# Online Joint Optimization of Sponsored Search Ad Bid Amounts and Product Prices on e-Commerce

Shoichiro Koguchi[1], Kazuhide Nakata[1][a], Ken Kobayashi[1][b], Kosuke Kawakami[2], Takenori Nakajima[2] and Kevin Kratzer[2]

[1] *Department of Industrial Engineering and Economics, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku 152-8550, Tokyo, Japan*

[2] *HAKUHODO Technologies Inc., Akasaka Biz Tower, 5-3-1, Akasaka, Minato-ku, Tokyo, Japan*

*{koguchi.s.aa, nakata.k.ac, kobayashi.k.ar}@m.titech.ac.jp,*

*{kosuke.ka logies.co.jp*

Keywords: Bid Amount Optimization, Revenue Management, Dynamic Pricing, e-Commerce.

Abstract: With the rapid development of the e-commerce market, sellers are increasingly required to devise effective strategies to maximize sales and profits within limited resources. This paper insists that demand on e-commerce platforms can be partially controlled through advertising bid amounts. We examine the simultaneous control of demand and product pricing via advertising bids. Specifically, this study proposes a method for the joint optimization of advertising bid amounts and product prices to maximize sellers' sales and profits. Previous research has often focused on either advertising bid amounts or product prices, with little consideration of their simultaneous optimization. In contrast, our study develops an optimization method that accounts for the interdependencies between advertising bid amounts, advertising budgets, product prices, and inventory control. This comprehensive approach enables sellers to optimize advertising bid amounts and product prices simultaneously by considering these interrelated factors. Moreover, the proposed method demonstrates high scalability and is well-suited to real-world e-commerce markets, allowing for adaptation to various market conditions. Simulation results indicate that the proposed method significantly enhances sales and profits compared to approaches that do not incorporate price variability.



## 1 INTRODUCTION

The global e-commerce (EC) markets have been growing, driven by advancements in information technology and changing consumer behavior (Chen et al., 2016).

As the market expands, sellers on e-commerce platforms need to find more effective strategies. Among the multitude of products available, it is essential for sellers first to gain consumer awareness, which requires effective advertising strategies. Additionally, setting an appropriate product price is crucial to encouraging consumers to complete a purchase after being made aware of a product. In other words, advertising strategies and product pricing are two key elements that sellers must optimize. However, determining the appropriate settings for both of these elements is a complex task. Currently, both advertising bid amounts and product prices are often set based on specialists' expertise or the sellers' intuition and experience. Thus, this paper aims to support sellers in effectively setting both advertising bid Amounts and product prices.

This paper focuses on sponsored search ads, a type of programmatic advertising commonly adopted on many e-commerce platforms. Programmatic advertising refers to ads traded via bidding systems through platforms like Google Ads or Amazon Ads. It is popular due to its simplicity in allowing advertisers to launch campaigns even with a small budget and its flexibility to adjust ad bids, budgets, and delivery settings. In fact, in 2021, programmatic advertising accounted for 87.4% of internet advertising expenditures in Japan (DENTSU INC., 2024). Sponsored search ads are a form of programmatic advertising that appears on the search results pages of search engines. When consumers enter a keyword into the search engine, ads related to that keyword are displayed at the top of the search results (Figure 1).

[a] https://orcid.org/0000-0002-5479-100X
[b] https://orcid.org/0000-0002-6609-7488



Koguchi, S., Nakata, K., Kobayashi, K., Kawakami, K., Nakajima, T. and Kratzer, K.
Online Joint Optimization of Sponsored Search Ad Bid Amounts and Product Prices on e-Commerce.
DOI: 10.5220/0013118300003893

In *Proceedings of the 14th International Conference on Operations Research and Enterprise Systems (ICORES 2025)*, pages 67-78
ISBN: 978-989-758-732-0; ISSN: 2184-4372

Therefore, advertisers must set appropriate ad bids for each potential keyword to ensure their ads are shown. Keyword selection and bid settings are critical elements for sponsored search ads to function effectively.


Figure 1: Sponsored Search Advertising on Search Screens in E-commerce.

On the other hand, appropriate revenue management is necessary to encourage consumers to make purchases. For example, according to the research by Jamil et al. (2022), product pricing significantly impacts consumers' purchase intentions, and proper price setting directly influences sales. Setting a high product price increases the revenue per purchase, but the number of consumers willing to buy the product is expected to decrease due to the higher price. Conversely, setting a lower product price will likely result in more purchases, but the revenue per purchase will decrease. In addition to this trade-off, it is essential to set an appropriate price that considers factors such as inventory levels by each company's objective.

Based on these considerations, this paper aims to improve sellers' sales and profits by examining the online optimization of both ad bid amounts and product prices over multiple periods. The contributions of this paper are as follows:

- We mathematically model the optimization of bid amounts and product prices in the context described in Section 2. These two elements are closely intertwined in the purchase process, making it essential to account for their interdependence rather than treating them separately.
- Based on the related work (Majima et al., 2024), we proposed a method that simultaneously optimizes ad bid amounts and product prices over multiple periods using machine learning-based predictions. The method reduces the problem to a Mixed-Integer Programming (MIP) problem through discretization, allowing for efficient computational solving.
- The performance of the proposed method was validated through simulations using real-world data from Hakuhodo Technologies. These numerical experiments demonstrated not only the effectiveness of the proposed method but also its flexibility and scalability.

The structure of this paper is as follows. Section 2 explains revenue management in e-commerce sites. Specifically, it describes the process of generating sales and profits in e-commerce and highlights the necessity of simultaneously optimizing advertising bids and product prices. Section 3 reviews related research. It introduces previous studies and theories on advertising bid optimization and revenue management, providing the background knowledge for this study. Section 4 details this paper's proposed method. It explains the proposed approach's modeling, the formulation of the sales maximization problem, and the formulation of the profit maximization problem. Section 5 presents the results of numerical experiments conducted to verify the effectiveness of the proposed method. It discusses the construction of the simulator, the experimental setup, and the simulation results, confirming that the proposed method achieves superior outcomes compared to other models. Finally, Section 6 provides the conclusion.

# 2 REVENUE MANAGEMENT ON e-COMMERCE

## 2.1 Sales and Profit Generation Process on e-Commerce Sites

In this paper, the terms related to advertising operations are defined as follows:

- Impression
  When a user enters a keyword into a search engine, the advertisement is displayed on the search results page.
- Click
  When the user clicks on the displayed advertisement.
- Conversion
  When the user purchases a product associated with the clicked advertisement.

The sales and profit generation process in e-commerce sites is illustrated in Figure 2. First, an advertisement is displayed on the search results page after a user searches. Next, whether the user clicks on the displayed advertisement depends on the click-through rate. Similarly, whether the user purchases a product after clicking on the ad depends on the conversion rate. At this point, sales are the product of the number of conversions and the product price, while advertising costs are incurred based on the number of clicks and the ad bid amount. Additionally, inventory costs are incurred based on the remaining inventory. Among these processes, the two variables that sellers can control are the ad bid amount and the product price.

In the next section, we will describe in detail the mechanism by which the number of impressions and advertising costs arises, as shown in Figure 2, and clarify the impact that ad bids have on the sales and profit generation process in e-commerce sites.

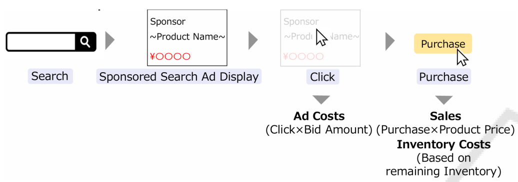


Figure 2: The flow until the seller generates profit.

## 2.2 Sponsored Search Ads

This section provides a detailed explanation of the type of advertisements handled in this paper.

### 2.2.1 The Process by Which Sponsored Search Ads Are Displayed

Sponsored search ads are displayed based on the keywords users enter into a search engine. For example, if a user searches for the keyword "wine," advertisements related to wine will be displayed.displaying the ad that wins the auction

Next, the process by which sponsored search ads are displayed is illustrated in Figure 3. When a user inputs a keyword into a search engine, an auction is held for the ads associated with that keyword. The advertising platform conducts this auction automatically, displaying the ad that wins the auction. However, many advertising platforms do not disclose the detailed mechanisms of these auctions, making it difficult for advertisers to measure the competitiveness of each ad and keyword precisely. For example, in Google Ads and Amazon Ads, factors such as the bid amount, the relevance of the ad text to the search, the estimated click-through rate, and the quality of the landing page contribute to a hidden score (quality score) calculated by the platform, which influences the auction results (AmazonAds, 2022; GoogleAds, 2024), but the specific algorithm is not disclosed.

As a result, advertisers must estimate the number of clicks and conversions they can expect for a given keyword and bid amount based on the limited information available to them.

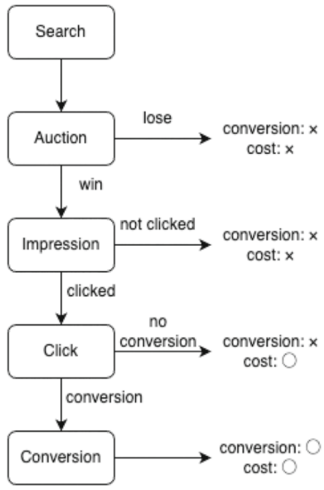


Figure 3: The process leading to the display of sponsored search advertisements (quoted from Majima et al. (2024)).

### 2.2.2 Information Available to Advertisers

Figure 4 illustrates the interaction between advertisers and the advertising platform. Advertisers set bid amounts for each keyword associated with an advertisement. Additionally, advertisers can flexibly adjust bid amounts throughout the day. Although real-time adjustments for each search and auction are impossible, advertisers can access detailed data from the advertising platform, allowing them to track impressions, clicks, conversions, and advertising costs at a more granular level than daily reports (Yang et al., 2020).

Additionally, advertisers set a budget for each advertisement. If the budget is depleted during the campaign period, the ad will no longer be displayed unless the advertiser replenishes the budget.

## 2.3 The Necessity of Simultaneously Determining Ad Bid Amounts and Product Prices

Revenue management, focusing on price adjustments, plays a crucial role in strategies for maximizing sales and profits for sellers. Revenue management optimizes the trade-off between product demand and pricing to maximize both revenue and profits. Moreover, it enables optimization that considers inventory levels, allowing for more efficient profit maximization by integrating inventory management with pricing strate-

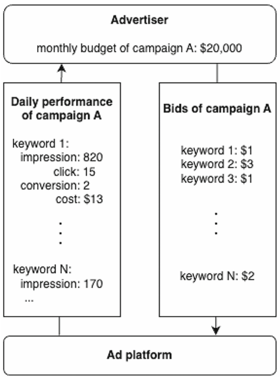


Figure 4: Interaction between advertisers and ad platforms (quoted from Majima et al. (2024)).

gies. In practice, on e-commerce sites like Amazon, sellers can set their product prices, and pricing decisions are critical factors influencing the seller's revenue and profits (Schlosser and Richly, 2019).

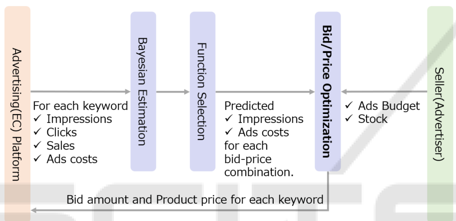


Figure 5: Overview of the proposed method by Majima et al. (2024).

However, several studies have highlighted the difficulty of demand forecasting in revenue management (Besbes and Zeevi, 2015; Koupriouchina et al., 2014). Therefore, this paper focuses on the fact that a significant portion of purchases on e-commerce platforms occurs through advertisements and considers controlling product demand via ad bid amounts. Specifically, by adjusting ad bid amounts, sellers can control the number of ad impressions and advertising costs while optimizing product prices, ultimately helping to improve overall sales and profits.

# 3 RELATED WORK

## 3.1 Bid Amount Optimization

The optimization of bid amounts in Internet advertising has been studied under various conditions, such as different advertising platforms, constraints, and scenarios. Nuara et al. (2022) formulated the problem of optimizing bid amounts and daily budgets across multiple platforms in cost-per-click advertising campaigns as a semi-bandit problem (Chen et al., 2013). Their study aims to maximize the expected revenue of an advertiser while adhering to daily budget constraints across all campaigns. Their algorithm is based on GP-UCB (Gaussian Process Upper Confidence Bound) (Srinivas et al., 2010), predicting conversions and advertising costs for each bid amount and performing optimization through dynamic programming. GP-UCB is a Bayesian optimization technique that uses Gaussian processes.

Inspired by the work of (Nuara et al., 2022; Majima et al., 2024), proposed a method focusing on keyword-level bid optimization over multiple periods. Figure 5 illustrates the overall framework of their approach. They combine Bayesian inference with bandit algorithms, modeling relationships between advertising metrics using Bayesian networks to predict the conversions and advertising costs associated with each keyword. Their method then solves an optimization problem that maximizes the number of conversions under budget constraints for each period, dynamically adjusting bid amounts while considering uncertainty. The study by Majima et al. (2024) is closely related to the advertising setup in this paper and serves as a foundational approach.

However, these studies assume constant product prices, equating the seller's revenue maximization with the maximization of conversions. This paper proposes a method that simultaneously optimizes both the bid amount and the product price, considering that click-through rates and conversion rates can vary with product prices. To the author's knowledge, no previous research has explored the simultaneous optimization of bid amounts and product prices, making this problem setup a significant contribution to this paper.

## 3.2 Revenue Management

Research on revenue management has become increasingly active in recent years, driven by advancements in data analysis technologies and the widespread use of the Internet. Since this paper focuses on setting product prices for each period, the field of dynamic pricing within revenue management is most relevant. Dynamic pricing is a pricing strategy that continuously adjusts prices based on fluctuating demand or available supply. This approach is commonly used in pricing for airlines, hotels, and event tickets, as well as in e-commerce pricing.

A dynamic pricing framework using deep reinforcement learning was proposed by Liu et al. (2019) for e-commerce platforms and demonstrated its effectiveness. They modeled the problem as a Markov decision process and evaluated it through online experiments on Tmall.com. However, appropriately representing the complex relationship between bid amounts and product prices using deep reinforcement learning is challenging, making this method difficult to apply to the current research.

In addition, Li and Zheng (2023) proposed a dynamic pricing model that combines external information with inventory constraints. Their model explores a method for dynamically setting prices under unknown demand functions by utilizing newly observed external information at the beginning of each period. This study maximizes expected cumulative revenue while balancing inventory management and pricing decisions.

Gharakhani et al. (2022) proposed a model for optimizing pricing, inventory management, and advertising frequency. Their model considers a time-dependent inventory cost function and explores a method for dynamically adjusting product prices and inventory levels for each product. This approach seeks to maximize revenue while accounting for the impact of inventory costs.

These studies highlight that considering inventory constraints and inventory costs plays a vital role in maximizing revenue and improving the efficiency of inventory management. This paper combines these insights with the foundational work of Majima et al. (2024) to explore the simultaneous optimization of bid amounts and product prices.

# 4 PROPOSED METHOD

## 4.1 Modeling

In this section, we explain the proposed model. We assume that the seller sells products over $T$ discrete periods and determines the optimal advertising bid amount and product price for each period. The advertising budget $R$ is the total budget across all periods, and the product stock is replenished at any quantity for each period. Additionally, any remaining stock at the end of a period will be carried over to the next period.

Based on these conditions, this paper considers solving a multi-period optimization problem as an online optimization problem for each period. In each period, based on the process by which sales and profit are generated on an e-commerce site (see Section 2.1), the flow through which the seller obtains sales and profit is assumed as shown in Figure 6.

First, after a user performs a search, whether or not an ad appears on the search results page and its position depends on the bid amount (generally, ads

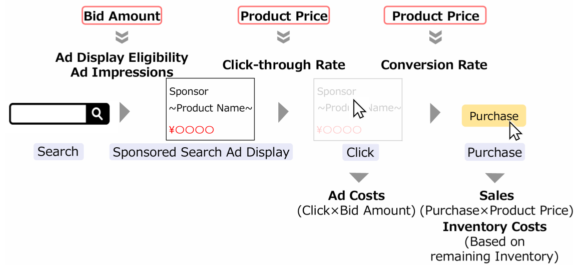


Figure 6: Flow until profit is generated for the seller.

displayed in higher positions tend to receive more impressions). Next, whether the user clicks on the displayed ad depends on the click-through rate (CTR), which is influenced by the product price. Similarly, the conversion rate (CVR), which indicates whether the user purchases the product after clicking, also depends on the product price. Furthermore, advertising costs are incurred based on the number of clicks, and sales are determined by multiplying the number of conversions by the product price. Additionally, inventory costs are incurred based on the remaining inventory level.

Based on the above flow, the expected sales $u(b,p)$ for the seller is modeled as follows:

$$u(b,p) = p \cdot \phi_{\mathrm{CTR}}(p) \cdot \phi_{\mathrm{CVR}}(p) \cdot v(b). \tag{1}$$

In equation (1), $b$ represents the bid amount, and $p$ represents the product price. The terms $\phi_{\mathrm{CTR}}(p)$ and $\phi_{\mathrm{CVR}}(p)$ refer to functions for the click-through rate and conversion rate, both of which are influenced by the product price. Lastly, $v(b)$ is a function representing the number of impressions determined by the bid amount.

The expected profit for the seller is then modeled as follows:

$$r(b,p) = u(b,p) - c(b,p). \tag{2}$$

In equation (2), $c(b,p)$ represents the cost incurred before the seller generates profit.

Additionally, budget constraints to ensure that advertising costs do not exceed the seller's advertising budget, as well as inventory constraints to ensure that the number of conversions does not exceed the available inventory, are set as follows:

$$c(b,p) \leq R, \tag{3}$$

$$\phi_{\mathrm{CTR}}(p) \cdot \phi_{\mathrm{CVR}}(p) \cdot v(b) \leq S. \tag{4}$$

In equation (3,4), $R$ and $S$ represent the advertising budget and inventory quantity in period $t$, respectively.

## 4.2 Formulation of Sales Maximization

Based on Section 4.1 and previous research, we formulate the sales maximization problem. In particular, we construct a model suitable for the problem setting of this paper concerning the study by Majima et al. (2024).

However, since the number of impressions $v(b)$ and the advertising cost $c(b, p)$ cannot be determined a priori, it is necessary to predict these values. We employ the Bayesian estimation method (Bayesian AdComB-PT) used in Majima et al. (2024) to predict the number of impressions and advertising costs.

Furthermore, as we cannot assume the convexity of the functions for impressions and advertising costs, it is difficult to obtain a globally optimal solution using continuous optimization methods. Therefore, we treat these functions by discretizing them. The problem is formulated as a discrete optimization problem by discretizing the combinations of bid amounts and product prices.

### 4.2.1 Explanation of Parameters

We explain the parameters used in the formulation.

$N$ denotes the number of keywords, representing the total number of keywords used in advertising. Next, $P$ represents the number of price candidates, indicating the number of possible product prices. $B$ is the number of bid candidates, which refers to the number of potential advertising bid amounts. These parameters are fundamental in building an advertising strategy.

The current inventory is represented by $S$, showing the amount of stock on hand. The remaining advertising budget is denoted by $R$, representing the budget left for advertising. The product price is expressed as $p_l$, representing the price set for each product.

$v_{i,j,l}$ refers to the predicted number of impressions for keyword $i$, bid amount $b_j$, and product price $p_l$. Similarly, $c_{i,j,l}$ represents the predicted advertising cost under the same conditions. The sales volume is denoted by $u_{i,j,l}$, representing the number of units sold for keyword $i$, bid amount $b_j$, and product price $p_l$. $x_{i,j,l}$ is a binary variable, taking a value of 1 if the combination of keyword $i$, bid amount $b_j$, and product price $p_l$ is selected, and 0 otherwise. Furthermore, $\phi_{\text{CTR}}(p_l)$ represents the click-through rate (CTR) at product price $p_l$, and $\phi_{\text{CVR}}(p_l)$ represents the conversion rate (CVR) at product price $p_l$. The inventory cost function is defined by $h(\cdot)$, which calculates costs based on the inventory level. The functions for CTR, CVR, and inventory costs will be discussed later. The parameters introduced so far are summarized below:

$N$ : Number of keywords
$P$ : Number of price candidates
$B$ : Number of bid candidates
$S$ : Current inventory
$R$ : Remaining budget
$p_l$ : Product price
$v_{i,j,l}$ : Predicted number of impressions
$c_{i,j,l}$ : Predicted advertising cost
$u_{i,j,l}$ : Sales volume
$x_{i,j,l}$ : Binary variable
$\phi_{\text{CTR}}(p_l)$ : Click-through rate at product price $p_l$
$\phi_{\text{CVR}}(p_l)$ : Conversion rate at product price $p_l$
$h(\cdot)$ : Inventory cost function

### 4.2.2 Objective Function

We now explain the objective function. Since the goal of optimization is to maximize sales, the objective function is the product of the predicted sales $u_{i,j,l}$ and the decision variable $x_{i,j,l}$:

$$\sum_{i=1}^{N}\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l} \cdot u_{i,j,l} \cdot p_l. \tag{5}$$

Additionally, $u_{i,j,l}$ (the predicted number of conversions) is calculated as the product of the predicted number of impressions, click-through rate, and conversion rate:

$$u_{i,j,l} = \phi_{\text{CTR}}(p_l) \cdot \phi_{\text{CVR}}(p_l) \cdot v_{i,j,l}. \tag{6}$$

### 4.2.3 Click-Through and Conversion Rate Functions

Click-through rate (CTR) and conversion rate (CVR) are defined as functions dependent on product price. This reflects the assumption that when a product is searched for on an e-commerce site, consumers make click and conversion decisions based on the product price.

Many studies (Li and Zheng, 2023; Bolton, 1989; Lee et al., 2023) use a log-linear function to compute price elasticity for their own products during specific periods. This paper adopts a similar method. In this case, the price elasticity of a product is equivalent to the slope of the regression function's $\beta$ coefficient, representing the change in sales volume for a given change in price.

$$\log Q_A = \alpha + \beta \cdot \log P_A \tag{7}$$

In equation (7), $Q_A$ represents the sales volume of product A, $\alpha$ is the intercept, and $\beta$ is the price sensitivity of product A.

In this paper, we assume that each company can calculate its products' price elasticity. Therefore, we treat the regression coefficients as parameters set by each company.

Since both click-through rate and conversion rate take values between 0 and 1, we use a sigmoid function to represent them. Based on this, the functions for CTR and CVR are defined as follows:

$$\begin{aligned}\phi_{\text{CTR}}(p) &= \sigma(\alpha_{\text{CTR}} + \beta_{\text{CTR}} \log(p))\\ \phi_{\text{CVR}}(p) &= \sigma(\alpha_{\text{CVR}} + \beta_{\text{CVR}} \log(p))\end{aligned} \tag{8}$$

In equation (8), $\sigma(z) = \frac{1}{1+e^{-z}}$ is the sigmoid function, and $\alpha_{\text{CTR}}, \beta_{\text{CTR}}, \alpha_{\text{CVR}}, \beta_{\text{CVR}}$ are parameters.

#### 4.2.4 Constraints

This section explains the four types of constraints used in the optimization.

- **Budget Constraint**
  This constraint ensures that the advertising cost $c_{i,j,l}$ stays within the given advertising budget for each period. While it is possible to set a lower limit as well as an upper limit on the budget, in the sales maximization problem, the optimization is expected to consume as much budget as possible, so no lower limit is set. To prevent the budget from being exhausted at the start of the period, the budget for each period is set as the remaining budget divided by the number of remaining periods:

$$\sum_{i=1}^{N}\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l} \cdot c_{i,j,l} \leq \frac{R}{T-t+1}. \tag{9}$$

- **Inventory Constraint**
  This constraint limits the number of conversions $u_{i,j,l}$ to stay within the available inventory for each period:

$$\sum_{i=1}^{N}\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l} \cdot u_{i,j,l} \leq S. \tag{10}$$

  For simplicity, this paper sets only an upper inventory limit, but depending on the objectives, the problem can also be formulated to approach the optimal inventory level.

- **Same Price for Each Product Constraint**
  This constraint ensures that the same product price is used across different keywords searched by users for the same product. In equation (11), $p'_l$ is a binary variable indicating whether a specific price is applied to the product. The following constraint holds under the assumption that each keyword is assigned a single bid amount:

$$N \cdot p'_l = \sum_{i=1}^{N}\sum_{j=1}^{B} x_{i,j,l} \quad \forall l \in \{1,2,\ldots,P\}. \tag{11}$$

- **Single Bid Amount and Product Price Selection per Keyword Constraint**
  This constraint ensures that only one combination of bid amount and product price is selected for each keyword:

$$\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l} = 1 \quad \forall i \in \{1,2,\ldots,N\}. \tag{12}$$

#### 4.2.5 Formulation

Based on the above, the sales maximization problem for each period handled in this paper can be summarized as follows. The variables are $x_{i,j,l}$ and $p'_l$:

$$\begin{aligned}
\max \quad & \sum_{i=1}^{N}\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l} \cdot u_{i,j,l} \cdot p_l\\
\text{subject to} \quad & \sum_{i=1}^{N}\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l} \cdot c_{i,j,l} \leq \frac{R}{T-t+1},\\
& \sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l} = 1 \quad \forall i \in [N],\\
& N \cdot p'_l = \sum_{i=1}^{N}\sum_{j=1}^{B} x_{i,j,l} \quad \forall l \in [P],\\
& \sum_{i=1}^{N}\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l} \cdot u_{i,j,l} \leq S,\\
& x_{i,j,l} \in \{0,1\}, \quad p'_l \in \{0,1\},\\
& \forall i \in [N], \quad \forall j \in [B], \quad \forall l \in [P].
\end{aligned} \tag{13}$$

The optimization problem (13) is an integer programming problem and can be solved using a general-purpose integer programming solver. This model allows finding the optimal combination of bid amount and product prices to maximize sales while efficiently utilizing the advertising budget.

### 4.3 Formulation of the Profit Maximization Problem

In Section 4.2, we formulated the maximization problem to maximize the seller's sales. The sales maximization problem is effective, especially when the goal is to exhaust the budget within the period or reduce inventory as much as possible. This applies to situations where the focus is on increasing product awareness. On the other hand, in actual business, it is often essential to consider the costs incurred before generating sales and making efficient decisions regarding bid amount and product prices. Therefore, in this chapter, the objective of the optimization is profit maximization, balancing sales and costs. Specifically, based on the process of generating sales and profits on e-commerce sites (2.1), the costs considered in this paper include advertising costs and inventory costs. Advertising costs depend on bid amounts and product prices, while inventory costs depend on the duration and size of product storage.

#### 4.3.1 Objective Function

Since our goal is to maximize total profit, the objective function is defined as the predicted profit (predicted sales - advertising costs - inventory costs) multiplied by the decision variable $x_{i,j,l}$, which is expressed as follows:

$$\sum_{i=1}^{N}\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l}\left(u_{i,j,l}\cdot p_l - c_{i,j,l} - h\left(S - u_{i,j,l}\right)\right) \quad (14)$$

#### 4.3.2 Inventory Cost Function

The inventory cost $h(\mathrm{S})$ is defined as a linear function of the inventory level. Specifically, the inventory cost increases depending on the amount of goods held in stock from this period to the next. The proportional coefficient is proportional to the size of the product and is a parameter that the seller can set. This reflects the fact that e-commerce sites like Amazon set storage fees based on the size and quantity of the inventory (Amazon, 2024).

#### 4.3.3 Formulation

Based on the above, the profit maximization problem for each period handled in this paper is defined as shown in (15), where the variables are $x_{i,j,l}$ and $p'_l$. As in (13), the optimization problem (15) can also be solved using a general-purpose integer programming solver. This model allows one to find the optimal combination of bid amount and product prices to maximize sales and minimize inventory costs while efficiently using the advertising budget.

$$\begin{aligned}
\max \quad & \sum_{i=1}^{N}\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l}\left(u_{i,j,l}\cdot p_l - c_{i,j,l}\right. \\
& \left. - h(S - u_{i,j,l})\right) \\
\text{subject to} \quad & \sum_{i=1}^{N}\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l}\cdot c_{i,j,l} \le \frac{R}{T-t+1}, \\
& \sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l} = 1 \quad \forall i \in [N], \\
& N\cdot p'_l = \sum_{i=1}^{N}\sum_{j=1}^{B} x_{i,j,l} \quad \forall l \in [P], \\
& \sum_{i=1}^{N}\sum_{j=1}^{B}\sum_{l=1}^{P} x_{i,j,l}\cdot u_{i,j,l} \le S, \\
& x_{i,j,l} \in \{0,1\}, \quad p'_l \in \{0,1\}, \\
& \forall i \in [N], \quad \forall j \in [B], \quad \forall l \in [P].
\end{aligned} \quad (15)$$

## 5 NUMERICAL EXPERIMENTS

### 5.1 Experimental Setup

In this study, we evaluated the performance of the proposed method using a simulation-based experimental.

Since it was difficult to access a real-world operation, we adopted a simulation-based approach. The simulator employed in this study was designed to replicate the operational environment of e-commerce platforms based on real advertising log data. Furthermore, the simulator was extended from the one proposed in Majima et al. (2024) to align with the objectives of this study. Specifically, while Majima et al. (2024) assumes constant product prices, we modified the simulator to account for the effects of price fluctuations on sales, CTR, and CVR.

To the best of our knowledge, there are no existing methods that can be directly applied to our problem setting. Thus, we conducted comparative experiments under various scenarios. We showed the specific experimental setting in Table 1. For everyday items, we assume a low price elasticity. In contrast, luxury goods are assumed to have high price elasticity. This distinction is critical as it allows us to explore various pricing patterns and evaluate their impact on sales performance. In our experiments, we set the parameters to $N = 10$, $B = 20$, $P = 20$, and $T = 10$. The Gurobi Optimizer (Optimization, 2022) was used to solve mixed-integer programming problems. For the current problem, it was solved within 10 seconds. Additionally, even when increasing the values of $N$, $B$, $P$, and $T$, Gurobi solved the problem without any issues.

Additionally, this paper compares and evaluates the following six models:

- **Model 1 (Proposed Method)**
  A model that optimizes both bid amounts and product prices.
- **Model 2 (Lowest Price Fixed Model)**
  A model that optimizes only the bid amounts, fixing the product price at the lowest among the candidates.

Table 1: Experimental Settings.

| Experimental Setting | Assumed Product | Objective Function | Budget | Supply Inventory per Period |
|---|---|---|---|---|
| Setting A | Everyday Item | Sales Maximization | 5,000,000 | 200 |
| Setting B | Luxury Good | Sales Maximization | 2,000,000 | 50 |
| Setting C | Everyday Item | Profit Maximization | 5,000,000 | 100 |
| Setting D | Luxury Good | Profit Maximization | 2,000,000 | 25 |

- **Model 3 (Highest Price Fixed Model)**
  A model that optimizes only the bid amounts, fixing the product price at the highest among the candidates.
- **Model 4 (Lowest Bid Fixed Model)**
  A model that optimizes only the product prices, fixing the bid amount at the lowest among the candidates.
- **Model 5 (Highest Bid Fixed Model)**
  A model that optimizes only the bid amounts, fixing the bid amount at the highest among the candidates.
- **Model 6 (Random Model)**
  A model that randomly selects bid amounts and product prices.

## 5.2 Optimization Results

In this section, we present the simulation results for the sales maximization problem and profit maximization problems, demonstrating the proposed method's effectiveness.

The experimental results using the simulator are illustrated in Tables 2, 3, 4, and 5. Tables 2 and 3 present the results for the sales maximization problem, while Tables 4 and 5 show the results for the profit maximization problem. The results indicate the improvement rates based on the metrics from the random model (Model 6). Empty cells indicate that the model was not executable.

The proposed method (Model 1) demonstrates that optimizing both the bid amount and product price can achieve the highest sales and profit. The random model (Model 6) was found to be ineffective, as it failed to utilize advertising budgets and inventory efficiently. Models 2 and 3 optimized bid amounts, but Model 2's low prices increased CTR and CVR while leaving advertising budgets underutilized due to inventory constraints. Model 3, with high prices, reduced CTR and CVR, limiting conversions. Models 4 and 5, which fixed bid amounts, either overspent or underutilized advertising budgets. The proposed method addressed these issues, overcame these challenges, and achieved improvements in both sales and profits. The foundational research for this paper (Majima et al., 2024) focused on optimizing only the bid amounts, but it has been demonstrated that optimizing the product price in tandem further enhances overall sales.

In the next sections, we will discuss the proposed method's optimal solution and validity. The experimental results in these discussions will be based on experimental settings A and C, but similar trends are observed in experimental settings B and D.

## 5.3 Validation of the Optimal Solution of the Proposed Method

In this section, we will examine the optimal solution of the proposed method to validate its effectiveness. First, we illustrate the scatter plots of the remaining advertising budget and the optimized bid amounts for each period in Figures 7 (sales maximization problem) and 8 (profit maximization problem). The figures' notation "round n" represents the points for the n-th period. As illustrated, both in the sales maximization problem and the profit maximization problem, a higher remaining advertising budget corresponds to a higher bid amount. This can be interpreted as an attempt to increase sales by improving the number of impressions when there is a sufficient advertising budget. As a result, the proposed method produced reasonable outcomes. However, in the profit maximization problem, due to the optimization considering advertising costs, there is a tendency to choose lower bid amounts compared to the sales maximization problem, which can also be deemed realistic.

Next, we illustrate the relationship between inventory levels(Stocks) and the optimized bid amounts as well as product prices for each period in Figures 9 and 10 (sales maximization problem), and Figures 11 and 12 (profit maximization problem). As illustrated, both in the sales maximization problem and the profit maximization problem, it was observed that higher inventory levels were linked to higher bid amounts while also indicating a tendency to opt for lower product prices. This aligns with realistic expectations and confirms that valid results were obtained through the proposed method.

Table 2: Experimental Results for Setting A.

| | Model1 | Model2 | Model3 | Model4 | Model5 | Model6 |
|---|---|---|---|---|---|---|
| Remaining Budget | −89% | −73% | −100% | +130% | - | 0% |
| Remaining Inventory | −93% | −87% | −71% | +254% | - | 0% |
| Sales | **+47%** | +32% | +32% | −68% | - | 0% |

Table 3: Experimental Results for Setting B.

| | Model1 | Model2 | Model3 | Model4 | Model5 | Model6 |
|---|---|---|---|---|---|---|
| Remaining Budget | −20% | −8% | −35% | +89% | - | 0% |
| Remaining Inventory | −51% | −45% | +9% | +95% | - | 0% |
| Sales | **+22%** | +16% | 0% | −44% | - | 0% |

Table 4: Experimental Results for Setting C.

| | Model1 | Model2 | Model3 | Model4 | Model5 | Model6 |
|---|---|---|---|---|---|---|
| Remaining Budget | +368% | +354% | +348% | +524% | - | 0% |
| Remaining Inventory | +8800% | +10500% | +20600% | +52700% | - | 0% |
| Profit | **+32%** | +24% | +16% | +31% | - | 0% |

Table 5: Experimental Results for Setting D.

| | Model1 | Model2 | Model3 | Model4 | Model5 | Model6 |
|---|---|---|---|---|---|---|
| Remaining Budget | +1160% | +1280% | +680% | +1590% | - | 0% |
| Remaining Inventory | +550% | +743% | +493% | +1243% | - | 0% |
| Profit | **+6%** | +2% | +4% | −20% | - | 0% |



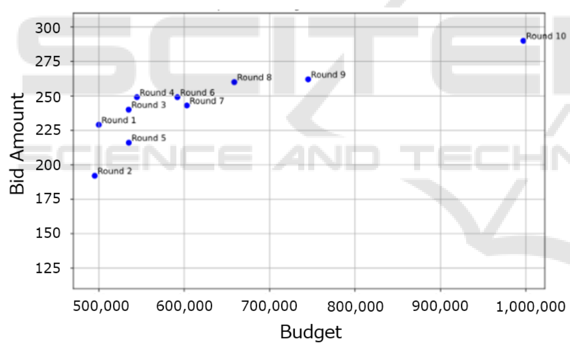


Figure 7: Relationship between the remaining advertising budget (Budget) and the optimal solution (Bid Amount) for each period (Sales Maximization Problem).

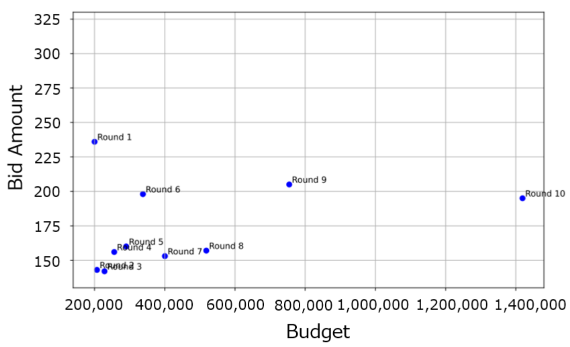


Figure 8: Relationship between the remaining advertising budget and the optimal solution (Bid Amount) for each period (Profit Maximization Problem).

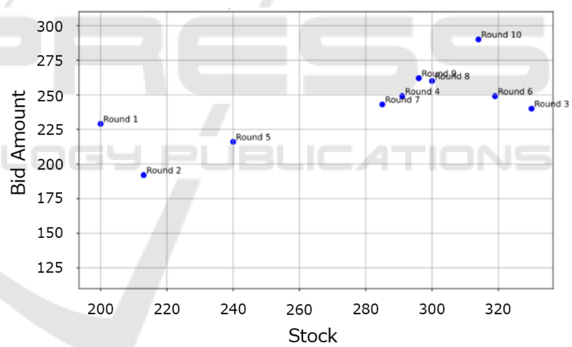


Figure 9: Relationship between inventory levels (Stocks) and the optimal solution (Bid Amount) for each period (Sales Maximization Problem).

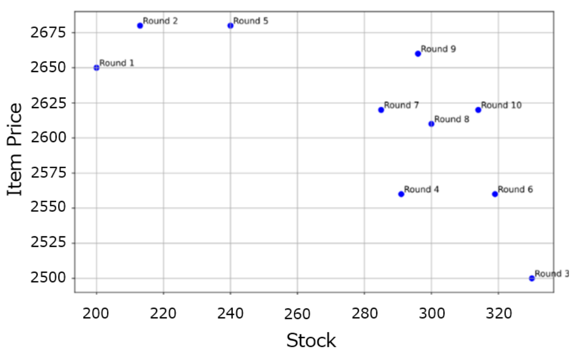


Figure 10: Relationship between inventory levels and the optimal solution (Product Price) for each period (Sales Maximization Problem).

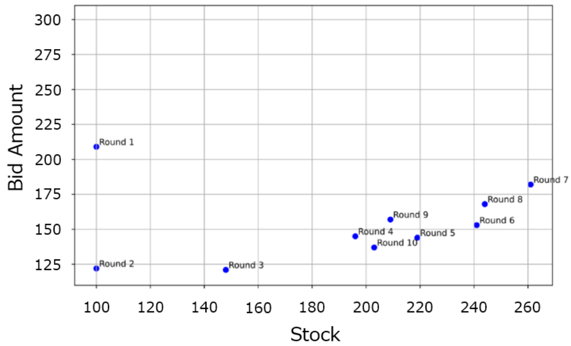


Figure 11: Relationship between inventory levels and the optimal solution (Bid Price) for each period (Profit Maximization Problem).

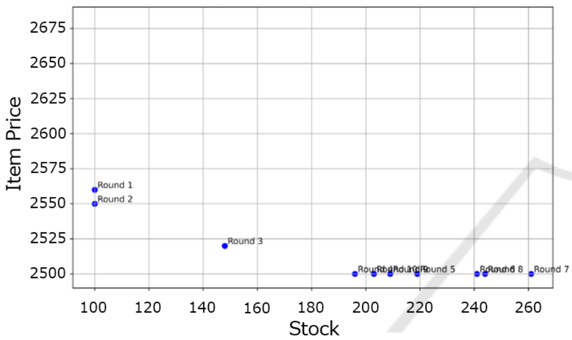


Figure 12: Relationship between inventory levels and the optimal solution (Product Price) for each period (Profit Maximization Problem).

# 6 CONCLUSION

In this paper, we proposed a joint optimization method for advertising bid amounts and product prices to maximize the sales and profits of sellers on e-commerce platforms. Unlike previous studies that focused on either bid amounts or product prices individually, our approach aims to achieve a more realistic and effective optimization by considering the interdependence between the two. The proposed method uses a machine learning model to predict the number of impressions and advertising costs, thereby optimizing the bid amounts, advertising budget, product prices, and inventory control while considering their mutual interdependencies. This enables sellers to comprehensively optimize both bid amounts and product prices simultaneously.

The results of numerical experiments illustrate that the proposed method significantly improves sales and profits compared to conventional methods that only optimize bid amounts. In particular, the effective allocation of inventory management and advertising budget proves crucial, as properly handling these factors leads to sales and profit maximization. Specifically, the proposed method, which optimizes both bid amounts and product prices, achieved higher sales and profits than other models, such as fixed-price models, fixed-bid models, and random models. Furthermore, we explored the relationship between bid amounts and product prices, illustrating that the proposed method selects the optimal bid amounts and product prices based on the remaining advertising budget and inventory levels for each round.

Moreover, in the profit maximization problem, we incorporated advertising costs and inventory holding costs into the optimization, constructing a model that is closer to real-world operations. We also examined the differences in remaining inventory levels depending on the magnitude of inventory holding costs, further illustrating the validity of the proposed method. These results demonstrate that the proposed method provides a practical and effective strategy for sellers in the real e-commerce market.

As for future work, We think that constructing more accurate demand functions is a potential direction. In this paper, demand is modeled as solely dependent on price, but in reality, numerous factors influence demand. By incorporating these factors, it will be possible to more accurately predict the baseline demand and price sensitivity, thereby refining the price elasticity curve. Specifically, following the approach used by Li and Zheng (2023), we will consider incorporating customer demographics and real-time market information (e.g., sales events) into demand forecasting. For example, we can construct a demand function that reflects these variables by collecting external data such as customer demographics, purchase history, social media trends, and competitor pricing. This approach allows us to express baseline demand and price sensitivity as coefficients in a logarithmic-linear function.

Additionally, a model that accounts for the uncertainty in predicting impressions and advertising costs could be explored. For this, applying robust optimization methods may be effective. Robust optimization considers the worst-case scenario for uncertain parameters, enabling optimal decision-making even under uncertainty. This approach would help mitigate the risks associated with inaccurate predictions of advertising costs and impressions, ensuring stable performance. Specifically, this approach sets a range of uncertainty in determining bid amounts and product prices and finds the optimal solution within that range. This would help avoid extreme losses even if predictions deviate from expectations.

Furthermore, there is room to improve budget control methods. For instance, a strategy could be devised to spend a more significant portion of the budget at the start of the period and intensively deploy ads to boost future sales. This approach could lead to early market share acquisition and long-term profit increases while also ensuring the budget is fully utilized. However, from another perspective, it may be more optimal to allocate less of the budget at the start of a period with high uncertainty and conserve the budget until the situation becomes more apparent, maximizing overall profits across the entire period. More effective advertising strategies can be realized by flexibly adjusting budget allocation within the period.

By introducing these methods, we can construct more accurate optimization models, contributing to the maximization of sellers' sales and profits on e-commerce platforms. Extending the proposed method and validating its effectiveness in real-world environments rather than through simulations could lead to developing strategies that maximize sellers' sales and profits.

## REFERENCES

Amazon (2024). Inventory Storage Fees in Amazon Stores. https://sell.amazon.co.jp/en/pricing (Accessed on 06/17/2024).

AmazonAds (2022). How Does Bidding Work with Amazon Ads? https://advertising.amazon.com/en-us/library/videos/campaign-bidding-sponsored-products (Accessed on 06/17/2024).

Besbes, O. and Zeevi, A. (2015). On the (surprising) sufficiency of linear models for dynamic pricing with demand learning. *Management Science*, 61(4):723–739.

Bolton, R. N. (1989). The relationship between market characteristics and promotional price elasticities. *Marketing Science*, 8(2):153–169.

Chen, L., Mislove, A., and Wilson, C. (2016). An empirical analysis of algorithmic pricing on amazon marketplace. In *Proceedings of the 25th International Conference on World Wide Web*, pages 1339–1349.

Chen, W., Wang, Y., and Yuan, Y. (2013). Combinatorial multi-armed bandit: General framework and applications. In *Proceedings of the 30th International Conference on Machine Learning*, pages 151–159.

DENTSU INC. (2024). 2023 Advertising Expenditures in Japan: Detailed Analysis of Expenditures on Internet Advertising Media. https://www.dentsu.co.jp/en/news/release/2024/0312-010705.html (Accessed on 06/17/2024).

Gharakhani, B., Ghandehari, M., and Ansari, A. (2022). A mathematical model for optimizing pricing-inventory, and advertising frequency decisions with a multivariate demand function and a time-dependent holding-cost function. *International Journal of Management Science and Engineering Management*, pages 1–17.

GoogleAds (2024). About Ad Quality. https://support.google.com/google-ads/answer/156066 (Accessed on 06/17/2024).

Jamil, D. A., Mahmood, R. K., and Ismail, Z. S. (2022). Consumer purchasing decision: Choosing the marketing strategy to influence consumer decision making. *Journal of Marketing Research*.

Kouprouchina, L., van der Rest, J.-P., and Schwartz, Z. (2014). On revenue management and the use of occupancy forecasting error measures. *International Journal of Hospitality Management*, 41:104–114.

Lee, K. H., Abdollahian, M., Schreider, S., and Taheri, S. (2023). Supply chain demand forecasting and price optimisation models with substitution effect. *Mathematics*, 11(2502).

Li, X. and Zheng, Z. (2023). Dynamic pricing with external information and inventory constraint. Technical report, UC Berkeley IEOR Department.

Liu, J., Zhang, Y., Wang, X., Deng, Y., and Wu, X. (2019). Dynamic pricing on e-commerce platform with deep reinforcement learning. *arXiv preprint arXiv:1912.02572*, pages 1–11.

Majima, K., Kawakami, K., Ishizuka, K., and Nakata, K. (2024). Keyword-level bayesian online bid optimization for sponsored search advertising. *Operations Research Forum*, 5.

Nuara, A., Trovò, F., Gatti, N., and Restelli, M. (2022). Online joint bid/daily budget optimization of internet advertising campaigns. *Artificial Intelligence*, 305:103663.

Optimization, G. (2022). Gurobi optimizer reference manual. https://www.gurobi.com (Accessed on 06/17/2024).

Schlosser, R. and Richly, K. (2019). Dynamic pricing under competition with data-driven price anticipations and endogenous reference price effects. *Journal of Revenue and Pricing Management*, 18:451–464.

Srinivas, N., Krause, A., Kakade, S., and Seeger, M. (2010). Gaussian process optimization in the bandit setting: No regret and experimental design. In *Proceedings of the 27th International Conference on Machine Learning*, pages 1015–1022.

Yang, W., Xiao, B., and Wu, L. (2020). Learning and pricing models for repeated generalized second-price auction in search advertising. *European Journal of Operational Research*, 282(2):696–711.