A Modified Sandpile Model for Simulating Lava Fountains at Mt Etna

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Abstract: This study aims to achieve two primary objectives. Firstly, it offers empirical evidence on the statistical distribution of inter-event times for lava fountains at Mt. Etna between 2011 and 2022, revealing that these times follow a power-law distribution, which supports the hypothesis that volcanic energy release exhibits dynamics characteristic of Self-Organized Criticality (SOC) systems. Secondly, it introduces a modified version of the classic Bak-Tang-Wiesenfeld (BTW) model, specifically adapted to simulate the inter-event times of lava fountains in volcanic environments like Mt. Etna. Although the proposed model is straightforward and offers initial insights, it remains preliminary. Further development is needed to enhance its accuracy and extend its applicability to more complex volcanic systems.

1 INTRODUCTION

Many natural systems exhibit self-organization at the edge of phase transitions, a phenomenon known as self-tuned phase transitions. Such systems have been observed across various domains, including laboratory fusion plasmas, solar physics, and magnetospheric physics (Watkins et al., 2015). In geophysics, many systems display SOC characteristics (Corral and Gonzales, 2019). It is now widely recognized in seismology that seismic energy release follows a power-law distribution, with different regions exhibiting distinct representations of the Gutenberg-Richter law of earthquake magnitude (Corral and Gonzales, 2019). The presence of power laws is a strong indicator of Self-Organized Criticality (SOC) (Jensen, 1998), suggesting that seismic energy release is a classic example of a SOC system.

Given the SOC characteristics observed in seismic energy release, it is plausible to hypothesize that volcanic energy release might follow similar dynamics. Volcanic activity encompasses a wide range of eruption styles, with significant variations in the volume of ejected material and the intervals between eruptions. Previous studies have identified power-law relationships between the Volcanic Explosivity Index (VEI) and eruption frequency, drawing a parallel to the Gutenberg-Richter law observed in earthquake magnitude distributions (Butters et al., 2017). This suggests a potential SOC behavior in volcanic activity as well.

This study specifically investigates the inter-event times of lava fountains at Mt. Etna, aiming to demonstrate their adherence to a power-law distribution. Mt. Etna is one of the most active volcanoes in the world, making it an ideal candidate for studying volcanic energy release through lava fountains. To model this phenomenon, we adapt the Sandpile model (Bak et al., 1987), a cellular automaton originally developed to illustrate SOC, which has been widely used to model critical phenomena in complex systems. Cellular automata, with their discretized time, space, and internal states, can produce complex structures despite their simple local evolution rules. They have been applied to model non-equilibrium phenomena such as crystal growth, fluid dynamics, and transport processes governed by the Navier-Stokes equations. In the context of volcanic systems, cellular automata have been employed to simulate intricate phenomena, including lava flows (Vicari et al., 2007) and magma ascent (Piegari et al., 2011). In this study, we explore an adapted version of the BTW model to interpret the inter-event times of Lava Fountains (LF) at Etna, providing further experimental evidence that volcanic energy release may follow power-law distributions.

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2 LAVA FOUNTAIN PHENOMENON

Lava fountains, such as those observed at Mt Etna, can be understood as resulting from the gradual accumulation of stress within the volcanic system. This stress eventually triggers the ascent of molten material, with gases playing a crucial role in facilitating this rise. However, the mechanisms that control such events are not fully understood (La Spina et al., 2017), and their modeling using partial differential equations (PDEs) presents significant challenges.

Firstly, our understanding of the physical properties within a volcanic system is limited. The subsurface conditions, such as the structure and composition of the magma chamber and conduit system, are poorly constrained by available observational data, making it difficult to accurately parameterize these features in mathematical models.

Secondly, magma itself is not a simple fluid. It exhibits non-Newtonian behavior, where its viscosity varies depending on factors like temperature and crystal content. The presence of solid particles further complicates its flow dynamics, introducing challenges that standard fluid dynamics equations struggle to capture accurately.

Furthermore, the interaction between rising magma and the surrounding rock, coupled with the complex degassing processes, introduces additional layers of complexity. These interactions can affect the magma's ascent rate, pressure buildup, and ultimately, the explosivity of eruptions. The exsolution of gases from magma plays a crucial role in driving ascent and eruption processes, but these processes are highly non-linear and not fully understood.

Additionally, the stochastic nature of volcanic activity, influenced by numerous random factors, complicates the prediction of precise behaviors using deterministic models. This inherent variability, coupled with the multitude of interacting processes, often renders traditional PDE-based models inadequate for capturing the full range of volcanic behaviors.

Given these challenges, alternative modeling approaches like the Sandpile model (Bak et al., 1987), which is adapted in this study, offer valuable insights into the statistical properties of volcanic phenomena. This model allows for the exploration of eruption dynamics without requiring detailed physical parameters, making it a powerful tool for studying the distribution of time intervals between eruptions.

3 DATA AND METHOD

3.1 Experimental Data

The dataset for this study comprises the start and end dates of 118 lava fountain episodes at Mt Etna, recorded between 2011 and 2022 (Calvari and Nunnari, 2022). These episodes have been meticulously documented, offering a detailed chronicle of volcanic activity during this period.

To analyze the temporal distribution of these lava fountain episodes, we calculated the inter-event times, defined as the difference in days between the start of one lava fountain and the start of the next. This metric quantifies the intervals between successive volcanic activities, providing insights into the underlying temporal patterns (Calvari and Nunnari, 2022).

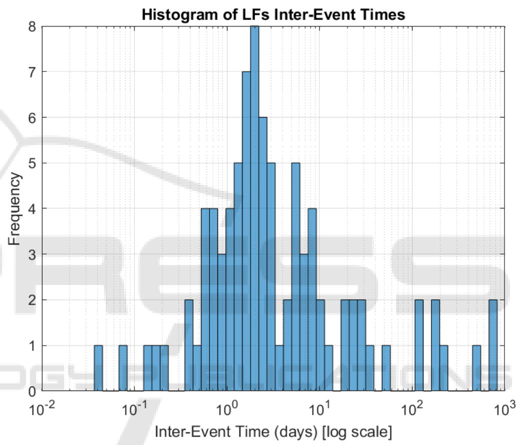

Figure 1: Histogram of LFs inter-event times. The x-axis represents the inter-event time in days, while the y-axis represents the frequency of occurrences. This histogram illustrates the distribution of intervals between consecutive lava fountain episodes at Mt Etna from 2011 to 2022, highlighting the variability and periodicity of volcanic activity.

Figure 1 presents the histogram of LFs inter-event times, offering a visual representation of the temporal distribution of lava fountain episodes. This histogram reveals the variability and periodicity in the volcanic activity at Mt Etna over the studied period, which is crucial for understanding the system's dynamics.

3.2 Some General Features of SOC Systems

Self-organized criticality (SOC) is a property of dynamical systems that naturally evolve to a critical state, where a minor event can lead to significant consequences. This concept has been applied to a wide range of physical, biological, and social systems. Here are some general features of SOC systems:

- **Scale Invariance.** SOC systems exhibit behavior that is independent of the scale at which they are observed. This means that similar patterns or structures can be found at different magnitudes, indicating a fractal-like nature. For a function of the type $f(x) = x^{\alpha}$, the relative variation $\frac{f(kx)}{f(x)} = k^{\alpha}$ does not depend on x.
- **Power-law Distributions.** The size and frequency of events in SOC systems typically follow a power-law distribution. This implies that small events are exceedingly common, while large events are rare but possible. One method to experimentally determine if a phenomenon could be generated by an SOC system is to measure a characteristic of the phenomenon, in space or time, and check if its probability distribution follows a power-law. However, even if a power-law distribution is observed, this does not constitute rigorous proof of SOC behavior, as the absence of a characteristic scale might be obscured or not evident in the dataset. This means that while observing a power-law distribution may suggest SOC behavior, it is not definitive evidence of it; other underlying factors or data limitations could obscure the true nature of the system.
- **Threshold Dynamics.** SOC systems often have a critical threshold that, when reached, can trigger a chain reaction or cascading events. The system naturally evolves towards this critical point without external tuning.
- **Long-range Correlations.** In SOC systems, distant parts of the system can become correlated, meaning that a change in one part of the system can influence another part, regardless of the physical distance.
- **Intermittent Activity.** The activity in SOC systems is typically sporadic, with periods of relative calm interspersed with bursts of intense activity. This intermittent behavior is a hallmark of critical systems.

Understanding these features helps in identifying and analyzing SOC behavior in various complex systems, from earthquakes to financial markets.

3.2.1 The BTW Model for Simulating SOC Systems

The BTW (Bak-Tang-Wiesenfeld) Sandpile model is a mathematical framework commonly used to simulate self-organized criticality (SOC) across various systems, including volcanic activity. The model operates on a grid, where each cell can hold a certain number of grains. When a cell exceeds a critical threshold, it "topples," distributing grains to neighboring cells and potentially causing a chain reaction of toppling events, known as an avalanche.

In its standard form, the Sandpile model operates as follows:

1. **Initialization.** A square grid of size pile_width × pile_width is initialized. Each cell in the grid is randomly assigned up to three grains.
2. **Grain Addition.** Grains are added to the grid one at a time at random positions. If adding a grain causes a cell to exceed three grains, an avalanche is triggered.
3. **Avalanche Process.** During an avalanche, four grains are removed from the overloaded cell: one grain is added to each of its four neighboring cells (top, right, bottom, left). If any of these neighboring cells exceed three grains as a result, they also topple, continuing the avalanche until no cell exceeds three grains.
4. **Boundary Conditions.** Grains that topple off the edge of the grid are lost and not considered in subsequent steps.
5. **Simulation Loop.** The process of adding grains and resolving avalanches continues until the specified number of grains has been added to the grid.

3.2.2 Adaptation of the BTW Model for Simulating Lava Fountains

To better simulate lava fountains, the adapted BTW model introduces critical modifications to the boundary conditions and the resistance matrix, which are necessary to capture the unique behaviors observed in volcanic activity. Below, we outline the major differences between the standard BTW model and our adapted version.

Consider a 2D grid of size $N \times N$. Each cell in the grid is denoted by (i, j) with $i, j \in \{1, 2, \ldots, N\}$. The rules governing the accumulation and release of stress are as follows:

1. **Stress Accumulation Rule.** Stress, representing the buildup of volcanic pressure, is incrementally added to a random position in the bottom row during each simulation step:

$$S(N, j) \to S(N, j) + 1,$$

where j is randomly chosen from $\{1, 2, \ldots, N\}$.

2. **Resistance Matrix.**

- The resistance matrix $R(i, j)$ decreases from the bottom to the top of the grid.

– The resistance on the lateral boundaries (left and right edges) is set to infinity:

$$R(i,j) = \begin{cases} \infty, & \text{if } j = 1 \text{ or } j = N \\ R_{\min} + \frac{i-1}{N-1} \times (R_{\max} - R_{\min}), & \text{otherwise} \end{cases}$$

A visual representation of the resistance matrix is shown in Figure 2.

Figure 2: An example of the resistance matrix adapted for simulating volcanic activity, illustrating how resistance decreases with height and influences the dynamics of stress accumulation and release.

3. **Stress Redistribution Rule.** When the stress $S(i,j)$ in a cell exceeds the resistance $R(i,j)$, an event occurs:

$$S(i,j) \to S(i,j) - 4,$$

and the excess stress is redistributed to the four neighboring cells:

$$\begin{aligned} S(i+1,j) &\to S(i+1,j)+1, \\ S(i-1,j) &\to S(i-1,j)+1, \\ S(i,j+1) &\to S(i,j+1)+1, \\ S(i,j-1) &\to S(i,j-1)+1. \end{aligned}$$

This redistribution of stress can trigger further events in neighbouring cells, leading to a chain reaction, referred to as an avalanche.

4. **Boundary Conditions.** The stress in cells at the top and bottom rows is lost if it attempts to redistribute outside the grid:

$$S(i,j) \to 0 \text{ for } i = 1 \text{ or } i = N.$$

5. **Stop Condition.** The simulation is terminated when the accumulated stress in the entire grid reaches a threshold value.

These modifications, particularly the introduction of a resistance matrix that decreases with height and the specific boundary conditions, are critical for replicating the self-organized criticality observed in volcanic systems. The next subsection will explore the implications of these changes for understanding the dynamics of lava fountains at Mt Etna.

4 RESULTS

This section presents the analysis of lava fountain (LF) inter-event times and their compatibility with a power-law model. Section 4.1 evaluates whether the observed inter-event times of LFs at Mt. Etna fit a power-law distribution, providing the corresponding model parameters. Section 4.2 examines whether the inter-event times from simulated LFs, using the adapted sandpile model, exhibit similar power-law behavior, thereby assessing the model's ability to replicate the characteristics of the actual volcanic events.

4.1 A Power-Law for the Mt Etna LF Inter-Event Times

To determine if the lava fountains at Mt. Etna exhibit features characteristic of a Self-Organized Criticality (SOC) system, we analyzed the inter-event times of lava fountains recorded from 2011 to 2022. Following the analysis by (Calvari and Nunnari, 2022), the inter-event times were fitted to a power-law distribution, resulting in the parameters $\alpha = 1.72$, $x_{\min} = 5.39$, $\sigma_\alpha = 0.19$, and $\sigma_{x_{\min}} = 6.49$. The p-value of 0.46 indicates that the power-law model is statistically plausible, suggesting that the inter-event times lack a characteristic scale, which is consistent with the behavior of SOC systems.

Figure 3 shows the empirical complementary cumulative distribution function (CCDF) for the LF inter-event times and the corresponding power-law fit, highlighting the fit's alignment with the data.

4.2 Simulating the Adapted Sandpile Model

To assess whether the adapted sandpile model can replicate the inter-event times of lava fountains observed at Mt. Etna, we conducted simulations over 50,000 cycles on a 64x64 grid. We tracked both the number of avalanches and the grains expelled from the top central portion of the grid, which we identified as simulated lava fountain events. The times between these events are considered as the inter-event times of simulated lava fountains.

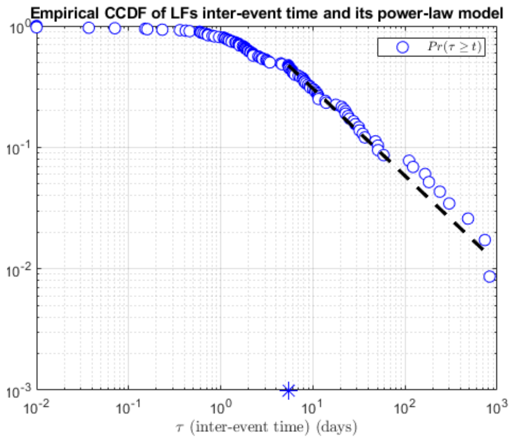

Figure 3: Empirical complementary cumulative distribution function (CCDF) (blue circles) and its power-law model fit (dotted black line) for the inter-event times of lava fountains (LF) at Mt. Etna between January 2011 and July 2022. The symbol '*' indicates the $x_{\min}$ for the considered dataset.

4.2.1 Simulating the Avalanche Size

Figure 4 illustrates the distribution of observed avalanches by size as recorded by the model. This distribution is a key indicator of the model's behavior and its potential alignment with SOC characteristics.

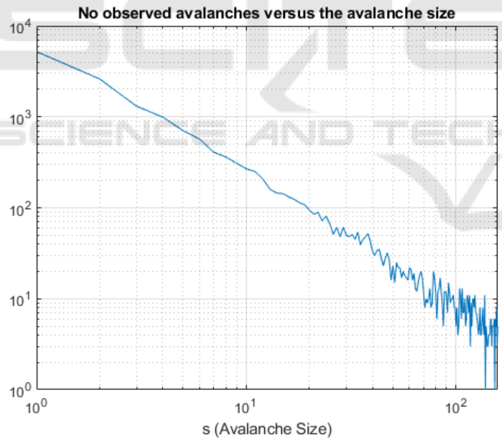

Figure 4: Number of observed avalanches versus their size for the adapted 64x64 grid size sandpile model.

The empirical CCDF for avalanche sizes, along with the fitted power-law model, is presented in Figure 5. The parameters obtained ($\alpha = 1.70$, $x_{\min} = 8$) and the p-value of 0.60 suggest that the model reproduces the avalanche dynamics characteristic of the original Bak-Tang-Wiesenfeld (BTW) sandpile model, reinforcing the model's validity.

4.2.2 Simulating the LF Inter-Event Times

Over 50,000 simulation cycles, the model recorded 68 lava fountain events, allowing for the analysis of

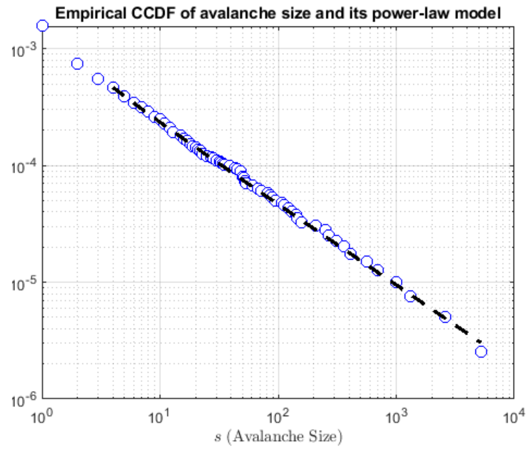

Figure 5: Empirical CCDF of avalanche sizes and the corresponding power-law fitting model.

inter-event times. The power-law fit for the simulated LF inter-event times yielded parameters of $\alpha = 1.59$, $x_{\min} = 47$, $\sigma_\alpha = 0.25$, and $\sigma_{x_{\min}} = 180.64$. The p-value of 0.09 suggests marginal plausibility, indicating that while the model approximates the observed distribution, it may not fully replicate the statistical characteristics of the actual LF inter-event times at Mt. Etna. This result suggests limitations in the model's ability to fully capture the dynamics of the volcanic system.

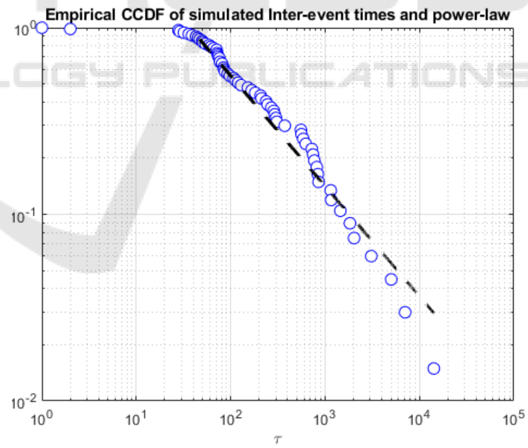

Figure 6: CCDF of the simulated LF inter-event times using the adapted 64x64 grid size sandpile model.

5 CONCLUSIONS

This study demonstrates that the proposed sandpile model effectively reproduces the avalanche dynamics characteristic of the Bak-Tang-Wiesenfeld (BTW) model, yet it only partially succeeds in simulating the inter-event times of the lava fountain phenomena observed at Mt. Etna. Several factors contribute to the model's partial success in replicating real-world lava fountain dynamics:

Firstly, the simplified stress propagation mechanism in the model likely fails to fully capture the complex dynamics inherent in real lava fountains. This simplification may overlook critical interactions and feedback processes. Future refinements could include integrating more sophisticated stress redistribution algorithms or exploring alternative stress propagation models that better reflect the complexity of volcanic processes.

Secondly, the limited grid size (64x64) and the number of simulation cycles (50,000) may have constrained the model's ability to accurately simulate the complexity of lava fountains. Larger grids and extended simulation periods could capture a broader range of inter-event times, potentially leading to a better approximation of real-world dynamics. Future studies should consider scaling up these parameters to assess their impact on the accuracy of the simulation results.

Furthermore, the comparison with the more complex model by (Piegari et al., 2011) underscores the inherent trade-offs between model simplicity and accuracy. While our model offers a tractable framework for initial investigations, it may lack the detailed mechanisms necessary for higher fidelity simulations. This balance between simplicity and detail highlights the need for ongoing model refinement and validation to better capture the nuances of volcanic activity.

Overall, this work represents a valuable initial attempt to simulate a complex geophysical phenomenon using a simplified model. The insights gained from this study offer a foundation for future research, which could focus on refining the model to better capture the nuances of lava fountain dynamics and exploring alternative approaches to improve simulation accuracy. Continued development and validation of the model will be crucial in enhancing its ability to reproduce real-world lava fountain phenomena and contribute to our understanding of these complex systems.

Future research directions may include:

- Further refining the stress propagation mechanism to better mirror the dynamics observed in actual lava fountains, possibly by incorporating more complex algorithms or alternative modeling approaches.
- Systematically testing the model with larger grid sizes and extended simulation periods to explore how these factors influence the fidelity of the simulated lava fountain dynamics, potentially leading to more accurate and reliable predictions.
- Investigating additional external factors, including environmental conditions and geological variations, that could significantly influence lava fountain behavior, offering a more holistic understanding of the processes involved.

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