Effective Inventory Control Under Very Large Unknown Deterioration Rate and Volatile, Almost Unpredictable Customer Demand

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Abstract: We consider a periodically reviewed perishable Supply Chain (SC) whose dynamics shows the following elements of complexity: the goods are affected by a very large, uncertain deterioration factor (DF), the customer demand is highly unpredictable and volatile. The problem we face is to define an effective Inventory Replenishment Policy (IRP) conciliating the conflicting requirements of maximizing the satisfied customer demand and containing the Bullwhip Effect (BE). The method we propose is situated in the general framework of min-max Model Predictive Control (MPC) applied to SC management. We exploit the flexibility and generality of min-max MPC to define a specifically tailored method to address the peculiarity of the current, extremely complex issue. Especially, we demonstrate the advantages of using a short prediction horizon and point-wise constraints on the IRP.

1 INTRODUCTION

If not suitably taken into account, perishable goods may lead to a serious performance degradation of the SC management policy (Chaudary et al., 2018). The complexity of the related control problem motivated many authors to develop appropriate perishable IRPs in the MPC framework (Gaggero and Tonelli, 2015; Taparia et al., 2020; Lejarza and Baldea, 2020a; Lejarza and Baldea, 2020b; Hipolito et al., 2022) because of its appealing features (Rossiter and Bishop, 2004). The aforementioned papers assume an exactly known DF.
The extension to uncertain DF in a min-max MPC framework was proposed in (Ietto and Orsini, 2022a; Ietto and Orsini, 2022b; Ietto and Orsini, 2023a; Ietto and Orsini, 2023b; Jetto and Orsini, 2024; Ietto and Orsini, 2024) assuming that over a sufficiently long period of time the future customer demand is constrained inside a known compact set. In these latter contributions the authors have proposed a polynomial B-splines parametrization of the control law because this kind of functions admit a parsimonious parametric representation in terms of the so called "control points" (De-Boor, 1978). This appealing property allows reformulating the min-max MPC as an estimation problem with a greatly reduced numerical complexity: it is enough to estimate the few control points univocally defining the optimal control law (i.e. the optimal IRP). The longer the control interval, the greater the numerical advantage.

Here we consider the more critical issue of defining an effective IRP in the case of goods with a very large, uncertain DF and a volatile, highly unpredictable customer demand.
If the problem is not appropriately addressed, the simultaneous presence of these two negative factors would cause a dramatic increase in the BE. With the syntagm "effective IRP" we mean a replenishment policy optimally conciliating the following antagonist requirements:
-R1) maximizing the amount of fulfilled demand avoiding overstocking,
-R2) containing the BE.
To the best of our knowledge this topic has not yet been dealt with.
In this paper we deal with this problem in the same previously mentioned min-max MPC framework. Nevertheless, the particular features of this involved issue impose a specially customized design procedure. To this purpose we act in two directions:

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removing the B-splines parametrization of the IRP and using an MPC with a short prediction horizon.

We show that avoiding B-splines parametrization makes it possible to derive point-wise constraints on the control law that are more suitable for the present problem: the new constraints are based on the current values of the upper and lower boundaries delimiting the actual demand. This is essential to satisfy R2.
The aftereffect of giving up the B-splines parametrization is the increased numerical complexity of the procedure to solve the min-max optimization problem. Using a short horizon min-max MPC is useful to reduce this side effect and is also justified by the large uncertainty affecting the future customer demand.
Theoretical considerations involving stability and feasibility as well as numerical results prove correctness and effectiveness of the proposed alternative.
The paper is organized as follows. The SC model and the assumptions on the customer demand are given in Section 2, the min-max MPC problem is formulated in Section 3. A numerically simpler constrained robust Least Squares (LS) reformulation of the min-max MPC problem is described in Section 4. Simulation results and concluding remarks are reported in Sections 5 and 6 respectively.

2 UNCERTAIN SC MODEL

2.1 Inventory Level Equation

For ease of exposition, but without any loss of generality, we consider a single-echelon periodically reviewed SC given by the series connection of a retailer with a manufacturer. The latter is modeled as a pure delay time. We assume:
A1) inside each review period $[kT,\,(k+1)T)$, $k \in Z^+$, the retailer performs the following operations: updates the inventory value, receives goods from manufacturer, dispatches goods to the customer, places a replenishment order. These operations are synchronized at the beginning of the review period;
A2) the manufacturer fully satisfies each non null replenishment order issued by the retailer with a time delay $L = \ell T$, $\ell \in Z^+$;
A3) the goods arrive at the retailer new and deteriorate while kept in stock;
A4) inside each review period the stocked goods deteriorate with a large uncertain DF $\alpha \in [\alpha^-, \alpha^+] \subset (0,1)$.

Hence, inside the k-th review period, the inventory level equation has the following form

$$y(k+1) = \rho(y(k)+u(k-L)-h(k)), \quad y(0) = 0, \quad (1)$$

where:
-$y(k+1)$ is the inventory level at the end of the k-th review period;
-$u(k-L)$ is the replenishment order issued at time $(k-L)$;
-$h(k)$ is the fulfilled part of the customer demand $d(k)$. It is given by

$$h(k) \overset{\triangle}{=} \min\{y(k)+u(k-L), d(k)\} = d(k)-z(k), \quad (2)$$

for some $z(k) \in [0,\, d(k)]$ that represents the amount of possibly unsatisfied demand;
- $\rho = 1-\alpha$ is the uncertain decay factor belonging to $[\rho^-,\,\rho^+] = [1-\alpha^+,\,1-\alpha^-] \subset (0,1)$.

2.2 Assumption on the Future Customer Demand

As often observed in practical cases, the customer demand shows a dynamic with characteristics of large volatility and unpredictability (Abolghasemia et al., 2020). This makes it very difficult to obtain accurate forecasts through a mathematical model (Carlson and Doyle, 2002).
For this reason and according to the robust control approach, the demand forecast that we use to implement the proposed MPC is only based on the following very intuitive assumption:
A5) $d(k) \le \bar{d} < \infty$, $k \in Z^+$, at any k and over an M-steps prediction horizon $P_k \overset{\triangle}{=} [k+1, k+M]$, the unknown future trajectory $d(k+j)$, $j = 1, \cdots, M$, belongs to a compact set D_k limited below and above by two known trajectories: $d^-(k+j)$ and $d^+(k+j)$, $j = 1, \cdots, M$. The assumed large unpredictability and volatility are taken into account assuming arbitrary oscillations of $d(k+j)$, $j = 1, \cdots, M$, inside a D_k, characterized by large values of $|d^+(k+j) - d^-(k+j)|$ and a short M. The future trajectory $d(k+j), j = 1, \cdots, M$, can be written as

$$d(k+j) = d(k+j|k) + \delta d(k+j|k) \quad (3)$$

where $d(k+j|k)$ is the predicted demand that coincides with the central trajectory of D_k and $\delta d(k+j|k)$ is the corresponding prediction error. This choice minimizes the ℓ_2 norm of $\delta d(k+j|k)$.
A typical example of customer demand over the k-th prediction horizon is illustrated in Fig 1.

3 THE CONTROL PROBLEM

3.1 Min-Max MPC Formulation

For any $k \in Z^+$, let $H_k \overset{\triangle}{=} [k, \cdots, k+N-1]$, be the k-th control horizon of length $N \le M-L+1$ and

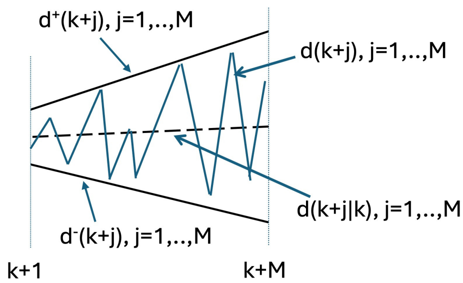

Figure 1: A typical example of volatile customer demand over $P_k = [k+1, k+M]$.

$U_k \triangleq [u(k|k), \cdots, u(k+N-1|k)]$ be the optimal predicted sequence of replenishment orders to be computed.
The point-wise bounds $u^-_{k,i}$ and $u^+_{k,i}$ on $u(k+i|k)$, $i = 0, \cdots, N-1$, are computed at the beginning of each H_k before solving the min-max MPC.
The robust min-max MPC problem is formally defined as follows:

$$\min_{U_k} \max_{\rho \in [\rho^-, \rho^+]} J_k \tag{4}$$

$$\text{subject to: } (1)-(3) \text{ and}$$

$$u^-_{k,i} \le u(k+i|k) \le u^+_{k,i} \quad i = 0, \cdots, N-1 \tag{5}$$

The cost functional J_k is defined in the following way

$$J_k = \sum_{i=0}^{N-1} e^T(k+L+i|k) q_i(k) e(k+L+i|k) + (\Delta u(k|k))^T \lambda(k) \Delta u(k|k)$$

with:

$$e(k+L+i|k) \triangleq (y(k+L+i|k) + u(k+i|k)) - d^+(k+L+i) \tag{6}$$

$$\Delta u(k|k) \triangleq u(k|k) - u(k-1)$$

$$y(k+L+i|k) = \rho^{L+i} y(k) + \sum_{\ell=0}^{L-1} \rho^{L+i-\ell} u(k+\ell-L) + \sum_{\ell=0}^{i-1} \rho^{i-\ell} u(k+\ell|k) - \rho^{L+i} h(k) - \sum_{\ell=1}^{L+i-1} \rho^{L+i-\ell} h(k+\ell|k) \tag{7}$$

$$h(k+\ell|k) = d(k+\ell|k) + \delta d(k+\ell|k) - z(k+\ell|k)$$

According to the receding horizon paradigm, over each H_k, only the first sample $u(k|k)$ of U_k is issued by the retailer to the manufacturer (namely $u(k) = u(k|k)$).
Some remarks on J_k are now in order:

- the tracking error definition (6) is motivated by R1: the necessity of fulfilling any possible customer demand compatible with A5 without incurring overstocking.
- The term $\Delta u^T(k|k) \lambda(k) \Delta u(k|k)$ and the hard constraints (5) have been introduced to satisfy R2: limiting large deviations on the IRP reduces the unavoidable costs related to frequent order quantity changes. Forcing the control effort to fluctuate within a predefined amplitude range allows us to contain the BE. How to set the limits $u^-_{k,i}$ and $u^+_{k,i}$, $i = 0, \cdots, N-1$, of this range is explained here beneath

3.2 The Point-Wise Limits on the Control Effort

The assumptions on the future customer demand do not allow computing the hard constraints on the control effort using the same arguments based on the notion steady-state response used in (Jetto and Orsini, 2024).
Here, the bounds $u^-_{k,i}$ and $u^+_{k,i}$, $i = 0, \cdots, N-1$ are calculated by an induction process starting from the following assumption:
A6) At a generic time instant $k-1$ we have already determined $u^-_{k-1,i}$ and $u^+_{k-1,i}$, $i = 0, \cdots, N-1$, in such a way that

$$e(k-1+L+i|k-1) \ge 0 \;\; i = 0, \cdots, N-1, \tag{8}$$

$\forall d(k+j) \in D_{k-1}, j = 1, \cdots, M$ and $\forall \rho \in [\rho^-, \rho^+]$

Remark 1 Note that A6 is a very weak assumption, because (8) can be trivially satisfied for $k = 0$ choosing

$$u^-_{0,i} = u^+_{0,i} = d^+(L+i), \;\; i = 0, \cdots, N-1 \tag{9}$$

△

Now, we show that A6 implies that an analogous condition also holds at the next time instant.
Consider the one-step prediction form of (1)

$$y(k+L+i|k) = \rho\,[y(k-1+L+i|k-1) + u(k-1+i|k-1) - h(k-1+L+i|k-1)] \tag{10}$$

By (2) and (8), $y(k+L+i|k)$ can be rewritten as:

$$y(k+L+i|k) = \rho\,[y(k-1+L+i|k-1) + u(k-1+i|k-1) - d(k-1+L+i)] \tag{11}$$

Recalling (6), assumption A6, Remark 1 and (11) it can be readily seen that

$$e(k+L+i|k) \ge u(k+i|k) - d^+(k+L+i) + \rho\left[d^+(k-1+L+i) - d(k-1+L+i)\right] \quad i = 0, \cdots, N-1 \tag{12}$$

By (12), the minimum $u(k+i|k)$ such that

$$e(k+L+i|k) \geq 0 \;\; i=0,\cdots,N-1, \qquad (13)$$

$$\forall d(k+j) \in D_k, j=1,\cdots,M \text{ and } \forall \rho \in [\rho^-, \rho^+]$$

is

$$u(k+i|k) = d^+(k+L+i) - \rho^- \left[d^+(k-1+L+i) - d(k-1+L+i)\right]$$

As

$$d(k-1+L+i) \in [d^-(k-1+L+i), d^+(k-1+L+i)]$$

we derive the following limits $u^-_{k,i}$ and $u^+_{k,i}$ on $u(k+i|k)$, $i=0,\cdots,N-1$

$$u^-_{k,i} = d^+(k+L+i) - \rho^- \left[d^+(k-1+L+i) - d^-(k-1+L+i)\right] \qquad (14)$$

$$u^+_{k,i} = d^+(k+L+i) \qquad (15)$$

The amplitude $A_{k,i}$ of $[u^-_{k,i}, u^+_{k,i}]$ is

$$A_{k,i} = \rho^- \left[d^+(k-1+L+i) - d^-(k-1+L+i)\right] \qquad (16)$$

Equations (14)-(16) provide an estimate of the BE in terms of limits on the IRP. Some theoretical considerations on this result are now in order.

Remark 2

- the point-wise upper bound $u^+_{k,i}$ coincides with the maximum admissible value for the current customer demand. This prevents an amplification of $u(k+i|k)$ with respect to any possible maximum customer demand compatible with A5;
- condition (16) evidences that the amplitude $A_{k,i}$ of $[u^-_{k,i}, u^+_{k,i}]$ decreases progressively as ρ^- tends to 0^+. This is a very positive effect because reduces the negative impact of large perishability on the BE (Minner and Transchel, 2017).

We are now in a position to state conditions to obtain an anti-BE effect, i.e. an IRP taking values inside over subset of the demand variability range.

Theorem The above point-wise constraints imply a contraction occurs with respect to the bounds on the customer demand namely

$$u(k+i|k) \in [u^-_{k,i}, u^+_{k,i}] \subset [d^-(k+L+i), d^+(k+L+i)] \qquad (17)$$

if and only if

$$\Delta_{k,i} > \rho^- \Delta_{k-1,i} \qquad (18)$$

where $\Delta_{k,i} \triangleq d^+(k+L+i) - d^-(k+L+i)$ and $\Delta_{k-1,i} \triangleq d^+(k-1+L+i) - d^-(k-1+L+i)$.

Proof By (14),(15), condition (17) holds if and only if

$$d^+(k+L+i) - \rho^- \left[d^+(k-1+L+i) - d^-(k-1+L+i)\right] > d^-(k+L+i)$$

namely (18) holds.

4 SOLVING THE MIN-MAX MPC AS A CONSTRAINED ROBUST LS PROBLEM

In this section we reformulate the min-max optimization problem (4),(5) as a constrained robust LS estimation problem that can be numerically solved much more efficiently using interior point methods (Lobo et al., 1998). We define the following vectors:

- $\mathbf{u}_k \triangleq [u(k|k), \cdots, u(k+N-1|k)]^T$
- $\nu_l \triangleq [0,\cdots,0,1,0,\cdots,0]$ where the element 1 is in the l-th position $(1 \leq l \leq N)$
- ν_0 is the $(1 \times N)$ null row vector.

This allows rewriting each element $u(k+i|k)$, $i=0,\cdots,N-1$, of the optimal predicted sequence $\mathcal{U}_k$ as

$$u(k+i|k) = \nu_{i+1}\,\mathbf{u}_k \qquad (19)$$

We now show that the column vector $\mathbf{u}_k$ can be computed as the solution of the constrained robust LS estimation problem defined beneath.

As $\rho \in [\rho^-, \rho^+]$, an equivalent representation of ρ is

$$\rho = \bar{\rho} + \delta\rho \qquad (20)$$

where $\bar{\rho}$ is the central value of $[\rho^-, \rho^+]$ and $\delta\rho$ is the perturbation with respect to the nominal $\bar{\rho}$. From (20) it follows that

$$\rho^\ell = \bar{\rho}^\ell + \Delta\rho_\ell \qquad (21)$$

where $\Delta\rho_\ell$ is the sum of all terms containing the $\delta\rho$'s, in the explicit expression of ρ^ℓ.

Starting from (19) and (21), an equivalent representation of the predicted tracking error given by (6) is

$$e(k+L+i|k) = (b_{k,i} + \delta b_{k,i}) - (D_{k,i} + \delta D_{k,i})\mathbf{u}_k$$

where

$$\begin{aligned} b_{k,i} \;&\triangleq\; \bar{\rho}^{L+i} y(k) + \sum_{\ell=0}^{L-1} \bar{\rho}^{L+i-\ell} u(k+\ell-L) \\ &-\; \bar{\rho}^{L+i} h(k) \\ &-\; \sum_{\ell=1}^{L+i-1} \bar{\rho}^{L+i-\ell} d(k+\ell|k) - d^+(k+L+i) \end{aligned}$$

$$\begin{aligned} \delta b_{k,i} \;&\triangleq\; \Delta\rho_{L+i} y(k) + \sum_{\ell=0}^{L-1} \Delta\rho_{L+i-\ell} u(k+\ell-L) \\ &-\; \Delta\rho_{L+i} h(k) - \sum_{\ell=1}^{L+i-1} \Delta\rho_{L+i-\ell} d(k+\ell|k) \\ &-\; \sum_{\ell=1}^{L+i-1} \rho^{L+i-\ell} \delta d(k+\ell|k) \\ &+\; \sum_{\ell=1}^{L+i-1} \rho^{L+i-\ell} z(k+\ell|k) \end{aligned}$$

$$D_{k,i} \triangleq \begin{cases} -\nu_{i+1} & i=0 \\ -\left[\sum_{\ell=0}^{i-1}(\bar{\rho}^{(i-\ell)}\nu_{\ell+1})+\nu_{i+1}\right] & i \geq 1 \end{cases}$$

$$\delta D_{k,i} \triangleq \begin{cases} -\nu_{i} & i=0 \\ -\left[\sum_{\ell=0}^{i-1}\Delta\rho_{i-\ell}\nu_{\ell+1}\right] & i \geq 1 \end{cases}$$

Also the term

$$\Delta u(k|k) = u(k|k) - u(k-1) \triangleq \nu_1 \mathbf{u}_k - u(k-1)$$

in J_k can be rewritten as

$$\Delta u(k|k) = (b_{u_k} + \delta b_{u_k}) - (D_{u_k} + \delta D_{u_k})\mathbf{u}_k$$

where:
$b_{u_k} = -u(k-1)$, $D_{u_k} = -\nu_1$, $\delta b_{u_k} = 0$ and $\delta D_{u_k} = \nu_0$. The definition of appropriate extended vectors $\underline{e}_k$, $\underline{u}_k^-$, $\underline{u}_k^+$, $\underline{b}_k$, $\delta\underline{b}_k$ and matrices $\underline{D}_k$ and $\delta\underline{D}_k$

$$\underline{e}_k = \begin{bmatrix} q_0^{1/2}(k)e(k+L|k) \\ \vdots \\ q_{N-1}^{1/2}(k)e(k+L+N-1|k) \\ \lambda^{1/2}(k)\Delta u(k|k) \end{bmatrix}$$

$$\underline{u}_k^- \triangleq \begin{bmatrix} u_{k,0}^- \\ \vdots \\ u_{k,N-1}^- \end{bmatrix} \quad \underline{u}_k^+ \triangleq \begin{bmatrix} u_{k,0}^+ \\ \vdots \\ u_{k,N-1}^+ \end{bmatrix} \tag{22}$$

$$\underline{b}_k = \begin{bmatrix} q_0^{1/2}(k)b_{k,0} \\ \vdots \\ q_{N-1}^{1/2}(k)b_{k,N-1} \\ \lambda^{1/2}(k)b_{u_k} \end{bmatrix} \quad \delta\underline{b}_k = \begin{bmatrix} q_0^{1/2}(k)\delta b_{k,0} \\ \vdots \\ q_{N-1}^{1/2}(k)\delta b_{k,N-1} \\ \lambda^{1/2}(k)\delta b_{u_k} \end{bmatrix}$$

$$\underline{D}_k = \begin{bmatrix} q_0^{1/2}(k)D_{k,0} \\ \vdots \\ q_{N-1}^{1/2}(k)D_{k,N-1} \\ \lambda^{1/2}(k)D_{u_k} \end{bmatrix} \quad \delta\underline{D}_k = \begin{bmatrix} q_0^{1/2}(k)\delta D_{k,0} \\ \vdots \\ q_{N-1}^{1/2}(k)\delta D_{k,N-1} \\ \lambda^{1/2}(k)\delta D_{u_k} \end{bmatrix}$$

allow us to reformulate the min-max MPC (4)-(5) as the following constrained robust LS estimation problem:

$$\min_{\mathbf{u}_k} \max_{\|\delta\underline{D}_k\|_s \leq \beta_k \, \|\delta\underline{b}_k\| \leq \xi_k}$$

$$\|(\underline{b}_k + \delta\underline{b}_k) - (\underline{D}_k + \delta\underline{D}_k)\mathbf{u}_k\|^2 \tag{23}$$

$$\text{subject to} \quad \underline{u}_k^- \leq \mathbf{u}_k \leq \underline{u}_k^+ \tag{24}$$

Exploiting a result of (Lobo et al., 1998), it has been shown in (Jetto and Orsini, 2024) that at any k, the solution $\mathbf{u}_k$ of the constrained robust LS estimation problem (23)-(24) can be determined minimizing the following sum of euclidean norms

$$\min_{\mathbf{u}_k} \|\underline{b}_k - \underline{D}_k \mathbf{u}_k\| + \beta_k\|\mathbf{u}_k\| + \xi_k \tag{25}$$

where $\mathbf{u}_k$ must satisfy (24).

Remark 3 As ξ_k is independent of $\mathbf{u}_k$, only the upper bound β_k on $\|\delta\underline{D}_k\|_s$ in (25) needs to be determined at each k. The numerical value of β_k is determined putting $\rho = \rho^+$. △

The theoretical considerations that justify this approach, mentioned in the Introduction, are reported in the following remark.

Remark 4 Feasibility of constraints (5) derives from: (19) and the consistency of (24) w.r.t. (23). The internal asymptotic stability of the controlled SC derives from: i) $0 < \rho < 1$, ii) $d(k) \leq \bar{d} < \infty$, $k \in Z^+$, iii) constraints (5). As the stated properties of stability and feasibility are independent of the length of the prediction horizon, it follows that assumption A5 can be limited to very short intervals P_k, only depending on the actual knowledge about the limits on the future customer demand.

5 NUMERICAL SIMULATIONS

We consider a highly perishable single echelon SC whose dynamics equation (1) is characterized by a lead time $L = 1$ and a very large uncertain DF $\alpha \in [\alpha^-, \alpha^+] = [0.45, 0.5]$ or, equivalently an uncertain decay factor $\rho \in [\rho^-, \rho^+] = [0.5, 0.55]$.

According to A5, at each $k \in Z^+$ and over a very short M-steps prediction interval P_k with $M = 2$, the unknown future demand is arbitrarily varying inside a given compact set $\mathcal{D}_k$. The profile of the whole, assumed, actual customer demand $d(k)$ is the irregular continuous line shown in Fig. 2. The dashed lines are the boundaries of the compact set $\mathcal{D}$ enclosing $d(k)$. The dynamic equation (1) of the actual SC is implemented assuming an actual decay factor $\rho = 0.53$.

The parameters defining the min max MPC algorithm are the length $N = 2 = M - L + 1$ of H_k (control horizon) and the weights of (6): $q_i(k) = \frac{1}{(0.01 \cdot d^+(k+L+i))^2} e^{-i}$, $\lambda(k) = \frac{1}{(0.01 \cdot u(k-1))^2}$ for $k \geq 1$, $\lambda(0) = 1$ chosen according to the guidelines given in (G.F. Franklin, 1990). The simulation is stopped at time $k = 40$. The ordering signal $u(k)$ obtained is reported in Fig. 3. This figure clearly shows the limitation of the BE: $u(k)$ has a smoother waveform than the customer demand and is contained in a narrower range. The actual ($d(k)$) and fulfilled ($h(k)$) customer demands are shown in Fig. 4. The almost total overlap of the two curves evidences the effectiveness of the proposed method: the percentage of Unsatisfied Customer Demand defined as

$$UCD = \frac{\sum_{k=0}^{40}|d(k) - h(k)|}{\sum_{k=0}^{40} d(k)} \times 100 \tag{26}$$

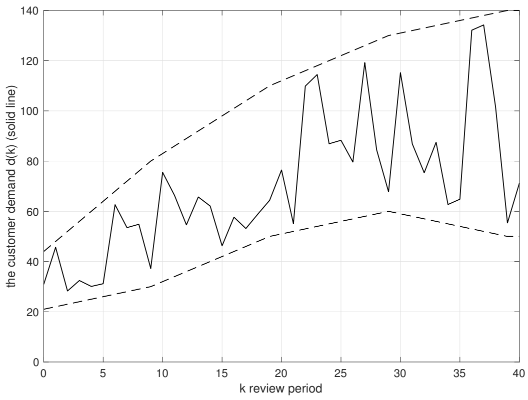

Figure 2: The actual customer demand d(k) (solid line). The dashed lines delimit the compact set D given by the consecutive contiguous overlapping of all the "a priori" given sets D_k's.

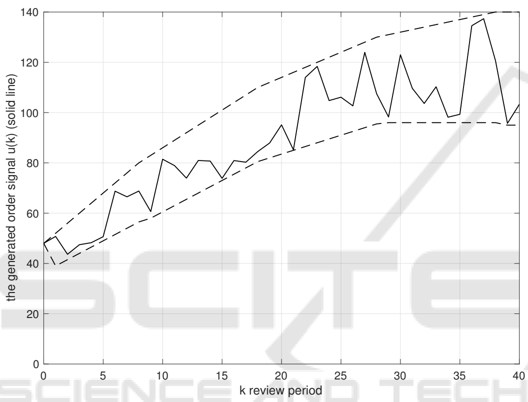

Figure 3: The generated ordering signal $u(k)$ (solid line) and the boundaries trajectories (dashed lines) computed by (14)-(15).

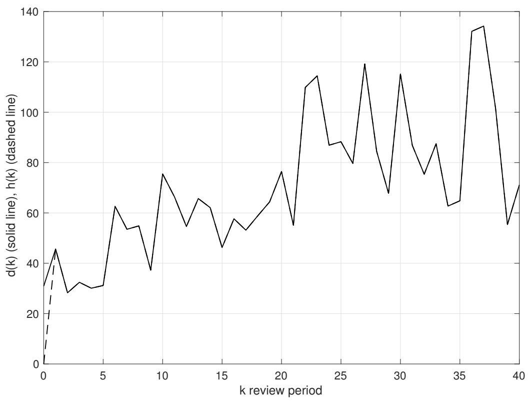

Figure 4: The customer demand $d(k)$ (solid line) and the fulfilled customer demand $h(k)$ (dashed line). The two trajectories are overlapped for $k \geq L$.

results to be $UCD = 1.1\%$.

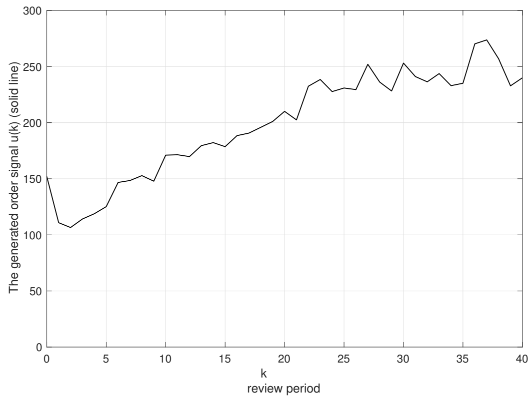

Figure 5: (OUT policy) The generated ordering signal $u(k)$ (solid line).

5.1 Comparison with the Order up to (OUT) Policy

With reference to the same SC, we compare our method with an OUT policy. To take into account the presence of perishable goods, of a time delay and of an uncertain future customer demand, we propose a version of the OUT policy where the predicted inventory level $y(k+L+1|k)$ coincides with the possible maximum value of the demand $d^+(k+L+1)$ in accordance with A5. The replenishment policy is computed solving (7) with respect to the single sample $u(k|k) \triangleq u(k)$ setting $i = 1$. To guarantee customer satisfaction according to a precautionary worst case approach we also assume:

- $y(k+L+1|k) = d^+(k+L+1)$;
- $h(k+\ell|k) = d(k+\ell) = d^+(k+\ell)$, $\ell = 1, \cdots, L$,
- $\rho = \rho^-$

Solving (7) gives

$$\begin{aligned} u(k) &= \frac{1}{\rho^-}\Big(d^+(k+L+1) + (\rho^-)^{L+1} h(k) \\ &+ \sum_{\ell=1}^{L} (\rho^-)^{L+1-\ell} d^+(k+\ell) - (\rho^-)^{L+1} y(k) \\ &- \sum_{\ell=0}^{L-1} (\rho^-)^{L+1-\ell} u(k+\ell-L)\Big) \end{aligned} \tag{27}$$

With the same fulfilled demand (the UCD performance index is the same) the OUT policy yields a higher replenishment order (compare figure 3 with 5) leading to performance degradation in terms of BE containment and excessive inventory.

6 CONCLUSIONS

Suitably exploiting the flexibility and generality of min-max MPC we defined an optimal and robust IRP to effectively counteract the negative effects of the assumed very critical operating conditions. In this context, the result on the point-wise hard constraints represents a more general key theoretical contribution towards the solution of the very long-standing problem of controlling the BE. Numerical simulations show the effectiveness of the method in reconciling the two opposing requirements R1 and R2.

7 FUTURE WORK

Possible and promising developments of this approach concern the extension to the case of an uncertain time varying decay factor.

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