# A Comparison of Adaptive PID, Adaptive Dual-PID and Adaptive Fractional PID Controllers for a Nonlinear System with Variable Parameters

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Keywords: Adaptive PID , Dual-Adaptive PID, Adaptive Gain FO-PID , Nonlinear Processes, Variable Parameters.

Abstract: This paper presents a comparative analysis of three control strategies: Adaptive PID, Dual-Adaptive PID, and Adaptive Gain FO-PID controllers. These controllers were evaluated on nonlinear dynamic systems with varying parameters, considering set point variations, disturbances, and measurement noise. Performance was quantified using key metrics such as settling time, overshoot, Integral of Squared Error (ISE), and Integral of Squared Control Output (ISCO). The results demonstrate that the Adaptive Gain FO-PID consistently outperforms the other methods, highlighting its superior ability to manage the complexities of nonlinear systems.

## 1 INTRODUCTION

Industrial processes are often characterized by a high level of complexity arising from various factors such as nonlinear behavior, uncertainties in system parameters, and unmodeled dynamics, all of which can complicate the design and implementation of effective control strategies(Liptak, 2018). In addition to these challenges, many industrial systems exhibit poorly understood or difficult-to-characterized plant properties, further increasing the difficulty of accurately modeling the process(Smith and Corripio, 2005). Furthermore, time delays are a common feature in industrial processes, which can severely limit the performance and stability of traditional control techniques, requiring the development of more advanced and robust control methods to address these multifaceted challenges (Mejia et al., 2022).

PID control remains the benchmark algorithm for addressing a vast majority of control challenges across industry, academia, and everyday applications. Even today, it dominates industrial control, accounting for more than 90% of solutions, while also serving as the foundational model for teaching feedback control principles in universities and technical institutions. Furthermore, it is widely adopted as a standard automated solution in numerous household devices and utilities, including smartphones, cruise control systems in cars, ovens, microwaves, drones, air conditioning units, heating systems, electric bikes, Segways, hoverboards and elevators (Borase et al., 2021; Hägglund and Guzmán, 2024).

The classical PID controller is typically tuned by trial and error or based on the model parameters near a specific operating point (O'dwyer, 2009). However, in highly non-linear systems, PID control may struggle to maintain a stable response, resulting in poor tracking and disturbance rejection performance (Anchitipan and Camacho, 2021). As noted in (Yurkevich, 2011), the effectiveness of PID control is often correlated with the dynamic characteristics of the system, and many industrial processes can be adequately represented by simplified first- or second-order dynamic models (O'dwyer, 2009).

Given the popularity of the PID controller, some improvements have been made that incorporate some modifications to the original algorithm. These methodologies, specifically developed to address the complexities of dynamic processes, can potentially enhance the performance of PID controllers beyond traditional outcomes. There are various modifications such as non-linear PID (Han, 2009), two-degree-of-freedom PID (Taguchi and Araki, 2000), Fractional Order PID (Podlubny, 1994; Podlubny, 1999b), and Dual-Adaptive PID (Chotikunnan and Chotikunnan, 2023; Obando et al., 2023).

Despite their widespread use and success in var-

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ious industrial applications, both conventional PID controllers and PID with modifications enhancement face limitations, particularly in adapting to evolving process conditions (Samad, 2017). These challenges have motivated ongoing research into improved PID controller designs using adaptive control techniques and self-tuning methods to improve process control performance(Åström and Wittenmark, 2008).

Adaptive control aims to manage uncertain dynamic systems in real time using adaptation and learning mechanisms (Annaswamy and Fradkov, 2021). Adaptive PID control techniques have gained significant traction, with numerous studies exploring parameter adaptation, model reference adaptive control, and self-tuning methods to enhance system performance (Isermann and Isermann, 1991; Chaínho et al., 2005; Annaswamy and Fradkov, 2021; Liu et al., 2021). These approaches deliver improved dynamic behavior and precise regulation, which makes them highly suitable for industrial applications. Adaptive controllers can be realized on platforms like Microchip Technology's microcontrollers, specifically dsPIC microcontrollers, which integrate the capabilities of a Digital Signal Processor (DSP) and a Programmable Intelligent Computer (PIC). These dsPIC microcontrollers provide a computationally efficient and effective solution for process control. Furthermore, self-tuning Adaptive PID controllers, which continuously update parameters during closed-loop operation, further improve control performance in systems with variable dynamics (Huang et al., 2002). Adaptive controllers scheduled for gains, designed to adjust gains in response to changes in system parameters, have also demonstrated superior stability and control accuracy compared to classical PID controllers, ensuring robust performance under diverse operating conditions (Vesely̆ and Ilka, 2013).

Adaptive PID controllers have been extended to various industrial applications. For example, (Zhao et al., 2012) developed an Adaptive PID controller that automatically adjusts the parameters to accommodate changing environments, simplifying the interface between processes and control systems. Similarly, (Razmi et al., 2022) used Lyapunov-based adaptive rules to enhance load frequency control (LFC), while (Wase et al., 2023) applied a fuzzy gain scheduling PID controller to a CSTR, achieving robust disturbance rejection and improved performance.

This research focuses on comparing three Adaptive PID control strategies: PID, Dual-Adaptive PID, and fractional one. They are applied to a non-linear process with variable parameters. A performance evaluation is done using ISE and ISCO indices.

The remainder of this paper is organized as follows. Section 2 describes the fundamentals. Section 3 outlines the results and Section 4 conclusions.

## 2 FUNDAMENTALS

This section is dedicated to describing the three adaptive schemes used. First, a PID gain scheduling is presented, then a Dual-Adaptative and an Adaptive Gain FO-PID .

### 2.1 Gain Scheduling

Gain scheduling is a widely used technique in the design of controllers for nonlinear systems, particularly those with time-varying parameters. It involves dynamically adjusting the controller gains based on measurable parameters, ensuring stability and optimal performance across a wide range of operating conditions. This approach is frequently applied in PID controllers, offering enhanced robustness and efficiency in systems where dynamic changes necessitate continuous adjustments to the controller. By adapting to varying system dynamics, gain scheduling ensures consistent and reliable control (Milhim et al., 2010).

### 2.2 Adaptive PID

The architecture of the controller is depicted in Figure 1. A gain scheduling strategy is employed, where the controller gains are adjusted dynamically based on the system's current state. This is achieved using a Lookup Table that provides optimal gain values for different operating conditions, ensuring that the controller adapts in real-time to variations in the system's behavior (Huang and Shah, 1999).

The general form of a Adaptive PID controller with gain scheduling is expressed as:

$$u(t) = K_p(x)e(t) + K_i(x)\int_0^t e(t)\,dt + K_d(x)\frac{de(t)}{dt}. \quad (1)$$

In this formulation, $K_p(x)$, $K_i(x)$, and $K_d(x)$ represent the proportional, integral, and derivative gains, respectively. These gains are not fixed; instead, they vary as functions of the measurable parameter $x$, which is typically derived from the system's output, such as a sensor or transmitter reading (Smith and Corripio, 2005). By adjusting the gains according to the parameter $x$, the controller can respond more effectively to changes in the system dynamics, improving stability and performance across a range of operating conditions. This gain scheduling approach ensures that the controller maintains optimal performance even when the system is subject to significant disturbances or when operating across a wide range of setpoints. Such adaptability makes it particularly useful in processes where system dynamics are nonlinear or time-varying.

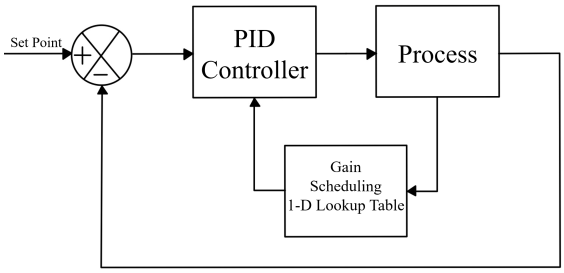


Figure 1: Adaptive PID Controller Design.

## 2.3 Dual-Adaptive PID

The controller scheme is depicted in Figure 2. This system implements a Dual-Adaptive PID control strategy that switches between a PD and a PI controller depending on the magnitude of the system error. The adaptive mechanism of this Dual-Adaptive PID follows a similar gain scheduling approach as described in the previous Adaptive PID controller. In this configuration, both PI and PD with Look up Table controllers are employed, with the ability to dynamically switch between them based on the error conditions during the simulation. The flowchart detailing the switching logic is shown in Figure 3.

The control law for the Dual-Adaptive PID is defined as:

$$u(t) = \begin{cases} K_p(x)e(t) + K_d(x)\frac{de(t)}{dt}, & \text{if } |e_{last}(t)| \geq |e(t)|, \\ K_p(x)e(t) + K_i(x)\int_0^t e(t)\,dt, & \text{if } |e_{last}(t)| < |e(t)|. \end{cases} \quad (2)$$

In this expression, the proportional gain $K_p(x)$, the integral gain $K_i(x)$, and the derivative gain $K_d(x)$ are functions of the measurable system parameter $x$. The switching condition is determined by comparing the current error $e(t)$ with the previous error $e_{last}(t)$. When the error decreases over time, the PD controller is active, and when the error is 0, the system switches to the PI controller. This dynamic switching mechanism allows the controller to balance responsiveness and stability, making it suitable for systems with varying dynamics and disturbances. By alternating between the PD and PI modes, the Dual-Adaptive PID controller provides a more flexible and efficient approach to maintaining desired performance across a range of operating conditions.

## 2.4 Adaptive Gain Fractional-Order PID

The design of the controller scheme is illustrated in Figure 4. This figure shows how Lookup Tables are

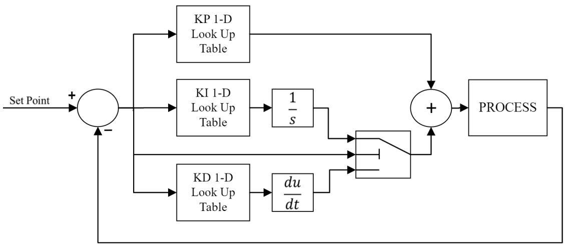


Figure 2: Dual-Adaptive PID Controller Design.

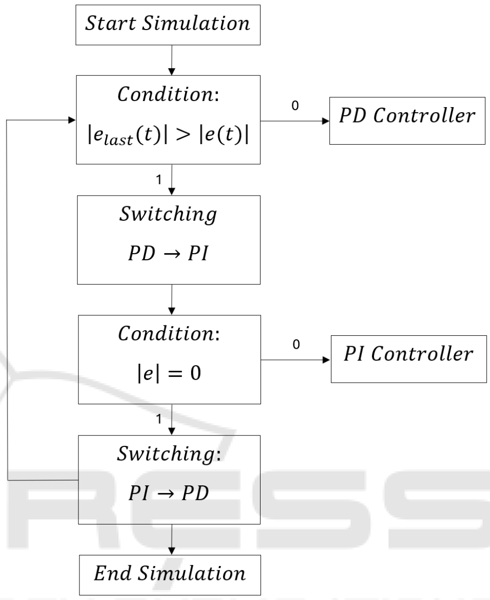


Figure 3: Switching condition flowchart.

utilized to determine the PID controller constants, with the added feature of fractional-order components by incorporating the optimal values of $\lambda$ and $\mu$ (Podlubny, 1994). The control law for an Adaptive Gain Fractional-Order PID (FO-PID) controller can be expressed as follows:

$$u(t) = K_p(x)e(t) + K_i(x)D_t^{-\lambda}e(t) + K_d(x)D_t^{\mu}e(t). \quad (3)$$

In this equation, $D_t^{-\lambda}$ and $D_t^{\mu}$ represent the fractional integral and derivative operators, respectively. These fractional orders introduce additional flexibility to control dynamics, enhancing performance. The values of $\lambda$ and $\mu$ were determined through trial and error to optimize the robustness and responsiveness of the system (Petráš, 2011), (Podlubny, 1999a).

The adaptive gain scheduling mechanism leverages Lookup Tables to dynamically adjust controller parameters $K_p(x)$, $K_i(x)$, and $K_d(x)$ based on the operating conditions of the system. This adaptability ensures that the Adaptive Gain FO-PID controller can maintain superior control performance across a range of dynamic environments and disturbances.

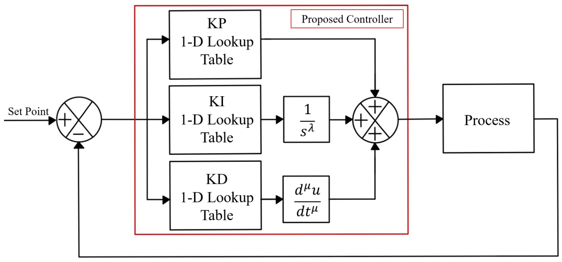


Figure 4: Adaptive Gain FO-PID Controller Design.

The FOMCON toolbox (Tepljakov et al., 2019) was used together with Simulink to implement the Adaptive Gain FO-PID controller.

# 3 NONLINEAR PROCESS UNDER STUDY

This section presents results and discussion regarding the application of adaptive approaches to a nonlinear process. Based on (Iglesias et al., 2007), a brief initial description of nonlinear systems is provided. This is followed by an explanation of the empirical modeling procedure and the tuning process. .

## 3.1 Nonlinear Process Model

The variable height mixing tank process illustrated in Figure 5 consists of three flows: an input of hot fluid $W_1(t)$ at temperature $T_1(t)$, an input of cold fluid $W_2(t)$ at temperature $T_2(t)$ controlled by the FC valve, and an output flow $W_3(t)$ at temperature $T_3(t)$ arising from the combination of the input streams. Specifically, the FC actuator regulates the cold stream to maintain $T_3(t)$. The temperature $T_4(t)$ is obtained by a transmitter positioned 125 feet downstream from the tank, with a range of 100 to 200 $^\circ F$.

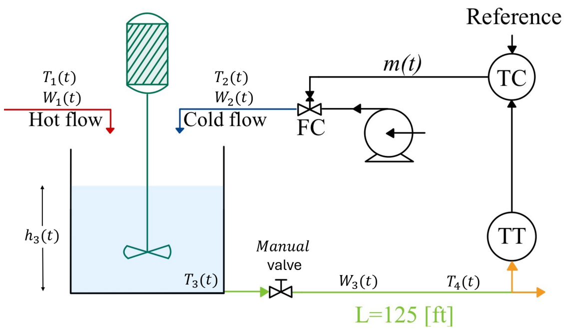


Figure 5: Mixing Tank Process.

The variable height mixing tank model is a modified version of the process described in (Camacho and Smith, 2000), adapted in (Iglesias et al., 2007), and later discussed in (Morales et al., 2021; Prado et al., 2022; Vásquez et al., 2023).

The control objective in this process lies in maintaining the mixing temperature $T3(t)$ against external disturbances of hot flow $W1(t)$ through the control output $U(t)$ at the FC valve position FC.

For this work, the following assumptions were made:

- The tank content and the pipeline are entirely isolated.
- The tank is fully mixed with a uniform internal temperature.
- The liquid volume varies but does not overflow.
- The main disturbance of the tank is the hot flow $W_1(t)$.

The system's mathematical model is obtained by utilizing the fundamental principles of energy and mass balance conservation laws, as detailed below.

Energy Balance:

$$W_1(t)C_{p_1}T_1(t)+W_2(t)C_{p_2}T_2(t)-W_3(t)C_{p_3}T_3(t)= A_3\rho C_{v_3}\frac{d(h_3(t)T_3(t))}{dt}. \tag{4}$$

Mass balance:

$$W_1(t)+W_2(t)-W_3(t)=A_3\rho\frac{dh_3(t)}{dt}. \tag{5}$$

The mixing output flow can be expressed as a function of level $h_3(t)$ in the following manner:

$$W_3(t)=11.8685C_{VL3}\sqrt{h_3(t)}. \tag{6}$$

The temperature $T_4(t)$ of the mixture, after being transferred from the mixing tank to the transmitter's site, is defined by:

$$T_4(t)=T_3(t-t_0(t)). \tag{7}$$

where $t_0$ represents the time delay for the product mixture with temperature $T_3(t)$ to arrive at the endpoint of the pipeline, with a temperature of $T_4(t)$. This delay is influenced by the length of the pipe and its cross section $A$. Thus, the time delay can be quantified by:

$$t_0(t)=\frac{LA\rho}{W_3(t)}. \tag{8}$$

Dynamics of the temperature transmitter can be described as follows:

$$\frac{dT0(t)}{dt}=\frac{1}{\tau_T}\left[\frac{T_4(t)-100}{100}-T0(t)\right]. \tag{9}$$

Table 1: Variables and stationary state values of the process.

| Variable | Description | Stationary value |
|---|---|---|
| $W_1$ | Hot current mass flow | 250 lb/min |
| $W_2$ | Cold current mass flow | 191.17 lb/min |
| $W_3$ | Output current mass flow | 441.17 lb/min |
| $C_{p1}$ | Heat capacity (hot flow side) | 0.8 Btu/lb-°F |
| $C_{p2}$ | Heat capacity (cold flow side) | 1.0 Btu/lb-°F |
| $C_{p3,v3}$ | Heat capacity at constant volume | 0.9 Btu/lb-°F |
| Ref. | Reference (Temperature) | 150°F |
| $T_1$ | Hot flow temperature | 250°F |
| $T_2$ | Cold flow temperature | 50°F |
| $T_3$ | Product temperature | 150°F |
| $T_4$ | Temp. considering time delay | – |
| $\rho$ | Content density | 62.4 lb/ft$^3$ |
| $A_3$ | Mixing tank section | 3.51692 ft$^2$ |
| $h_3$ | Liquid level | 4.26509 ft |
| $C_{CVL3}$ | Manual valve flow coeff. | 18 gpm/ ft$^{1/2}$ |
| $C_{CVL}$ | Control valve flow coeff. | 12 gpm/ psi$^{1/2}$ |
| $TO$ | Transmitter output signal | 0.5 p.u |
| $V_p$ | Valve position | 0.478 |
| $m$ | Controller output | 0.478 p.u |
| $G_f$ | Specific gravity | 1 |
| $\delta P_v$ | Pressure loss | 16 psi |
| $\tau_T$ | Sensor time constant | 0.5 min |
| $\tau_{V_p}$ | Valve time constant | 0.4 min |
| $t_o$ | Time delay | – |
| $A$ | Pipeline cross-section | 0.2006 ft$^2$ |
| $L$ | Pipeline length | 125ft |

where $\tau_T$ denotes the constant time of the temperature sensor. The servo-valve positioning is driven by the controller according to the following equation:

$$\frac{dV_p(t)}{dt} = \frac{1}{\tau_{VP}}\left[m(t) - V_p(t)\right]. \tag{10}$$

where $m(t)$ is the controller output of the process to be controlled. Based on the servo-valve position, the input cold flow can be calculated as follows:

$$W_2(t) = \frac{500}{60} C_{VL} V_p(t) \sqrt{G_F \Delta P_V}. \tag{11}$$

The variables of the non-linear model and the stationary state values with their respective units are summarized in Table 1.

## 3.2 Process Nonlinear Behavior

To evaluate the non-linear properties of the system, it is recommended to examine the variations in the parameters $K$, $\tau$, and $t_0$ as the input signal $m(t)$ to the valve undergoes a sequence of small step changes. It is crucial to consider that the attributes of the mixing tank are influenced by the operating conditions.

Therefore, the parameters of the First-Order Plus Dead-Time (FOPDT) model will alter at various operating points. Figure 6 illustrates the changes in the mixing tank process parameters in response to variations in the input signal. As the signal increases from 0.1 to 0.898 per unit with increments of +10%, the parameters trace the black curves. In contrast, the parameters trace the blue curves as the signal drops from 0.898 to 0.1 per unit with decrements of $-10\%$. It is noted that both $K$ and $t_0$ show an almost linear growth pattern, while the time constant ($\tau$) does not show a clear correlation with the plant input signal $m(t)$.

The ambiguity in $\tau$, combined with the hysteresis-like behavior in $K$ and $t_0$, undermines the effectiveness of the traditional Proportional-Integral-Derivative (PID) controller.

## 3.3 Empirical Modeling Procedure

Given the system's highly nonlinear behavior, one approach to implementing adaptive control schemes is by constructing the scheduling process using a collection of simple reduced-order models. The reaction curve method described in (Smith and Corripio, 2005; Liptak, 2018) is utilized to obtain these models. The derived reduced-order models are utilized to develop the gain scheduling scheme. It is important to remember that gain scheduling adaptive control is a method to dynamically adjust the controller's parameters in real-time based on variations in operating conditions or system dynamics. As the system moves from one operating region to another, the controller adjusts its gains by selecting the pre-determined set of gains associated with the current operating condition.

This approach is especially beneficial for managing nonlinear or time-varying systems, where the controller gains are contingent on the system's present state or environment.

The approximate or reduced FOPDT model of the process is derived using the reaction curve method (Smith and Corripio, 2005) and can be expressed as:

$$G(s) = \frac{Ke^{-t_0}}{(\tau s + 1)}. \tag{12}$$

Where: ($K$) Process Gain, ($\tau$) Time Constant, and ($t_0$) Delay, which are obtained directly from the procedure.

For example, if we consider that the controller output is 0.5 the resulting FOPDT model is presented in Equation (13).

$$G_{p_1}(s) = \frac{-0.8217}{0.73\,s + 1} e^{-3.6\,s}. \tag{13}$$

Figure 7 illustrates the actual response of the nonlinear mixing tank model along with the FOPDT model. It can be seen that the results closely match the operating point, validating the accuracy of the model,

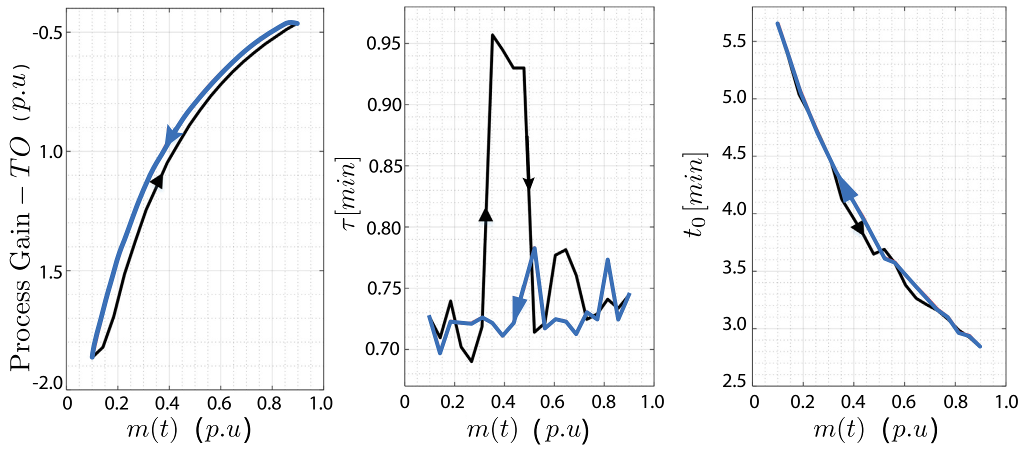


Figure 6: Nonlinear behavior of the characteristic parameters of the mixing tank process, (Iglesias et al., 2007).

but it changes depending on the controller output, as is seen in Figure 6.

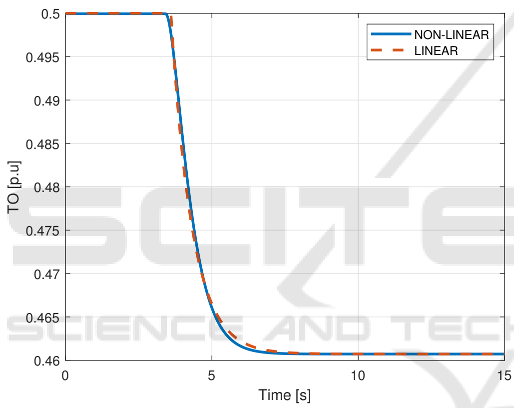


Figure 7: Comparative responses of the approximated FOPDT model and real system. The model output was depicted using a reaction curve test around the operation point.

In addition, $\frac{t_0}{\tau} = 4.9$, called the controllability ratio, indicates the difficulty of controlling a process (Obando et al., 2023; Mejia et al., 2022). Also known as normalized dead time or normalized time delay (Åström et al., 2006), smaller values of $\frac{t_0}{\tau}$ indicate processes that are easier to control, while larger values of $\frac{t_0}{\tau}$ represent systems that are harder to control. A value greater than one for $\frac{t_0}{\tau}$ signifies a difficult process with a dominant time delay.

# 4 CONTROLLERS EVALUATIONS

The controllers are then assessed for their response to change in setpoints and disturbance variations, and performance metrics are used to facilitate their comparative analysis.

## 4.1 Controller Tunings

The Dahlin equations were applied to tune the PID parameters, leveraging the values obtained from the Open-Loop Tuning method (Åström and Hägglund, 2006). The required PID gains, $K_P$, $K_I$, and $K_D$, were calculated by developing 20 approximations of FOPDT models, 10 for an increase of 10% and 10 for a decrease of 10% in system parameters. These models enabled the design of three 1-D Lookup Table blocks in Simulink, each corresponding to PID gains. The breakpoints in the Lookup Tables were configured to adapt to both reference changes and disturbances, allowing the system to dynamically select the optimal gain values. Furthermore, adjustments to fractional parameters $\lambda$ and $\mu$ were fine-tuned through trial and error to balance performance improvements with robustness.

## 4.2 Control Performance Indices

Process controller performance is often evaluated by comparing control quality with a standard or desired value (Liptak, 2018; Smith and Corripio, 2005). We are using here the integral of squared error (ISE), the integral of the squared variation of the control signal (ISCO), maximum overshoot, and settling time (Ts) to assess the proposed approach against PID and SMC controllers.

**Integral of the Squared Error (ISE).** Quantifies the system performance by integrating the squared error over a set time interval(Smith and Corripio, 2005; Liptak, 2018).

$$ISE = \int_0^\infty e(t)^2 dt. \tag{14}$$

**Integral of the Squared Variation of the Control Signal (ISCO).** This reflects the effort exerted by the control signal. It can be calculated using the following formula:

$$ISCO = \int u(t)^2 dt. \tag{15}$$

**Overshoot ($\mathbf{M}_p$).** It is defined as the deviation of the response at the time when a maximum peak appears with respect to the final or desired value of the response. It also called the maximum overshoot, is the amount of the output system that exceeds its target value as a percentage (%) (Smith and Corripio, 2005).
**Settling Time ($\mathbf{t}_s$).** The time required for the response to reach a steady state and remain within the specified tolerance bands around the final value. The normally used tolerance bands are 2% and 5% (Smith and Corripio, 2005).

### 4.3 Comparison of Controllers to Reference Change

Given the non-linearity of the mixing tank process and the varying non-linearity and hysteresis of the FOPDT model at different operating points, it is crucial to improve the conventional PID controller to address these issues. This research proposes developing some adaptive controllers as potential solutions. The following section details the use of these different controller configurations.

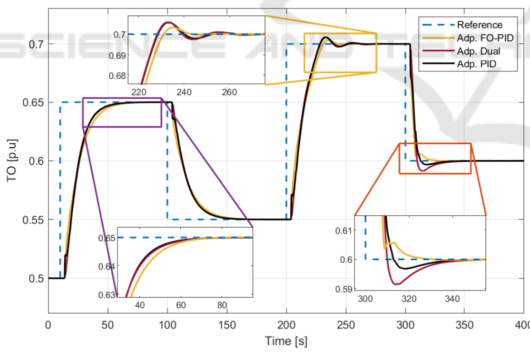


Figure 8: System response to Reference Changes.

Figure 8 illustrates the process response of a mixing tank under the application of Adaptive Gains FO-PID , Dual-Adaptive PID, and Adaptive PID controllers. While all controllers effectively guide the system toward the reference values, a more detailed analysis reveals significant differences in their performance.

The Adaptive PID controller performs similarly to the Dual-Adaptive PID controller during the reference shifts at 10 and 200 seconds. However, the most notable difference is observed during the 300-second reference shift, where the Adaptive PID demonstrates excellent reference tracking capability with minimal overshoot. Additionally, a detailed examination of the final reference shift reveals that the Adaptive PID controller maintains a stable response with less overshoot compared to the Dual-Adaptive PID.

The Dual-Adaptive PID controller exhibits slightly slower response times compared to the Adaptive PID controller. However, the most significant difference is observed during the 300-second reference shift, where the Adaptive PID controller demonstrates superior reference tracking capability. At this point, the Dual-Adaptive PID begins to outperform the Adaptive PID, switching during the overshoot to achieve faster response times. When the final reference change occurs, the Dual-Adaptive PID becomes even faster, transitioning from PD to PI during the overshoot to quickly reach the reference.

On the other hand, the Adaptive Gain FO-PID controller strikes a balance between settling time and overshoot, avoiding excessive stabilization times or high overshoot percentages. It is observed that, in response to a reference change, this controller demonstrates an immediate response compared to the other controllers, reducing its speed as it approaches the reference. These observations highlight the inherent trade-offs among the different adaptive control strategies concerning response time and overshoot. Understanding these trade-offs is crucial for optimizing control performance in mixing tank processes, ensuring that controllers are selected based on the specific performance criteria required for the application.

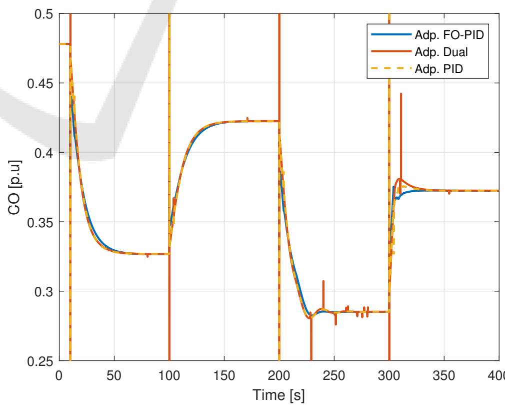


Figure 9: Control Action of Change Reference.

Figure 9 illustrates the response of various controllers to reference changes, specifically comparing the performance of the Adaptive Controllers with different control laws: Adaptive PID, Dual-Adaptive PID, and Adaptive Gain FO-PID.

The Adaptive PID controller provides intermediate behavior between the other two controllers. While it offers a balance between smoothness and response speed, it exhibits minor oscillations in the controller output.

The Dual-Adaptative PID controller exhibits a more abrupt response to reference changes due to the switching of controllers when the response deviates from the reference. This behavior can be disadvantageous in systems that are sensitive to rapid fluctuations.

In contrast, the Adaptive Gain FO-PID controller demonstrates the smoothest response to reference changes, making it advantageous for applications that require precise control and finely-tuned stability.

These observations highlight the distinct characteristics of each controller, emphasizing that each has its own advantages and limitations based on the specific requirements of the control application, such as in the case of a mixing tank with variable parameters.

Table 2: Comparison of Performance parameters for the last reference changes.

| Controllers | ISE | ISCO | $t_s$ [s] | $M_p$ [%] |
|---|---|---|---|---|
| Adap. FO-PID | 0.54 | 66.27 | 25.12 | 0.00 |
| Adap. Dual | 0.59 | 66.41 | 33.84 | 0.82 |
| Adap. PID | 0.58 | 66.40 | 35.91 | 0.30 |

Table 2 compares the performance of three control strategies Adaptive FO-PID, Dual-Adaptive PID, and Adaptive PID. The table evaluates critical metrics, including ISE, ISCO, $t_s$, and $M_p$, which collectively indicate the controllers' accuracy, control effort, speed of response, and stability when subjected to disturbances.

The Adaptive FO-PID stands out by achieving the fastest settling time with no overshoot, reflecting its ability to maintain precise and stable control under changing conditions. This makes it ideal for applications requiring minimal error and smooth transitions.

The Dual-Adaptive PID, although slightly slower in settling time, leverages its dynamic switching between control modes (PD and PI) to optimize its response, particularly by balancing speed and control accuracy. It does exhibit a minor overshoot, which is acceptable in scenarios prioritizing quick responses.

On the other hand, The Adaptive PID exhibits the longest settling time among the three controllers but compensates with low overshoot and consistent control. This balance between smoothness and stability makes it suitable for applications where gradual convergence to the reference is preferred over rapid changes. Overall, the table highlights the distinct strengths and trade-offs of each controller, showcasing the adaptability and control efficiency required for different system dynamics.

## 4.4 Comparison of Controllers Under Disturbances

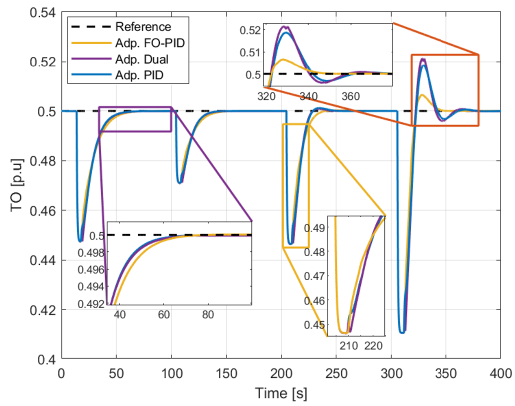


Figure 10: Comparison of disturbance rejection in the hot water.

Figure 10 illustrates the system's response to four disturbances in hot water introduction, applied at various values and times. This analysis evaluates the performance of three controllers: Adaptive Gain FO-PID, Dual-Adaptive PID, and Adaptive PID.

In the first disturbance, the Adaptive PID controller is shown to be much faster than the other controllers, while the Adaptive Gain FO-PID is the quickest to start rejecting the disturbance but is the slowest overall. This is due to its balance between performance and robustness.

In the second disturbance, all controllers exhibit a similar response, causing the parameters to continue varying. However, in the third disturbance, the controllers begin to differentiate. The Dual-Adaptive PID becomes faster than the Adaptive PID, also showing the overshoot of both controllers, while the Adaptive Gain FO-PID demonstrates robustness against this disturbance. The zoomed-in view of this disturbance better illustrates the controllers' behavior: the switching of the Dual-Adaptive PID, the peak of the derivative part of the Adaptive PID, and the quick response of the Adaptive Gain FO-PID.

Finally, with the variability of the parameters and the magnitude of the disturbance, the controllers face greater demands. The Adaptive PID and Dual-Adaptive PID show an increase in overshoot, with the Dual-Adaptive PID having the highest due to its faster passage through the reference point. At the maximum overshoot point, it performs a new switch to compensate and attempt to stabilize more quickly at the reference than the Adaptive PID. On the other hand, the Adaptive Gain FO-PID shows much greater robustness, with a much lower overshoot than the other controllers and stabilizing without oscillations more quickly. This characteristic of robustness is crucial for rejecting large magnitude disturbances and high variability in nonlinear systems.

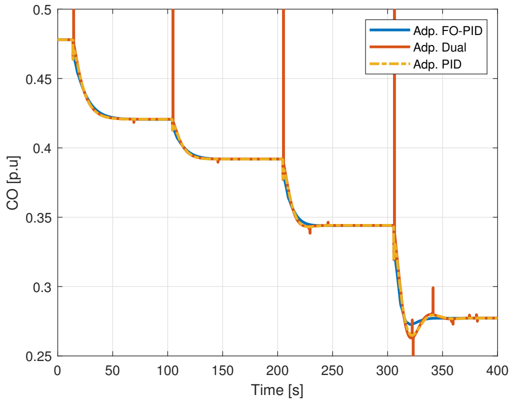


Figure 11: Comparison of control action in disturbances rejection in the hot water.

Figure 11 illustrates the control actions for each scenario. The Adaptive Gain FO-PID controller demonstrates the smoothest response to disturbances, making it particularly advantageous in applications where minimizing controller activity and power consumption is critical. This smooth response is due to its balanced approach to performance and robustness.

In contrast, the Dual-Adaptive PID controller exhibits a more abrupt response to disturbances. This is because it switches abruptly between controllers, as shown in the flowchart in Figure 3. Additionally, other impulses can be observed, which are the switches that occur when a new maximum error is detected. These abrupt changes can lead to higher power consumption and more wear on the system components.

The Adaptive PID controller displays an intermediate behavior, balancing smoothness and response speed. However, as the speed increases, it also introduces oscillations in the control output. This can be beneficial in scenarios where a quick response is needed, but it may also lead to instability if not properly managed.

These findings underscore the varying adaptive characteristics of the controllers. Each control strategy offers unique benefits and drawbacks depending on the specific demands of the application.

Table 3: Comparison of Performance Parameters for the Last Disturbance.

| **Controllers** | **ISE** | **ISCO** | $t_s$ **[s]** | $M_p$ **[%]** |
|---|---|---|---|---|
| Adap. FO-PID | 0.12 | 62.40 | 42.18 | 0.65 |
| Adap. Dual | 0.14 | 62.44 | 54.75 | 2.12 |
| Adap. PID | 0.13 | 66.41 | 56.95 | 1.86 |

Table 3 provides a comprehensive comparison of the performance parameters for disturbance rejection across three adaptive controllers.

The Adaptive FO-PID controller exhibits the best overall performance, achieving the lowest error, minimal control effort, and a short settling time. Its low overshoot makes it particularly effective in maintaining system stability during disturbance rejection, ensuring smooth and rapid recovery without significant oscillations. This controller is ideal for systems requiring high precision, fast stabilization, and minimal energy consumption.

The Dual-Adaptive PID controller, while demonstrating strong responsiveness, shows a slight increase in control effort and settling time compared to the FO-PID. Its dynamic nature, characterized by switching between control modes, results in faster corrective actions but introduces a higher overshoot. This makes the Dual-Adaptive PID a good choice in scenarios where fast disturbance rejection is more critical than maintaining minimal fluctuations, though it may lead to less stable recovery.

In contrast, the Adaptive PID controller offers a more moderate balance across the parameters. It achieves reasonable error minimization and control effort but lags behind the FO-PID in terms of settling time and exhibits more pronounced oscillations, reflected in a higher overshoot. While it can perform adequately in many applications, its slower response and higher energy consumption make it less efficient for systems requiring optimal disturbance rejection performance.

## 4.5 Comparison of Controllers with White Noise

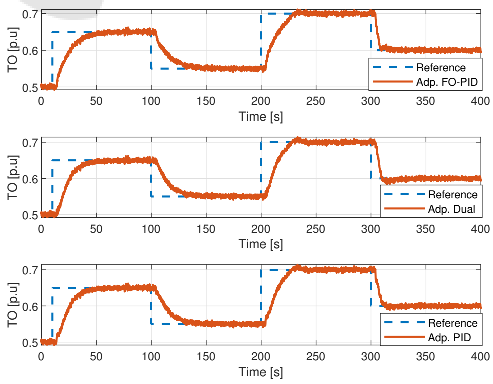


Figure 12: System Response to Reference Changes in Noisy Conditions.

Figure 12 illustrates the response of the mixing tank under noisy conditions, controlled by Adaptive Gain FO-PID , Dual-Adaptive PID, and Adaptative PID controllers. All controllers successfully guide the system to the reference values. However, the presence of noise does not result in significant differences in the system's response to the reference changes across the controllers. The most notable variations are observed in the control actions, where the controllers exhibit markedly different behaviors. Analyzing the control action is crucial, as significant variations are observed depending on the controller used.

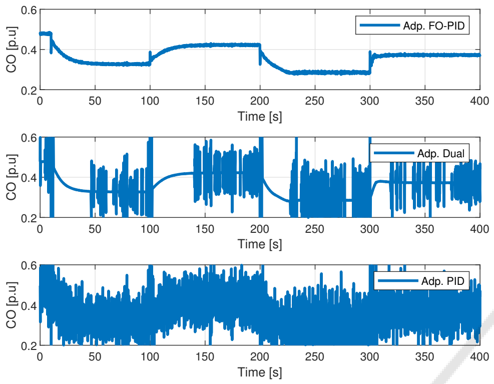


Figure 13: Control Actions in Response to Reference Changes Under Noisy Conditions.

Figure 13 compares the control actions of the Adaptive FO-PID, Dual-Adaptive PID, and Adaptive PID controllers under noisy conditions in response to reference changes.

The Adaptive FO-PID exhibits the smoothest and most stable control output, effectively rejecting noise and adapting to reference changes with minimal fluctuations. This performance is attributed to the fractional parameters in the integral and derivative gains, making this controller the best option in noisy environments.

In contrast, the Dual-Adaptive PID demonstrates how switching occurs when the error is not zero. This type of dual controller has an advantage in noisy conditions because it can reduce noise due to the PD controller's lack of an integral parameter, which otherwise increases noise. Additionally, when operating as a PI controller, the absence of a derivative gain further amplifies noise. These characteristics are evident in the control actions, showcasing the Dual-Adaptive PID's ability to handle noisy conditions effectively.

Lastly, the Adaptive PID controller displays the highest sensitivity to noise, with significant oscillations throughout the response period, indicating less stability compared to the other controllers.

This figure highlights the superior robustness of the FO-PID, the responsiveness but increased variability of the Dual-Adaptive PID, and the instability of the Adaptive PID under noisy conditions.

Table 4: Comparison of performance parameters for reference shifting under noise conditions.

| **Controllers** | **ISE** | **ISCO** |
|---|---|---|
| Adap. FO-PID | 0.55 | 66.28 |
| Adap. Dual | 0.61 | 66.43 |
| Adap. PID | 0.58 | 66.55 |

Table 4 provides a comparative analysis of the performance indices for three adaptive controllers Adaptive Gain FO-PID, Dual-Adaptive PID, and Adaptive PID based on two critical performance metrics: ISE , which quantifies the controller's ability to minimize tracking errors, and ISCO, which measures the control effort or energy consumption required to achieve the desired performance, particularly in noisy environments.

The Adaptive Gain FO-PID controller stands out with superior performance in both categories. It demonstrates exceptional error minimization, which indicates its high accuracy in maintaining the system's reference signal despite noise. Additionally, this controller achieves the highest energy efficiency, making it a balanced choice for applications where both precision and energy conservation are crucial. Its ability to optimize these two factors simultaneously makes it the most effective solution among the three controllers.

The Dual-Adaptive PID controller, while not as precise as the FO-PID in terms of error minimization, still delivers strong energy efficiency. Its slight increase in error is compensated by its low energy consumption, making it a viable alternative in cases where conserving control effort is prioritized over absolute accuracy. This balance between control precision and energy usage positions the Dual-Adaptive PID as a competitive option when compared to the FO-PID, especially in scenarios where performance trade-offs are acceptable.

On the other hand, the Adaptive PID controller, though capable of providing moderate error minimization, shows the highest energy consumption among the three controllers. This suggests that while it may be effective in reducing errors to a reasonable extent, its efficiency is compromised, leading to higher operational costs in terms of control effort. As a result, the Adaptive PID is less suited for applications that demand both high accuracy and energy efficiency, although it could still be a feasible choice in less demanding situations where energy consumption is not a critical concern.

## 5 CONCLUSIONS

This study provides insight into the performance of three adaptive control strategies under various conditions when applied to a nonlinear process with variable parameters and dominating delay. The tested controllers used were Adaptive PID, Dual-Adaptive PID, and Adaptive Gain FO-PID . The Adaptive Gain FO-PID consistently minimized the ISE and maintained a low ISCO, contributing to the life of the final control element with precise control and energy efficiency.

Adaptive Gain FO-PID achieved rapid stabilization with minimal overshoot and smooth disturbance response, making it suitable for tight control and robustness. Conversely, the Dual-Adaptive PID controller responded more aggressively to disturbances, resulting in higher ISE and ISCO values, which can be suitable for environments prioritizing quick adaptation but non-efficiency in energy. The Adaptive PID controller balanced error minimization and control effort, making it suitable for moderate error reduction, although its oscillations may limit effectiveness in dynamic or noise-sensitive environments.

These findings stress the importance of selecting the appropriate adaptive control strategy based on application requirements, balancing error minimization, energy efficiency, response time, and disturbance robustness. Therefore, the Adaptive Gain FO-PID presents the best for precision and performance, the Dual-Adaptive PID for rapid adaptation, and the Adaptive PID for a balanced approach.

Despite their advantages in performance for identification and control, fractional-order controllers require further development for practical implementations. Implementing Adaptive Gain FO-PID controllers involves fractional calculus, making them challenging since transitioning to fractional order requires advanced numerical methods, adding complexity and computational overhead. The gap between theory and practice should be reduced, and improvements in the knowledge of fractional calculus should be made so that plant operators can prove the advantages over conventional PID solutions.

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