Exploration of a Generalized Benders Decomposition Method for Solving Project Scheduling Problems with Resource Constraints

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Abstract: This research introduces a new Generalized Benders Decomposition-based Algorithm (GBDA) to solve the Multi-Mode Resource-Constrained Project Scheduling Problem (MRCPSP). The MRCPSP is a scheduling problem that besides precedence constraints, includes renewable and non-renewable resource constraints, as well as the selection of execution modes for the project activities. This mode selection determines the resource usage and duration of each activity. The GBDA splits the problem into a Master Problem (MP) and a Sub-Problem (SP) with a relaxation. Both problems are solved alternately, each one incorporating information from the other at each iteration, until a stopping criterion is met. Additionally, at each iteration, a non-relaxed SP is solved to obtain a solution for the original problem, and the best solution from all iterations is reported. The GBDA was tested, with three different stopping criteria, on benchmark instances from a public library and compared against solving the traditional formulation of the problem with an exact Mixed Integer Linear Programming (MILP) method. The GBDA found solutions of good quality in less than half the computing time than the exact method, with one of the stopping criteria. The analysis of the results provides valuable insights for future research.

1 INTRODUCTION AND LITERATURE REVIEW

The project scheduling branch of operations research deals with several types of optimization problems that require project activities to be sequenced subject to precedence constraints (i.e., some activities must be finished for other activities to start), usually with the objective or minimizing the duration of the project. One version of such project scheduling problems is the Resource-Constrained Project Scheduling Problem (RCPSP), which besides the usual precedence constraints, includes also resource constraints (i.e., activities require a certain amount of resources to be performed, and there is a maximum availability of those resources). This problem was introduced by (Dike, 1964), and since then, several authors have proposed variants extending or generalizing the RCPSP (Ding et al., 2023).

One well-known generalization of the RCPSP, proposed by (Elmaghraby, 1977), is the Multi-Mode Resource-Constrained Project Scheduling Problem (MRCPSP), which considers that project activities can be performed according to different execution modes, and that the resource consumption and duration of each activity depend on its selected execution mode, with a trade-off among them. If an activity is performed under an execution mode with a shorter duration, it will consume more resources, and vice-versa. Those resources can be renewable (i.e. their availability renews every time period), like workers or machinery; or non-renewable (i.e. they have a limited availability for the complete project), like monetary resources. The solution of the problem involves two decisions: the selection of an execution mode for each activity and the scheduling (i.e. selection of a start and finish time) of all activities.

There are several real-life applications of the MRCPSP, mostly in industrial and business contexts. Considering a manufacturing project as an example, an operation (project activity) can be performed faster (shorter activity duration) if more workers are assigned to it (greater renewable resource consumption). Or, if there is an operation that requires some

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material to be dried, for example, there could be two options (execution modes): to let it dry on its own (longer duration) or to use a drier machine (shorter duration) which costs more (greater non-renewable resource consumption). Besides manufacturing, software development and construction projects, among others, can also be modeled as MRCPSP.

The complexity of the MRCPSP was classified as NP-Hard by (Blazewicz et al., 1983). Several solution methods have been proposed to solve it, including exact mixed-integer linear programming (MILP) methods (Sprecher et al., 1997; Kyriakidis et al., 2012). However, due to the difficulty of solving large instances of the problem within computational times short enough to be practical using traditional exact methods (Sprecher and Drexl, 1998), some authors have proposed heuristic and metaheuristic methods to solve them. (Van Peteghem and Vanhoucke, 2014) provide a review of the most relevant metaheuristic methods proposed to solve the MRCPSP until 2012. Since then, more metaheuristic methods have been studied including: path relinking (Muritiba et al., 2018), variable neighbourhood search (Chakrabortty et al., 2019), simulated annealing (Shokoohyar and Amiri, 2021), memetic algorithms (Machado-Domínguez et al., 2021), genetic algorithms (Sebt et al., 2015; Zamani, 2019; Shokoohyar and Amiri, 2021; Afshar et al., 2022), multi-start iterated local search (Ramos et al., 2022), and ant colony optimization (Li and Zhang, 2013; de la Pisa et al., 2024). While there has been a substantial amount of research regarding metaheuristic methods for solving the MRCPSP, there have been considerably fewer attempts at using mathematical programming strategies to solve it efficiently.

A mathematical programming-based method used for solving MILP problems, known as Benders Decomposition (BD) and proposed originally by (Benders, 1962), is a technique that separates the decision variables of the problem in two sets, one of them handled by a Master Problem (MP), and the other one by a Sub-problem (SP). The values of the decision variables obtained by solving the MP become fixed parameters in the SP, and the solution of the SP provides optimality and feasibility cuts that are added to the MP, which is solved again, providing new values for the SP. This process iterates until a convergence criterion is reached. This method has been used to solve the p-median problem (Duran-Mateluna et al., 2023), assembly line balancing problems (Sikora and Weckenborg, 2023), location problems (Bayram and Yaman, 2018), vehicle routing problems (Fachini and Armentano, 2020), and multi-trip traveling repairman problems (Bruni et al., 2022), among others.

An extension of BD, capable of dealing with non-linear formulations by relying on the dual-Lagrangian formulation of the SP for generating the feasibility and optimality cuts, is known as Generalized Benders Decomposition (GBD) and was proposed by (Geoffrion, 1972). This approach has been used to solve pricing problems (Shams-Shoaaee and Hassini, 2020; Yaghin and Goh, 2021), inventory location problems (Tapia-Ubeda et al., 2018; Tapia-Ubeda et al., 2024), resource allocation problems (Li et al., 2019), and others.

There are some studies involving the utilization of BD or GBD for solving some versions of the MRCPSP or related problems, including: a matheuristic approach to solve the MRCPSP by (Ramos et al., 2024) combining GBD and heuristic scheduling strategies; the work by (Balouka and Cohen, 2021) and (Bold and Goerigk, 2022), who developed methods based on BD to solve stochastic versions of the MRCPSP; a BD approach to solve the single-mode RCPSP by (Bruni et al., 2017); the proposal by (Sadeghloo et al., 2023) of a goal programming method in combination with BD for the Multi-Project MRCPSP; and finally, the research by (Maniezzo and Mingozzi, 1999) regarding using BD only with the purpose of finding lower bounds for instances of the MRCPSP. It is worth noting that most of these studies use BD, not GBD, and solve a variety of problems similar to the MRCPSP. To the best of our knowledge, there are currently no published research articles that employ the same GBD-based solution method as the one proposed in this research, for solving the deterministic MRCPSP.

In order to address this gap in the scientific literature, the purpose of this research, is to propose a Generalized Benders Decomposition-based Algorithm (GBDA) to solve the MRCPSP, capable of finding good-quality solutions in short computing times. The proposed GBDA works by applying a partial relaxation to the problem; solving it with the GBD method; solving the original problem (without the relaxation) with an exact method at each iteration; and finding the best solution of all iterations.

The proposed GBDA was tested, with three different stopping criteria, on benchmark instances with 30 activities per project, obtained from the PSPLIB library published by (Kolisch and Sprecher, 1997). The results of the computational tests show a trade-off between computing time and solution quality in the three cases. With one of the tested stopping criteria, the GBDA was able to provide good quality solutions in less than half the time taken by a Baseline method, which solves the traditional formulation of the problem with an exact MILP method.

The development of the proposed GBDA, as a new method to solve the MRCPSP, as well as the insights from the analysis of the results of its implementation, provide valuable contributions to the research field, mainly as a starting point for future research along this scarcely explored, but promising path.

2 PROBLEM FORMULATION

The MRCPSP contains a set of project activities j or $k \in J$, which are numbered from 1 to p, plus two dummy activities: a "start" activity number 0 and a "finish" activity number $p+1$. These activities require to be sequenced considering a set of relationships of precedence among pairs of them $(j,k) \in PR$, and without exceeding resource availabilities. Dummy activities have a duration of 0 time units and no resource usage. There is a set of non-renewable resources $n \in NR$ with a maximum availability of QN_n. There is also a set of renewable resources $r \in RR$ with a maximum availability of QR_r, which renews itself each time period t. Activities can be performed according to different modes $m \in M$, which determine their duration d_{jm} and their consumption of non-renewable qn_{jmn} and renewable qr_{jmr} resources. Figure 1 shows a network diagram as a representation of a MRCPSP instance, consisting of 5 project activities, plus the two dummies, where activities are represented by nodes and relationships of precedence by arrows, and $PR = \{(0,1),(0,3),(1,2),(2,6),(3,4),(3,5),(4,6),(5,6)\}$.

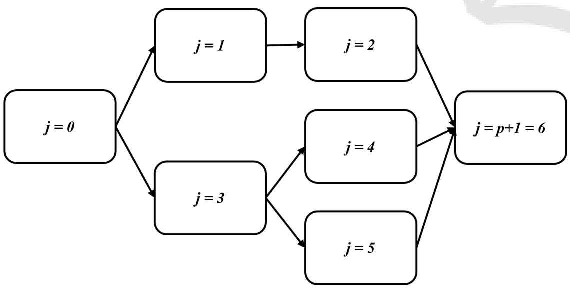

Figure 1: Network diagram or a MRCPSP instance.

A lower bound EF_j (earliest finish) and an upper bound LF_j (latest finish) for the finish time of each activity, as well as an upper bound T for the makespan of the project can be computed prior to solving the problem with the procedure described by (Lova et al., 2009). The solution of the problem includes two decisions. The first decision is the selection of an execution mode m for each activity j, modeled by the binary decision variable x_{jm}, which takes the value of 1 if activity j is executed in mode m and a value of 0 otherwise. The second decision is the assignment of a finish time t to each activity j, modeled by the binary decision variable y_{jt}, which takes the value of 1 if activity j finishes at time t and a value of 0 otherwise. The objective is to minimize the makespan of the project, which is equivalent to the finish time of the activity $p+1$.

Tables 1, 2 and 3 show a summary of the sets, parameters and variables of the mathematical model, respectively,

Table 1: MRCPSP mathematical model sets.

J	Project activities j or k
PR	Relationships of precedence (j,k)
M	Execution modes m
NR	Non-renewable resources n
RR	Renewable resources r

Table 2: MRCPSP mathematical model parameters.

p	Number of project activities
T	Upper bound for the project makespan
EF_j	Earliest finish time of activity j
LF_j	Latest finish time of activity j
d_{jm}	Duration of activity j in mode m
qn_{jmn}	Usage of non-renewable resource n by activity j in mode m
qr_{jmr}	Usage of renewable resource r by activity j in mode m
QN_n	Availability of non-renewable resource n
QR_r	Availability of renewable resource r

Table 3: MRCPSP mathematical model variables.

x_{jm}	Binary variable with a value of 1 if activity j is performed in mode m
y_{jt}	Binary variable with a value of 1 if activity j finishes at time period t.

Mathematical formulation:

Minimize:

$$\sum_{t=EF_{p+1}}^{LF_{p+1}} t\, y_{p+1,t}\,, \tag{1}$$

subject to:

$$\sum_{m \in M} x_{jm} = 1 \quad \forall j \in J, \tag{2}$$

$$\sum_{t=EF_j}^{LF_j} y_{jt} = 1 \quad \forall j \in J, \tag{3}$$

$$\sum_{t=EF_j}^{LF_j} t\, y_{jt} \leq \sum_{t=EF_k}^{LF_k} t\, y_{kt} - \sum_{m \in M} d_{km} x_{km} \quad \forall (j,k) \in PR, \quad (4)$$

$$\sum_{j \in J} \sum_{m \in M} qn_{jmn} x_{jm} \leq QNn \quad \forall n \in NR, \quad (5)$$

$$\sum_{j \in J} \sum_{m \in M} \sum_{q=t}^{t+d_{jm}-1} qr_{jmr} x_{jm} y_{jq} \leq QR_r \quad \forall r \in RR, \quad \forall t \in \{0,...,T\}, \quad (6)$$

$$x_{jm} \in \{0,1\}, \quad y_{jt} \in \{0,1\} \quad \forall j \in J \quad \forall m \in M \quad \forall t \in \{0,\ldots,T\}. \quad (7)$$

Equation (1), the objective function to be minimized, represents the finish time of the last project activity. Constraints (2) and (3) ensure that one and only one execution mode and finish time is assigned to each activity. Equation (4) guarantees that all the predecessors of an activity are finished before it starts. Constraint sets (5) and (6) specify that resource availabilities cannot be exceeded, for non-renewable and for renewable resources, respectively. Equations (7) state the domain of the decision variables.

3 METHODOLOGY

The mathematical formulation of the MRCPSP described in section 2 defines two decision variables: x_{jm} to assign an execution mode to each activity and y_{jt} to assign a finish time to each activity. This nonlinear formulation, as proposed by (Ramos et al., 2024), allows the problem to be addressed with a decomposition approach. The traditional linear formulation of the MRCPSP, proposed by (Talbot, 1982) includes only one decision variable, and thus, cannot be solved using a decomposition method.

Since the formulation to be addressed is nonlinear due to the quadratic formulation of equation (6), GBD is employed. Under this approach, the problem is decomposed into a MP that deals with the mode selection (variables x_{jm}) and a SP that deals with the activity scheduling (variables y_{jt}). This decomposition of the problem is employed as part of the proposed GBDA described in this section.

A requirement for GBD to ensure optimality is that the variables in the SP are continuous. For this reason, a partial relaxation is necessary in the formulation of the SP, by changing the domain of the y_{jt} variables from binary to continuous, otherwise the proposed method could not be employed. Once this partial relaxation is considered, the SP to be solved at each iteration of the GBDA is formulated as follows:

Minimize:

$$\sum_{t=EF_{p+1}}^{LF_{p+1}} t\, y_{p+1,t} \quad (8)$$

subject to:

$$\sum_{t=EF_j}^{LF_j} y_{jt} = 1 \quad \forall j \in J, \quad (9)$$

$$\sum_{t=EF_j}^{LF_j} t\, y_{jt} \leq \sum_{t=EF_k}^{LF_k} t\, y_{kt} - \sum_{m \in M} d_{km} x^i_{km} \quad \forall (j,k) \in PR, \quad (10)$$

$$\sum_{j \in J} \sum_{m \in M} \sum_{q=t}^{t+d_{jm}-1} qr_{jmr} x^i_{jm} y_{jq} \leq QR_r \quad \forall r \in RR, \quad \forall t \in \{0,...,T\}, \quad (11)$$

$$0 \leq y_{jt} \leq 1 \quad \forall j \in J \quad \forall t \in \{0,\ldots,T\}. \quad (12)$$

Where equations (8), (9), (10) and (11) are the equivalent for the SP of equations (1), (3), (4) and (6), respectively; equation (12) states the domain of the decision variables; and x^i_{jm} are the fixed values of the variables x_{jm} (mode selection) obtained by the solution of the MP at each iteration i of the GBDA. It is important to note that the solutions of the SP (variables y_{jt}) are only valid for the relaxed (continuous) formulation, and not for the original (binary) problem.

Subsequently, the formulation of the corresponding MP to be solved at each iteration of the GBDA is the following:

Minimize:

$$\gamma \quad (13)$$

subject to:

$$\sum_{m \in M} x_{jm} = 1 \quad \forall j \in J, \quad (14)$$

$$\sum_{j \in J} \sum_{m \in M} qn_{jmn} x_{jm} \leq QNn \quad \forall n \in NR, \quad (15)$$

$$\gamma \geq \sum_{t=EF_{p+1}}^{LF_{p+1}} t\, y^h_{p+1,t} - \sum_{j|(j,k)\in PR}\ \sum_{k|(j,k)\in PR} \lambda^h_{jk} \left(\sum_{m\in M} d_{km}\, x_{km} - \sum_{t=EF_k}^{LF_k} t\, y^h_{kt} + \sum_{t=EF_j}^{LF_j} t\, y^h_{jt}\right) - \sum_{r\in RR}\sum_{t=0}^{T} \mu^h_{rt} \left(\sum_{j\in J}\sum_{m\in M}\sum_{q=t}^{t+d_{jm}-1} qr_{jmr}\, x_{jm}\, y^h_{jq} - QR_r\right) \quad \forall h = 1,\ldots,i \tag{16}$$

$$x_{jm} \in \{0,1\} \quad \forall j \in J \quad \forall m \in M. \tag{17}$$

Where equation (13) is the objective function; equations (14) and (15) are the equivalent in the MP of equations (2) and (5); equation (16) represents the optimality cuts added at each iteration i of the GBDA derived from the dual Lagrangian equivalence obtained by solving the SP; y^i_{jt} are the values of the decision variables y_{jt} from the solution of the SP at iteration i; and λ^i_{jk} and μ^i_{rt} are the Lagrangian multipliers given by the dual values for constraints (10) and (11), respectively, obtained from the solution of the SP at each iteration i. Feasibility cuts are not needed because the MP guarantees feasibility regarding non-renewable resources when selecting execution modes (variables x_{jm}), which become fixed values in the SP which only deals with activity scheduling (variables y_{jt}).

Since the solution of the relaxed SP formulated by equations (8) to (12) provides continuous values for variables y_{jt}, which in the original problem are binary variables, to be able to obtain solutions for the original problem, a binary SP is also formulated containing equations (8) to (11) and equation (18), which replaces equation (12).

$$y_{jm} \in \{0,1\}, \quad \forall j \in J. \tag{18}$$

The solution of the MP (mode selection), along with the solution of its corresponding binary SP (activity scheduling), constitute a complete solution for the original problem. Consequently, at each iteration of the GBDA, after solving the MP and obtaining the values of the x_{jm} variables, the two versions of the SP are solved: the continuous SP is solved in order to obtain the values of the continuous y_{jt} variables and the optimality cut required by the MP for the next iteration; and the binary SP is solved to obtain a solution for the original problem including the binary values for the y_{jt} variables. The algorithm stops when the solution of the MP at two consecutive iterations is the same. Figure 2 shows a flowchart of the proposed GBDA.

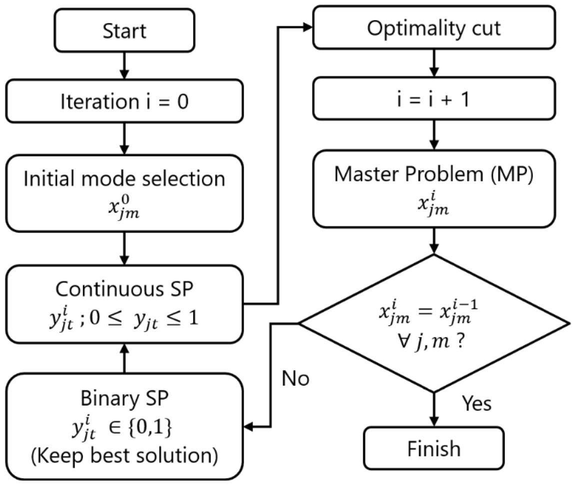

Figure 2: Flowchart of the GBDA.

4 EXPERIMENTATION

In order to assess the performance of the GBDA for solving the MRCPSP, several computational tests were performed on a personal computer with an 11th generation Intel i7 processor, a 2.8 GHz CPU and 32 GB of RAM. The AMPL software with the Gurobi optimization engine were employed. 50 Benchmark instances with 30 activities per project, 3 execution modes per activity, 2 types of renewable resources and 2 types of non-renewable resources, were used. They were obtained from the PSPLIB library published by (Kolisch and Sprecher, 1997).

Four different computational tests were carried out:

- Baseline: Solving the instances with the traditional linear formulation, using an exact MILP method directly with Gurobi, establishing a time limit of 100 minutes. The parameters for the optimization engine to establish the stopping criterion were set as follows:

```
option gurobi_options "timelim=6000"
```

- GBDA-A: Solving the instances with the GBDA described in section 3, considering a time limit of 100 seconds for each time the binary SP is solved. The parameters for the optimization engine to establish the stopping criterion were set as follows:

```
option gurobi_options "timelim=100"
```

- GBDA-B: Solving the instances with the GBDA, considering a time limit of 50 seconds for each time the binary SP is solved, and adjusting the Gurobi parameters to report optimality at a relative gap of 10%. The parameters for the optimization engine to establish the stopping criterion were set as follows:

```
option gurobi_options "timelim=50
mipgap=0.1"
```

- GBDA-C: Solving the instances with the GBDA, considering a time limit of 30 seconds for each time the binary SP is solved, and adjusting the Gurobi parameters to report optimality at a relative gap of 20%. The parameters for the optimization engine to establish the stopping criterion were set as follows:

```
option gurobi_options "timelim=30
mipgap=0.2"
```

The purpose of the Baseline method is to establish a benchmark against which to compare the GBDA. Since the GBDA solves the problem at each iteration, as opposed to the baseline method that solves it only once, a much shorter time limit per solution was defined, in order to obtain reasonable computing times for the GBDA. A time limit of 100 seconds per solution was set for test A of the GBDA. Considering the possibility that the solver could have already reached an optimal solution without being certain of it and still spend a considerable amount of time until optimality is proven, two other test of the GBDA were implemented: test B with a time limit of 50 seconds per solution and an relative MIP gap of 10%; and test C with a time limit of 30 seconds per solution and an relative MIP gap of 20%. The purpose of these different stopping criteria is to obtain solutions that could be optimal or sub-optimal with good quality, in a shorter amount of time.

5 RESULTS AND DISCUSSION

The results of the experiments were measured with the following variables: OPT_SOL, defined as the percentage of instances for which the optimal solution was found within the established time limits, in other words, the "success rate"; OF_DIF, defined as the average difference between the value of the objective function from the best solution found by the GBDA and the one from the best solution found by the Baseline method, expressed as a percentage; and TIME_DIF, defined as the average difference between the elapsed computing time needed by the GBDA and the one needed by the Baseline method to solve each instance, expressed as a percentage.

The results showing the performance of each of the methods are summarized in Table 4, while the complete results are presented in the appendix. A clear trade-off was found between the quality of the solutions found and the computational time taken by the GBDA in each of the tests. The values of the objective functions obtained with the GBDA range from 4.96% higher than the Baseline method to 8.96% higher, while the computational time taken ranges from 58.74% longer than the Baseline method to 49.22% shorter.

Table 4: Performance of the GBDA.

Method	OPT_SOL	OF_DIF	TIME_DIF
Baseline	94%		
GBDA-A	58%	4.96%	58.74%
GBDA-B	58%	5.69%	-5.95%
GBDA-C	58%	8.22%	-49.22%

The knowledge of this trade-off is valuable for decision-making. If a good-quality sub-optimal solution is acceptable, and time is limited, the GBDA-C method could be employed. On the other hand, if computing time is not a pressing issue and the optimal solution is highly preferred, the Baseline method is the best option. Table 4 also shows that reducing the time limit in combination with incrementing the relative MIP gap has a greater impact on the computing time (which is reduced considerably) than in the quality of the solution (which increases only a relatively small amount).

Additionally, to better understand the performance of the GBDA, the proportion of the total computational time devoted to the GBD method (i.e. solving the MP and the continuous SP), and the proportion of time devoted to solving the binary SP to obtain the solution of the original problem, were recorded separately. Their averages were calculated and assigned the variables TIME_GBD and TIME_BINSP. The resulting values are shown in Table 5.

Table 5: Proportion of time taken.

Method	TIME_GBD	TIME_BINSP
GBDA-A	8.74%	91.26%
GBDA-B	9.11%	90.89%
GBDA-C	18.07%	81.93%

It is clear that solving the binary SP employs most of the computing time taken by the GBDA, more than 90% in the case of GBDA-A and GBDA-B, and more than 80% in the case of GBDA-C, on average. While the GBD portion of the algorithm that deals with the MP and the continuous (relaxed) SP, is quite fast, once the relaxation is removed and the y_{jt} variables are considered binary as in the original problem, the SP requires considerably more computational time to be solved.

6 CONCLUSIONS

This research proposed a Generalized Benders Decomposition-based Algorithm (GBDA) to solve the Multi-Mode Resource-Constrained Project Scheduling Problem (MRCPSP). The algorithm divides the problem into a Master Problem (MP) and a relaxed (continuous) Sub-problem (SP) which are alternately solved, with the SP taking values from the solution of the MP, and with optimality cuts derived from the solution of the continuous SP being incorporated to the MP. This process iterates until convergence is met, resulting in an optimal solution for the relaxed problem. Additionally, at each iteration, a non-relaxed (binary) SP is solved to arrive at a solution of the original problem, finally reporting the best one across all iterations.

The GBDA, with three different stopping criteria, was tested and compared against a Baseline method, which solves the traditional MILP formulation with the Gurobi optimization engine. The results showed a trade-off between computing time and solution quality. With one of the tested stopping criteria, the GBDA took on average a little less than half the time taken by the baseline method, and provided solutions with an objective function value on average 8.22% above the one obtained by the baseline method. This is useful in the cases when arriving to a solution sooner has a higher priority than finding the optimal solution.

The main contributions of this research rely on the formulation of the GBDA as a new method to solve the MRCPSP. Besides being able to provide good-quality solutions in considerably less time than the traditional exact method, which is itself a useful contribution, the analysis of the proposed algorithm and its results provide important insights, valuable specially for future exploration. One of these insights is that most of the time taken by the algorithm is spent on solving the Binary SP. Future research could, thus, focus on exploring other decomposition variants or hybrid methods to improve the efficiency of solving the binary SP; or on using the relaxed GBD part of the algorithm, which proved to be very fast, as part of another algorithm or as some sort of pre-processing technique.

Another important finding is that increasing the optimality gap in Gurobi, had a much greater effect on the computational time than on the solution quality, which presents as well interesting opportunities for future exploration, for example by testing different combinations of time limits and MIP optimality gaps. The proposed algorithm could also be extended or modified to be used for similar problems or its variants, for example stochastic versions of the MRCPSP, multi-project scheduling, or other variants of the RCPSP.

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APPENDIX

Table 6 shows the complete results of the computational implementation for the 50 benchmark instances with the 4 methods.

The benchmark instances used for this research are publicly available at https://www.om-db.wi.tum.de/psplib/data.html.

Table 6: Results. a: Objective Function Value; b: Computational time (seconds).

	Baseline		GBDA-A		GBDA-B		GBDA-C	
Instance	OF[a]	Time[b]	OF	TIME	OF	TIME	OF	TIME
j3010_3	24	6.047	25	7.234	25	6.797	25	2.859
j3012_7	27	6.469	27	1.719	27	1.75	27	0.766
j3012_9	22	5.484	22	2.203	22	2.203	22	1.359
j3013_10	43	6000	49	4546.453	49	652.733	51	146.109
j3015_5	24	18.547	27	53.765	28	19.547	31	8.984
j3016_1	27	9.297	27	2.506	27	2.578	27	1.219
j3016_2	41	7.891	41	4.578	41	4.531	41	2.031
j3016_5	30	7.922	31	9.89	32	9.234	32	1.906
j3016_9	24	7.797	24	5.703	24	5.797	24	2.875
j3018_6	25	2.125	25	2.75	25	2.906	25	1.57
j3019_1	35	5.406	35	4.578	35	4.374	35	1.781
j3020_4	35	2.125	35	1.672	35	1.812	35	0.89
j3020_7	30	2.562	30	4	30	4.156	30	1.969
j3021_4	37	879.781	43	4025.594	44	224.297	45	44.937
j3021_5	37	6000	44	4443.797	44	1268.557	45	270.484
j3022_5	30	153.094	34	2339.563	36	45.86	39	35.844
j3022_9	34	11.484	34	13.203	34	10.859	34	5.344
j3023_10	21	7.703	21	8.578	21	7.312	21	3.968
j3025_1	34	5.547	38	14.016	38	11.031	39	1.028
j3025_8	33	5.875	39	23.969	39	17.688	43	7.284
j3026_2	22	2.328	27	8.375	27	7.828	29	3.297
j3026_5	22	4	22	3.078	22	2.75	22	1.453
j3027_3	28	1.75	28	1.749	28	1.546	28	0.843
j3027_10	29	1.625	29	1.735	29	1.781	29	1.062
j3028_1	37	3.688	37	1.641	37	1.547	37	0.875
j3028_5	23	3.609	23	1.031	23	1.172	23	0.609
j3031_1	25	4.125	28	10.359	29	7.312	30	5.14
j3031_2	43	5.828	43	4.03	43	3.687	43	2.109
j3032_8	29	2.578	29	3.125	29	3.172	29	1.375
j3039_2	42	81.656	43	14.734	43	11.328	43	5.422
j3039_4	49	58.469	51	7.64	51	6.296	62	3.202
j3040_9	43	14.688	43	4.406	43	4.25	43	2
j3041_7	39	203.312	43	50.437	46	31.938	43	21.234
j3043_7	33	4.078	33	3.922	33	3.703	33	2
j3044_3	34	11	34	2.875	34	2.672	34	1.5
j3045_6	47	6000	51	5579.797	50	1148.015	53	176.562
j3047_4	38	28.9	41	17.484	41	14.265	45	6.593
j3047_5	26	21.781	28	10.046	29	9.499	29	4.499
j3048_1	28	10.188	28	8.906	28	8.843	28	4.093
j3049_4	37	14.5	44	30.625	45	19.953	48	9.265
j3050_8	27	8.328	29	12.125	29	10.765	29	5.609
j3051_3	27	5.469	27	3.344	27	3.609	27	1.891
j3051_4	51	5.656	51	1.624	51	1.672	51	0.812
j3055_7	44	8.125	44	3.203	44	3.109	44	2.531
j3056_9	33	6.813	33	11.016	33	9.781	33	3.328
j3057_7	27	4.578	33	39.375	34	21.625	41	13.031
j3057_9	26	11.203	29	30.187	29	21.109	29	7.03
j3060_1	22	1.797	22	1.843	22	1.765	22	6.968
j3063_2	30	3.844	30	3.718	30	3.604	30	1.812
j3064_10	36	4.89	36	3.734	36	3.828	36	1.515