Effect of Phase Mismatch on the Dynamics of Bragg Solitons in a Semilinear Coupled Bragg Grating System with Cubic-Quintic Nonlinearity

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Abstract: We investigate the dynamics of quiescent Bragg solitons in a dual-core fiber Bragg grating system with a phase shift between the gratings where one core has cubic-quintic nonlinearity, and the other is a linear core. Since cubic-quintic nonlinearity is present in one core, our system demonstrates the existence of two distinct and disjoint families of quiescent Bragg solitons within the specified bandgap, classified as Type 1 and Type 2 solitons. Both types of quiescent solitons have been analyzed numerically to assess their stability. The stability analysis reveals that the presence of the phase mismatch between Bragg gratings enhances the overall stability for Type 1 solitons and leads to the formation of stable Type 2 solitons.

1 INTRODUCTION

Fiber Bragg gratings (FBGs) are optical devices where the refractive index of the core varies periodically (or aperiodically) along the optical fiber (Sankey et al., 1992; Kashyap, 2009). FBGs have been the subject of intense research over the past few decades due to their applications in optical filtering, sensing, and optical signal processing (Krug et al., 1995; Loh et al., 1996; Litchinitser et al., 1997). FBGs can be utilized for switching as well as pulse compression when they are operated in the nonlinear regime (Winful et al., 1979; Radic et al., 1995; Sankey et al., 1992). A fascinating characteristic of FBGs is the existence of a bandgap which prevents the propagation of any linear waves (Kashyap, 2009). Another interesting feature of FBGs is that they exhibit a strong dispersion which is the result of cross-coupling between forward and backward- propagating waves (Russell, 1991; De Sterke and Sipe, 1994). When the intensity is high enough, the grating-induced dispersion can be counterbalanced by the nonlinearity resulting in the generation of Bragg grating (BG) solitons (De Sterke and Sipe, 1994).

In recent decades, Bragg solitons have been the subject of significant research both theoretically (Sipe and Winful, 1988; Christodoulides and Joseph, 1989; Aceves and Wabnitz, 1989) and experimentally (Eggleton et al., 1996; De Sterke et al., 1997; Taverner et al., 1998). In particular, the dynamics and stability of BG solitons have been investigated in different types of nonlinearities such as Kerr nonlinearity (Aceves and Wabnitz, 1989; Eggleton et al., 1996), quadratic nonlinearity (Conti et al., 1997; Mak et al., 1998b), and cubic-quintic nonlinearity (Atai and Malomed, 2001; Islam and Atai, 2014; Islam and Atai, 2018; Atai, 2004; Dasanayaka and Atai, 2010). The study of gap solitons in cubic-quintic nonlinear media has become compelling due to their added complexity and control, along with the enriched diversity of soliton dynamics. Moreover, Bragg solitons dynamics have been explored in various optical structures including waveguide arrays (Mandelik et al., 2004), grating-assisted single core (Atai and Malomed, 2001) and dual core system (Mak et al., 1998a; Atai and Malomed, 2000; Mak et al., 2004).

It has been shown that a broad spectrum of solitons can be supported by semilinear coupled systems, which also have excellent switching features (Atai and Malomed, 2000; Shnaiderman et al., 2011; Chowdhury and Atai, 2016; Chowdhury and Atai, 2017). Furthermore, introducing a phase shift between the gratings has drawn significant attention for designing optical sensor and multiplexer (Srivastava et al., 2018). It has been shown that in coupled Bragg gratings with a phase shift where both cores have Kerr nonlinearity, asymmetric and quasisymmetric solitons can exist (Tsofe and Malomed, 2007).

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In this work, we investigate the existence and stability of quiescent BG solitons in a system of coupled FBGs where one core has cubic-quintic nonlinearity and the other core is linear and there is a phase mismatch between the gratings.

2 THE MODEL

The model describing the propagation of light in a semilinear coupled Bragg gratings with a phase mismatch where cubic-quintic nonlinearity is present in one core and the other core is linear can be written as:

$$
\begin{aligned}
&iu_t + iu_x + \left[|v|^2 + \tfrac{1}{2}|u|^2\right]u - \eta\left[\tfrac{1}{4}|u|^4 + \right.\\
&\left.\tfrac{3}{2}|u|^2|v|^2 + \tfrac{3}{4}|v|^4\right]u + v + \kappa\phi = 0,\\
&iv_t - iv_x + \left[|u|^2 + \tfrac{1}{2}|v|^2\right]v - \eta\left[\tfrac{1}{4}|v|^4 + \right.\\
&\left.\tfrac{3}{2}|v|^2|u|^2 + \tfrac{3}{4}|u|^4\right]v + u + \kappa\psi = 0,\\
&i\phi_t + ic\phi_x + \psi e^{i\frac{\theta}{2}} + \kappa u = 0,\\
&i\psi_t - ic\psi_x + \phi e^{-i\frac{\theta}{2}} + \kappa v = 0,
\end{aligned}
\tag{1}
$$

where u and v stand for the forward and backward-propagating waves in the nonlinear core and ϕ and ψ are their counterparts in the linear core respectively. $\kappa > 0$ is the linear coupling coefficient between two cores and $c \leq 1$ denotes the relative group velocity in linear core and the group velocity in the nonlinear core has been set to one. $\eta > 0$ governs the strength of quintic nonlinearity and $0 \leq \theta \leq 2\pi$ represents the phase mismatch between the gratings. From the experimental measurements in Ref. (Boudebs et al., 2003; Chen et al., 2006; Zhan et al., 2002), it is found that η can vary between the values of 0.05 and 0.62. Therefore, our investigation is limited to the range $0 \leq \eta \leq 1$.

To define the spectral bandgap within which soliton solutions may exist, it is crucial to analyze the linear spectrum of the model. Eqs. 1 give rise to the following dispersion relation:

$$
\begin{aligned}
\omega^4 &= 2\kappa^2\left(2\cos^2\left(\tfrac{\theta}{4}\right) - 1\right) - c^2\left(k^2+1\right)k^2\\
&+\omega^2\left[\left(c^2+1\right)k^2 + 2\kappa^2 + 2\right] - \kappa^4\\
&+k^2\left(2c\kappa^2 - 1\right) - 1,
\end{aligned}
\tag{2}
$$

where k represents the wave-number. Analysis of Eq. 2 demonstrates that the linear spectrum contains three bandgaps namely, the upper, the central, and the lower bandgaps. The bandgap spectrum is illustrated in Figure 1. When $\theta = 0$, the width of the central bandgap can be defined by $|\omega| \leq (1-\kappa)$. It is found that the central gap widens as θ is increased. Specifically, under the highest phase mismatch condition (i.e. $\theta = 2\pi$), the central gap merges with the upper and lower gaps, forming a single bandgap at $k = 0$.

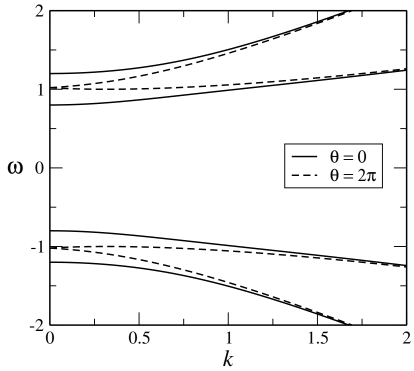

Figure 1: Linear spectrum at $c = 0.4$, $\kappa = 0.2$ and different values of θ.

3 SOLITON SOLUTIONS

Since no exact analytical solution exists for the model Eqs. 1, the quiescent soliton solutions have to be obtained numerically. To this end, we first substitute $\{u(x,t), v(x,t)\} = \{U(x), V(x)\}\exp(-i\omega t)$, and $\{\phi(x,t), \psi(x,t)\} = \{\Phi(x), \Psi(x)\}\exp(-i\omega t)$ into the Eqs. 1 and upon simplification we arrive at the following system of ordinary differential equations which is then solved using the relaxation method for different system parameters:

$$
\begin{aligned}
&\omega U + iU_x + \left(|V|^2 + \tfrac{1}{2}|U|^2\right)U - \eta\left[\tfrac{1}{4}|U|^4 + \right.\\
&\left.\tfrac{3}{2}|U|^2|V|^2 + \tfrac{3}{4}|V|^4\right]U + V + \kappa\Phi = 0,\\
&\omega V - iV_x + \left(|U|^2 + \tfrac{1}{2}|V|^2\right)V - \eta\left[\tfrac{1}{4}|V|^4 + \right.\\
&\left.\tfrac{3}{2}|V|^2|U|^2 + \tfrac{3}{4}|U|^4\right]V + U + \kappa\Psi = 0,\\
&\omega\Phi + ic\Phi_x + \Psi e^{i\frac{\theta}{2}} + \kappa U = 0,\\
&\omega\Psi - ic\Psi_x + \Phi e^{-i\frac{\theta}{2}} + \kappa V = 0.
\end{aligned}
\tag{3}
$$

In the case of $c = 0$, our numerical analysis indicates that the quiescent solitons completely fill all three bandgaps. However, when $c \neq 0$, quiescent soliton solutions are found only in the central bandgap. Moreover, due to the presence of cubic-quintic nonlinearity in one core, it is found that the model supports two different and disjoint families of solitons, namely Type 1 and Type 2 solitons. These soliton families are separated by a border which can be determined numerically. It is noteworthy that Type 1 and Type 2 solitons differ in their shapes, phases, and parities of their real and imaginary parts. In particular, near the border, the Type 2 solitons may have a nonsingular and sharp peak, featuring a distinctive two-tier profile. Figure 2 shows the examples of soliton profiles for Type 1 and Type 2.

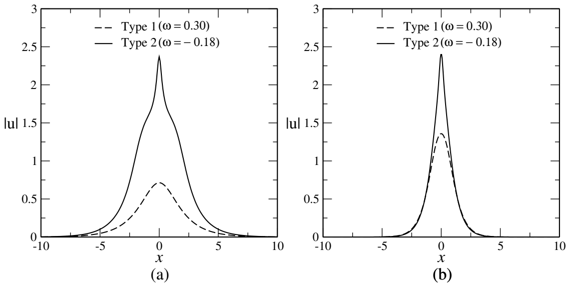

Figure 2: Type 1 and Type 2 soliton profiles for (a) $c = 0.4$, $\kappa = 0.5$, $\eta = 0.18$, $\theta = 0$; and (b) $c = 0.4$, $\kappa = 0.5$, $\eta = 0.18$, $\theta = 2\pi$.

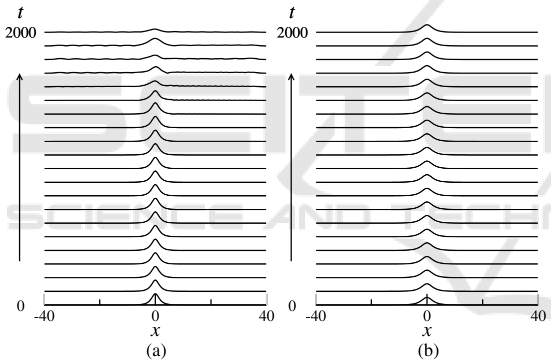

Figure 3: Propagation of Type 1 quiescent solitons with $c = 0.4$, $\kappa = 0.5$, $\theta = 0$. (a) $\eta = 0.12$, $\omega = 0.22$ (Unstable); and (b) $\eta = 0.14$, $\omega = 0.38$ (Stable).

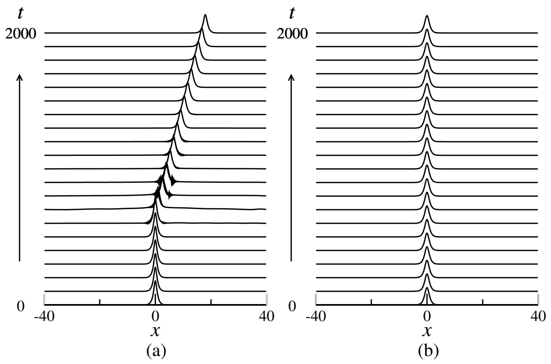

Figure 4: Propagation of Type 1 quiescent solitons with $c = 0.4$, $\kappa = 0.5$, $\theta = 2\pi$. (a) $\eta = 0.06$, $\omega = -0.48$ (Unstable); and (b) $\eta = 0.12$, $\omega = 0.22$ (Stable).

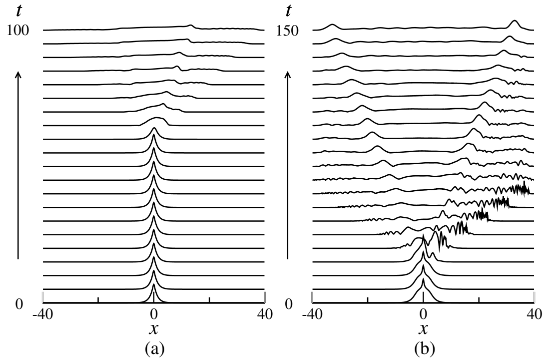

Figure 5: Propagation of Type 2 quiescent solitons with $c = 0.4$, $\kappa = 0.5$, $\theta = 0$. (a) $\eta = 0.72$, $\omega = -0.06$ (Unstable); and (b) $\eta = 0.44$, $\omega = 0.26$ (Unstable).

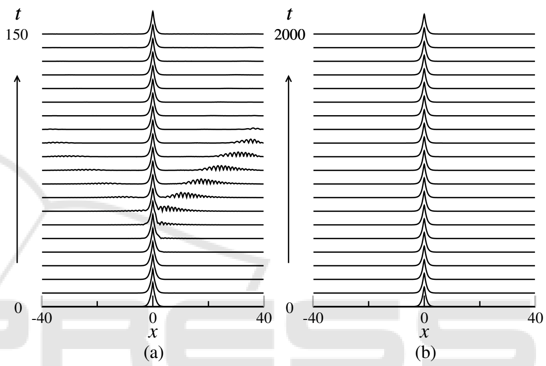

Figure 6: Propagation of Type 2 quiescent solitons with $c = 0.4$, $\kappa = 0.5$, $\theta = 2\pi$. (a) $\eta = 0.48$, $\omega = 0.38$ (Unstable); and (b) $\eta = 0.72$, $\omega = -0.06$ (Stable).

4 STABILITY ANALYSIS

To investigate the stability of the quiescent solitons, we have utilized the split-step Fourier transform method. Our analysis reveals the presence of both stable and unstable Type 1 and Type 2 solitons in our model depending on the values of system parameters. Figures 3 and 4 present the examples of stable and unstable Type 1 solitons propagation for $\theta = 0$ and $\theta = 2\pi$ respectively. It is apparent that the unstable Type 1 solitons typically lose some energies as radiation or may evolve into moving solitons after propagating a certain distance. Likewise, Figures 5 and 6 illustrate the evolution of Type 2 solitons for $\theta = 0$ and $\theta = 2\pi$ respectively. A notable observation is that Type 2 solitons exhibit strong instability and quickly decay into radiation especially when $\theta = 0$. On the other hand, in the case of $\theta = 2\pi$, stable Type 2 solitons are also observed. This demonstrates that the presence of phase mismatch has a stabilizing effect. The interplay of θ and other parameters on the stability of solitons is currently under investigation.

5 CONCLUSIONS

We have investigated the existence and stability of quiescent gap solitons within a system of coupled Bragg gratings where one core has cubic-quintic nonlinearity and the other core is linear. In addition, we have considered a phase shift between the gratings to explore the dynamics of BG solitons in the system. By examining the linear spectrum of the system, we have identified the bandgap region where the stationary gap solitons exist. Numerical methods have been employed to determine the soliton solutions which reveals the existence of two distinct families of Bragg solitons, categorized as Type 1 and Type 2. We have performed numerical analysis to assess the stability within each family of solitons and observed stable and unstable propagation for both types. In the absence of the phase shift, Type 2 solitons are generally unstable. Our findings indicate that the presence of a higher phase shift between the gratings expands the overall stability for Type 1 solitons and leads to the formation of stable Type 2 solitons.

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