_1t + b_2t^2 + b_3t^3 + b_4t^4 \quad (2)$$

where $s(t)$ represents the longitudinal position at time t .

The first derivative represents the rate of change of the longitudinal position with respect to time. The second measures the rate of change of longitudinal velocity. The third represents the rate of change of longitudinal acceleration.

4.2 Combining Profiles and Cost Function

Once the longitudinal and lateral profiles are generated, they are combined. A cost function is used to select the appropriate trajectory for the given objective. The function includes jerk, deviation from the reference path, lane change, and costs for weighting the fastest trajectory and shortest trajectory. By weighting these factors differently, trajectory designs for various purposes can be implemented. The results of these designs are presented in the following sections. Each trajectory's costs are combined and normalized, and the trajectory with the lowest total cost is selected. In the case of obstacle detection, the actual trajectory encountering an obstacle is prohibited rather than penalized by cost.

The elements of the cost function can be calculated as follows:

$$J(t) = \sum_{t=0}^T (d'''(t))^2 \quad (3)$$

The jerk cost $J(t)$ is calculated by summing the squares of the third derivatives of the lateral position with respect to time. This represents the cumulative lateral acceleration change over the planning horizon.

$$D_{\text{ref}} = \sum_{t=0}^T (d(t) - x_{\text{target}})^2 \quad (4)$$

The deviation from the reference route D_{ref} is calculated by summing the squared differences between the actual lateral position $d(t)$ and the target lane center x_{target} over the planning horizon.

$$Dist = \sqrt{(\text{GlobalRouteEnd}_{xy} - \text{TrajectoryEnd}_{xy})^2} \quad (5)$$

The distance cost $Dist$ is calculated as the Euclidean distance between the planned trajectory endpoint and the global route endpoint.

$$L_{\text{change}} = |d_i - x_{\text{target}}| \quad (6)$$

The lane change cost L_{change} is calculated as the absolute difference between the lateral displacement d_i of the trajectory and the target lane center x_{target} .

Table 1: Cost Function Parameters.

Cost Function Parameters		
Penalized Weights	Comfort stlye	Sporty stlye
Jerk	10.0	1.3
Speed	1.3	10.0
Deviation	3.0	3.0
Distance	0.01	0.01
Lane Change	3.0	3.0

Finally, the resulting trajectories are transformed from the Frenet frame to the global Cartesian coordinate system for visualization and tracking. The transformation from Frenet to global coordinates involves mapping a local position (given by s and d) on a reference path to its corresponding global position by calculating the global coordinates for the given arc length, applying the lateral offset to these coordinates, determining the orientation and segment lengths, and deriving the curvature from changes in orientation.

5 SIMULATION ENVIRONMENT

For the development and testing of the local trajectory planning method, a unique simulation environment was developed, which consists of the following components:

- A road generator that meets standard road design guidelines. It builds up the track from straight, curved, and clothoid transitional curves. The rural road environment was created by defining two lanes. The global path is the center line of the right lane in the direction of travel.
- A nonlinear planar single-track vehicle model containing a dynamic wheel model (Hegedűs et al., 2020) is also applied as an EGO vehicle to provide accurate behavior prediction.
- Lateral MPC controller running a linear dynamic model. The model states that there are tracking errors, which the controller aims to minimize. The yaw-rate profile is defined as a constraint.
- Longitudinal PID controller. The speed reference is determined by the speed profile.
- Transformation solutions that facilitate planning in the Frenet frame. Simplified traffic simulation.
- 2D graphical interface see in Fig. 3



Figure 3: Simulation Environment.

6 RESULTS

As a result of this study, we present a local trajectory planning method specifically designed for overtaking maneuvers in rural environments. A comprehensive software architecture was used for validation, including global route planning, a decision-making layer, local motion planning, and local feedback control. As mentioned in Section 4, the global route has been aligned and fitted to the reference path.

The maneuvering scenarios were tested in the presented simulation environment. Personalized decision-making was implemented using a finite-state machine based on a simplified version of the MOBIL (Minimizing Overall Braking Induced by Lane changes) model (Kesting et al., 2007), which is a lane-changing algorithm that assesses the advantages and consequences of a lane change for the ego vehicle and the vehicles around it. The model consists of an

incentive criterion that promotes lane changes benefiting traffic flow and a safety criterion that prevents lane changes from posing a risk to other vehicles. Our simplified MOBIL model defines three states, which maneuvers are shown in Figure 4:

- **Free-Driving State:** The ego vehicle remains in or returns to this state if there is no preceding vehicle. The primary operations involve controlling speed and following the reference route with the specified driving style. In this state, trajectories crossing into the opposite lane are strictly prohibited to ensure safety and compliance with traffic rules.
- **Tracking State:** This state occurs when another vehicle is in front of the ego vehicle, preventing a safe overtake. Here, Adaptive Cruise Control (ACC) is realized, allowing the ego vehicle to adapt to the speed of the preceding vehicle and maintain a safe distance. The Intelligent Driver Model (IDM) (Kesting et al., 2010) is utilized to calculate the desired acceleration based on the current speed, the distance and relative speed between the two vehicles.
- **Overtaking State:** This state is triggered when the decision-making process deems overtaking feasible, and the driver approves it. The steps involved in this maneuver are swerving, overtaking, and returning to the original lane.

6.1 Cost Function Based State Management

During the planning process, two distinct settings were identified beside the decision-based states: jerk minimization (comfort setting) and fastest trajectory execution (sporty setting). In the comfort setting, the cost function heavily weights the variation in lateral acceleration, whereas, in the sporty setting, the execution time is prioritized, as shown in Table 1. A comparative cost function diagram pair can be found in Figure 5.

The driving style can be customized using the cost parameters outlined above. For example, trajectories with lower lateral acceleration can be selected for a smoother ride, while a faster method yields sportier trajectories with higher jerks. This customization is reflected in the decision-making layer, where a sportier setting may lead to riskier decisions due to shorter trajectory execution times. For this, detailed maximum lateral slip values corresponding to different road IDs, which denote varying levels of curve difficulties in both comfort and sporty styles, are provided in Table 2.

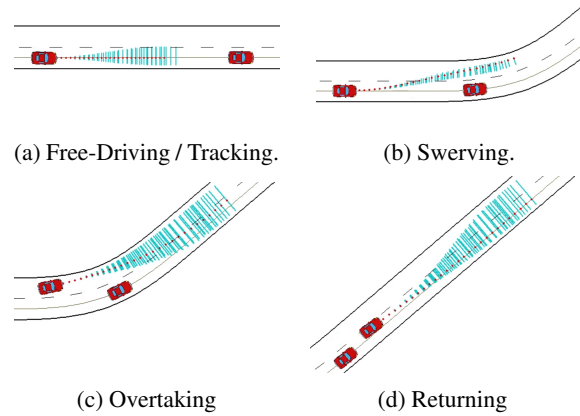


Figure 4: Local Trajectories during maneuvers.

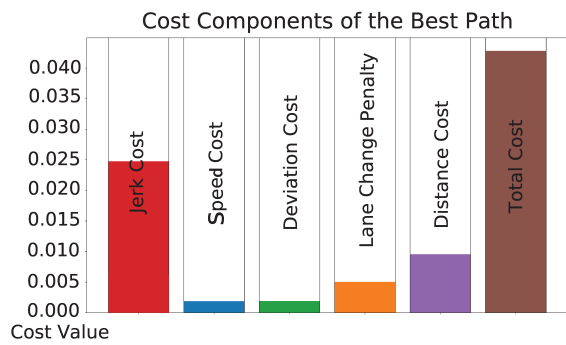
6.2 Control Strategies

The accurate and safe lateral tracking of planned trajectories is managed by an MPC (Model Predictive Controller) that handles the lateral and angular error. Meanwhile, the speed is maintained by a PID (Proportional-Integral-Derivative) controller. The target speed for the PID controller is determined by the motion planner and is maximized by the decision layer. These velocity profiles of the optimal trajectory are assigned to the control at each time step, tracing 75% of the trajectory before the next planning. By default, the vehicle plans and proceeds free-driving along the trajectory in one of the operationally switchable driving styles (comfort or sporty). If a vehicle is detected ahead within a safe braking distance, the behavioral layer switches to the tracking state, and the vehicle adaptively matches its speed to the preceding vehicle and maintains a safe distance until the overtaking maneuver is triggered. Upon activation, the vehicle switches to the overtaking state, increases its speed, and selects trajectories that allow for a quick and safe overtaking. Once the maneuver is complete, the vehicle returns to the previous state and continues free-driving on its path.

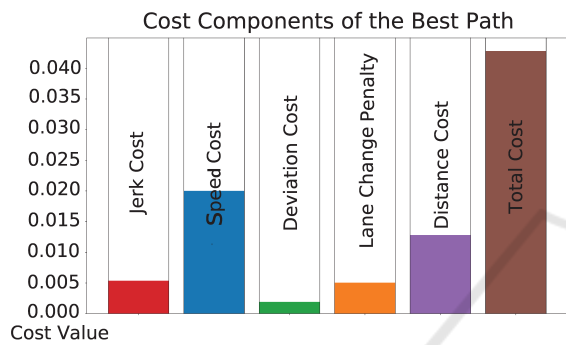
Table 2: Maximum lateral slip values for comfort and sporty styles across different difficulty of road sections.

Road ID	Comfort stlye	Sporty style
1	0.00772	0.01056
2	0.00171	0.01195
3	0.00746	0.01466
4	0.00838	0.01706
5	0.00996	0.01192

The driving style influences the lateral and longitudinal characteristics of the maneuvers. These trajectories are selected by the designer using the associated cost functions, ensuring the vehicle's behavior aligns with the desired performance and safety criteria.



(a) Cost Function Diagram of Comfort setting



(b) Cost Function Diagram of Sporty setting

Figure 5: Different Driving Styles given the same planning conditions.

7 CONCLUSIONS

In this paper, we presented a comprehensive approach to local motion planning for autonomous vehicles, with a particular focus on overtaking maneuvers in rural road environments. Our method demonstrated effectiveness, safety, and reliability, proving easy to tune for various driving styles. Currently, our simulation includes two vehicles, and we aim to expand this to incorporate traffic scenarios, allowing for overtaking in more complex environments. Our immediate goal is to implement real vehicle tests in a known environment to validate our approach further. Additionally, we aim to replace the polynomial fit design with a classical trajectory design method. This transition will allow us to determine curvature points and trajectory orientation more precisely, enabling the creation of unique, parameterizable trajectories. Such advancements will facilitate the design of optimized paths, including specific apex points of curves, thereby enhancing the versatility and performance of autonomous vehicle motion planning.

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