Youla-Kučera Parameterization: Theory and Applications

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Keywords: Linear Systems, Feedback, Stabilizing Controllers, Controller Parameterization, Multitask Controllers.

Abstract: Youla-Kučera parameterization is the parameterization of all linear controllers that stabilize a given linear plant.

EXTENDED ABSTRACT

Youla, Bongiorno, and Jabr (Polytechnic Institute of New York University) and Kučera independently discovered the parameterization formula in the late seventies, see (Youla et al., 1976a), (Youla et al., 1976b) and (Kučera, 1975), (Kučera, 1979).

Ten years later, Vidyasagar (1985) provided a comprehensive account of the result. Quadrat (2003) generalized the parameterization results from lumped-parameter systems to a class of distributed-parameter linear systems. The survey paper (Anderson, 1988) summarized the first twenty years of theoretical developments. In contrast, the recent survey paper (Mahtout et al., 2020) collects the latest developments and industrial applications and provides an impressive list of references.

Parameterization is essential when control systems are designed to be stable and meet additional performance specifications (Kučera, 1993). The specifications beyond stability are achieved by selecting an appropriate parameter. There is a one-to-one correspondence between the set of stabilizing controllers and the set of parameters. Furthermore, the parameter appears linearly in the closed-loop system transfer function, whereas the controller appears nonlinearly. Selecting the parameter instead of the controller thus simplifies the design significantly. The system is made stable first, and then the additional specifications can be accommodated, one at a time.

Performance specifications, such as optimality and robustness, are often conflicting and challenging to achieve using a single controller. In such a case, parameterization allows the designer to reconfigure the controller to reach satisfactory performance while guaranteeing overall system stability.

The Youla-Kučera parameterization is a fundamental result that launched an entirely new area of research and has been used to solve many control problems (Kučera, 2007), ranging from optimal control, robust control, disturbance and noise rejection, and vibration control to stable controller switching and fault-tolerant control.

There is a dual parameterization, which describes all linear systems stabilized by a given linear controller (Anderson, 1988). The parameter can then describe plant variations. This is useful for solving the problem of closed-loop plant identification (Hansen, et al., 1989). Open-loop identification is more straightforward, but it is often prohibitive to disconnect the plant. Identifying the dual parameter instead of the plant is a linear problem like open-loop identification.

This keynote presentation is a guided tour through the theory and applications of the Youla-Kučera parameterization. It explains the origins of the result, the derivation of the parameterization formula using the transfer functions, and the state-space representation of all stabilizing controllers (Nett et al., 1984), (Kučera, 2011). It also explains how to select the parameter to satisfy specific design requirements. New and exciting applications of the Youla–Kučera parameterization are then discussed: stabilization subject to input constraints (Henrion et al., 2001), output overshoot reduction (Henrion et al., 2005a), and fixed-order stabilizing controller design (Henrion et al., 2003), (Henrion et al., 2005b). A selection of applications in different control fields is presented,

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Kučera, V.
Youla-Kučera Parameterization: Theory and Applications.
DOI: 10.5220/0013113700003822

In *Proceedings of the 21st International Conference on Informatics in Control, Automation and Robotics (ICINCO 2024) - Volume 2*, pages 9-10
ISBN: 978-989-758-717-7; ISSN: 2184-2809

showing the efficiency of this approach in controlling complex systems (Cifdaloz et al., 2008), (Trangbaek and Bendtsen, 2009), and (Mahtout et al., 2018).

ACKNOWLEDGEMENTS

This work was co-funded by the European Union under Project Robotics and Advanced Industrial Production CZ.02.01.01/00/22_008/0004590.

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