A Stochastic Location-Routing Problem for the Optimal Placement of Lockers

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Abstract: This paper presents a Stochastic Location-Routing Problem aimed at optimizing the placement of parcel lockers in last-mile delivery. The model integrates locker location decisions with vehicle routing, taking into account customer preferences for home delivery or locker collection. It considers multiple scenarios of service requests to address the uncertainty in customer behavior. The problem is formulated as a two-stage stochastic program, where the first stage determines which lockers to activate, and the second stage optimizes vehicle routes based on the service preferences for each scenario. Computational experiments are based on a test problem used to validate the model's effectiveness. Proposed future extensions include integrating multi-period planning, introducing capacity constraints for both vehicles and lockers, enabling dynamic activation of lockers, and optimizing the algorithm for multi-core architectures to enhance computational efficiency. These advancements aim to enhance the model's applicability and scalability in tackling complex logistics challenges under uncertainty.

1 INTRODUCTION

In recent years, the rapid growth of online shopping across various industries has created ongoing challenges for companies handling the delivery of high volumes of parcels to increasingly demanding customers.

In this scenario, optimizing last-mile delivery, particularly in densely populated urban areas, has become essential for both business efficiency and environmental sustainability. To reduce delivery costs and minimize the ecological impact of last-mile logistics, innovative strategies are being introduced, with one prominent solution being the implementation of electronic, self-service lockers, placed in accessible public locations and serving as convenient pick-up points for online orders (Grabenschweiger et al., 2021).

The introduction of lockers offers benefits for both companies and customers. Customers gain the flexibility to customize their delivery experience by choosing between home delivery or secure locker collection at a convenient time. For companies, lockers enable the consolidation of deliveries to fewer locations, reducing inefficiencies such as missed deliveries when customers are unavailable. Furthermore, placing parcel lockers in urban areas increases delivery capacity without the need to expand the workforce, which also helps to reduce the environmental footprint (Orenstein et al., 2019).

Today, various types of lockers exist. Schwerdfeger and Boysen (2022) distinguishes between stationary and mobile infrastructures. Stationary lockers, which are the most common, remain fixed after installation, whereas Mobile Parcel Lockers (MPLs) can be relocated as needed.

Determining the optimal location for installing a parcel locker and planning deliveries at an operational level are both complex and interdependent decisions. It is important to note that the choice between delivery to a locker or directly to the customer's home is made by the customer, not the service provider. This introduces a level of uncertainty that must be factored into the decision-making process (Lai et al., 2022; Rossolov, 2023).

Under this perspective, this work proposes a stochastic model that integrates locker location and

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vehicle routing costs, taking into account customer time windows and variability in service type selection to optimize operations comprehensively.

The problem can be modeled as a Stochastic Location-Routing Problem (SLRP). The deterministic version of the problem (LRP) has been discussed in the literature for a long time. The earliest examples refer to Von Boventer (1961) and Maranzana (1964). However, LRP has gained substantial popularity in the scientific community only recently, as demonstrated in the surveys of Drexl and Schneider (2015) and Mara et al. (2021).

Indeed, the location of a logistics node, if determined without considering the impact on the resulting routes, may lead to sub-optimal decisions in terms of overall logistics costs. An integrated approach that accounts for both node location and route optimization is therefore essential to minimize operational costs and enhance the efficiency of the distribution network. This problem encompasses two NP-hard problems, making LRP itself an NP-hard problem (Nagy and Salhi, 2007).

This class of problems has been used in the literature to model applications in different contexts. Some recent examples in a deterministic setting are the following.

In Veenstra et al. (2021), lockers are used for pharmaceutical deliveries without considering time windows or locker capacities. Conversely, Liu et al. (2021) apply lockers in a grocery distribution context, taking into account both time windows and capacity constraints for lockers and depots. Wang et al. (2022a) propose an LRP model for last-mile distribution, where customers can be served either by small electric vehicles or through lockers, while considering the battery levels of the vehicles. Liu et al. (2015) extend the LRP into the Location-Inventory-Routing Problem, formulating a model that locates logistic nodes for both parcel distribution and returns collection. The model also incorporates inventory decisions, aiming to minimize the costs of location, transportation, inventory, and parcel returns.

In the literature, the concept of lockers can be extended to include similar options, such as Collection-and-Delivery Points (CDPs). These represent generic logistic nodes for parcel collection and delivery, which, unlike lockers, can include stores and private postal services that offer such services during their operating hours. Rautela et al. (2022) formulates a deterministic LRP model to locate CDPs within a complex logistics network.

A paper addressing a problem similar to the one under analysis is presented by Grabenschweiger and Dorner (2022). They examine a multi-period LRP for locker location, taking time windows into account. However, their model handles customer-to-locker assignments without considering uncertainty in customer service type selection. Liu et al. (2023b) explore a special case of LRP, focusing on the daily relocation of MPLs to minimize overall operational costs and enhance customer accessibility.

Although incorporating uncertain data makes models more realistic, only a small portion of the studies on LRP consider stochastic data, as highlighted by Mara et al. (2021). An example of a SLRP is presented by Aghalari et al. (2023), who formulate an SLRP model for locating charging stations for electric vehicles used in distribution. The model accounts for uncertainty in customer demand and environmental factors, which directly affect battery efficiency. The objective is to minimize location costs and the expected routing costs.

Other applications of SLRPs can be found in humanitarian logistics, as discussed by de Veluz et al. (2023), which focuses on the location of distribution and evacuation centers while considering various disaster scenarios. The model aims to minimize costs, the number of vehicles used, and travel times.

This study contributes to the body of knowledge on SLRPs as it is the first in the literature to address locker location while considering customer time windows, uncertainty in customer service selection, and multi-period planning.

The remainder of the paper is organized as follows. In Section 2 we report a formal discussion of the SLRP and the corresponding mathematical formulation. In Section 3 we describe the solution method proposed. In Section 4, we discuss the results of our computational experiments. Conclusions and future developments follow in Section 5.

2 MATHEMATICAL FORMULATION

We consider a complete directed graph $G = (N, A)$, where N represents the set of nodes. Node 0 specifically corresponds to the depot from which all routes must start. Additionally, let the sets C and L be subsets of N, where C represents the set of customers (with each customer indexed by c), and L represents the set of potential sites for locker placement (with each locker site indexed by l). Thus, we have $N = 0 \cup C \cup L$. A represents the set of arcs, where each arc (i, j) corresponds to the fastest path from node i to node j in G.

For each customer $c \in C$, let π_c represent the probability of serving the customer at their home. Consequently, $1-\pi_c$ corresponds to the probability that the customer will opt to collect their delivery from the nearest locker. These probabilities can be estimated using historical data on service preferences from the previous purchases of each customer.

A binary constant g_c is associated with each customer, where $g_c = 1$ if the customer is served at home and $g_c = 0$ if the service is provided through a locker. This allows the formulation of a stochastic programming problem to handle uncertainty by introducing a set S of scenarios. Each scenario $s \in S$ represents a possible service configuration for all customers, defined by a vector of $|C|$ binary elements, with an associated probability

$$p^{(s)} = \prod_{c \in C} p_c^{(s)},$$

where $p_c^{(s)} = \pi_c$ if $g_c = 1$ in scenario s, and $p_c^{(s)} = 1-\pi_c$ if $g_c = 0$.

The total number of scenarios is therefore equal to $2^{|C|}$.

The stochastic programming problem is structured as a two-stage model. In the first stage, decisions are made regarding the placement of lockers, which must be finalized before customers select their preferred service option (either home delivery or locker collection). The second stage involves making routing decisions, which occur after customers have chosen their service type.

The SLRP, in its most general form, can be multi-period, with T representing the set of time periods that define the planning horizon.

The input parameters of the problem are as follows:

- n: number of lockers to be activated ($n \leq |L|$);
- d_{ij}, $(i, j) \in A$: distance associated with arc (i, j);
- t_{ij}, $(i, j) \in A$: travel time along arc (i, j);
- γ_{ij}, $(i, j) \in A$: transportation cost along arc (i, j);
- δ_{cl}, $c \in C$, $l \in L$: transportation penalty cost incurred when a customer c is served by a locker l located at a distance $d_{cl} > \rho$, where ρ is a threshold set by the decision-maker. Specifically, ρ corresponds to the maximum distance a customer is willing to travel to a locker. The penalty cost $\delta_{cl} = 0$ if $d_{cl} \leq \rho$, and $\delta_{cl} = r(d_{cl} - \rho)$ otherwise, where r is a unit penalty cost;
- a_{ct}, $c \in C$, $t \in T$: binary coefficient equal to 1 if customer c must be served in time period t, 0 otherwise;
- $g_c^{(s)}$, $c \in C$, $s \in S$: binary coefficient indicating whether, in scenario s, customer c is served at home ($g_c^{(s)} = 1$) or via a locker ($g_c^{(s)} = 0$);
- e_i, $i \in N$: earliest time at which service can start at node i;
- l_i, $i \in N$: latest time by which service must start at node i;
- τ_i, $i \in N$: service time at node i;
- f_l, $l \in L$: activation cost of locker l;
- $p^{(s)}$, $s \in S$: probability associated with scenario s;
- k: number of available vehicles;
- M: arbitrarily large constant.

The decision variables are the following.
First-stage variables:

- w_l, $l \in L$: binary variable indicating whether a locker is opened in site l.

Recourse variables:

- $x_{ijt}^{(s)}$, $(i, j) \in A$, $t \in T$, $s \in S$: binary variable equal to 1 if arc (i, j) is traversed by a vehicle during time period t in scenario s, 0 otherwise;
- $y_{clt}^{(s)}$, $c \in C$, $l \in L$, $t \in T$, $s \in S$: binary variable equal to 1 if customer c is assigned to locker l and served during time period t in scenario s, 0 otherwise;
- $z_{ct}^{(s)}$, $c \in C$, $t \in T$, $s \in S$: binary variable equal to 1 if customer c is served at home during time period t in scenario s, 0 otherwise;
- $v_{it}^{(s)}$, $i \in N$, $t \in T$, $s \in S$: non-negative continuous variable representing the start time of the visit to node i during time period t in scenario s.

The objective of the SLRP is to minimize the total cost, which is influenced by three components:

- transportation costs associated with vehicle routes that visit both home customers and lockers:
$$z_1^{(s)} = \sum_{(i,j) \in A} \sum_{t \in T} \gamma_{ij}\, x_{ijt}^{(s)};$$
- penalty costs related to the service level, reflecting limitations on assigning customers to lockers that are too distant:
$$z_2^{(s)} = \sum_{c \in C} \sum_{l \in L} \sum_{t \in T} \delta_{cl}\, y_{clt}^{(s)};$$
- locker activation costs:
$$z_3 = \sum_{l \in L} f_l\, w_l.$$

Note that both the transportation costs and penalty costs depend on recourse decisions and thus vary across scenarios $s \in S$, whereas the locker activation costs are invariant across scenarios, as they are associated with the first-stage decisions.

The formulation of the two-stage model is the following:

$$\min \sum_{s\in S} p^{(s)} (z_1^{(s)} + z_2^{(s)}) + z_3 \quad (1)$$

subject to

$$\sum_{j\in N\setminus\{i\}} x_{ijt}^{(s)} \leq 1, \quad \forall\, i \in N\setminus\{0\}, \forall\, t \in T, \forall\, s \in S \quad (2)$$

$$\sum_{j\in N\setminus\{0\}} x_{0jt}^{(s)} \leq k, \quad \forall\, t \in T, \forall\, s \in S \quad (3)$$

$$\sum_{j\in N\setminus\{i\}} x_{ijt}^{(s)} = \sum_{j\in N\setminus\{i\}} x_{jit}^{(s)}, \quad \forall\, i \in N, \forall\, t \in T, \forall\, s \in S \quad (4)$$

$$y_{clt}^{(s)} \leq \sum_{j\in N\setminus\{l\}} x_{ljt}^{(s)}, \quad \forall\, c \in C, \forall\, l \in L, \forall\, t \in T, \forall\, s \in S \quad (5)$$

$$z_{ct}^{(s)} \leq \sum_{j\in N\setminus\{c\}} x_{cjt}^{(s)}, \quad \forall\, c \in C, \forall\, t \in T, \forall\, s \in S \quad (6)$$

$$\sum_{j\in N\setminus\{l\}} x_{ljt}^{(s)} \leq w_l, \quad \forall\, l \in L, \forall\, t \in T, \forall\, s \in S \quad (7)$$

$$\sum_{l\in L} w_l = n \quad (8)$$

$$\sum_{l\in L} (1 - g_c^{(s)})\, y_{clt}^{(s)} + g_c^{(s)}\, z_{ct}^{(s)} = a_{ct}, \quad \forall\, c \in C, \forall\, t \in T, \forall\, s \in S \quad (9)$$

$$v_{it}^{(s)} + \tau_i + t_{ij} - M\,(1 - x_{ijt}^{(s)}) \leq v_{jt}^{(s)}, \quad \forall\, i \in N, \forall\, j \in N\setminus\{0\}, \forall\, t \in T, \forall\, s \in S \quad (10)$$

$$v_{it}^{(s)} + \tau_i + t_{i0} \leq l_0, \quad \forall\, i \in N, \forall\, t \in T, \forall\, s \in S \quad (11)$$

$$v_{it}^{(s)} \leq l_i \sum_{j\in N\setminus\{i\}} x_{ijt}^{(s)}, \quad \forall\, i \in N\setminus\{0\}, \forall\, t \in T, \forall\, s \in S \quad (12)$$

$$v_{it}^{(s)} \geq e_i \sum_{j\in N\setminus\{i\}} x_{ijt}^{(s)}, \quad \forall\, i \in N, \forall\, t \in T, \forall\, s \in S \quad (13)$$

$$x_{ijt}^{(s)} \in \{0,1\}, \quad \forall\, (i,j) \in A, \forall\, t \in T, \forall\, s \in S \quad (14)$$

$$y_{clt}^{(s)} \in \{0,1\}, \quad \forall\, c \in C, \forall\, l \in L, \forall\, t \in T, \forall\, s \in S \quad (15)$$

$$z_{ct}^{(s)} \in \{0,1\}, \quad \forall\, c \in C, \forall\, t \in T, \forall\, s \in S \quad (16)$$

$$w_l \in \{0,1\}, \quad \forall\, l \in L \quad (17)$$

$$v_{it}^{(s)} \geq 0, \quad \forall\, i \in N, \forall\, t \in T, \forall\, s \in S. \quad (18)$$

In the objective function (1), the transportation and penalty costs are weighted according to the probabilities of the scenarios. Constraints (2) ensure that each node (except the depot) is visited at most once per time period in each scenario. Constraints (3) restrict the number of routes to the number of available vehicles in each time period and for each scenario. Equations (4) represent the flow balance constraints for each node, in every time period and for each scenario. Constraints (5) link the variables $y_{clt}^{(s)}$ to the variables $x_{ljt}^{(s)}$, ensuring that for each scenario s, if a customer c is served by locker l in time period t, the node corresponding to that locker must be visited during that time period. Constraints (6) link the variables $z_{ct}^{(s)}$ to the variables $x_{cjt}^{(s)}$, ensuring that for each scenario s, if a customer c is served at home in time period t, the node corresponding to that customer must be visited during that time period. Constraints (7) ensure that for each time period and scenario, if a locker is not opened, its corresponding node cannot be visited. Constraint (8) imposes the required number of lockers to be activated. Constraints (9) ensure that each customer is served either at home or through a locker, depending on the scenario s. Constraints (10)−(11) ensure temporal continuity of service and prevent the formation of sub-tours within the routes. Additionally, constraints (11) ensure that the time window for the depot is respected in every time period and scenario. Constraints (12)−(13) impose the satisfaction of time windows for each node, time period, and scenario. Finally, constraints (14)−(18) define the nature (binary or continuous) of the decision variables.

For simplicity, the remainder of this paper focuses on a single time period within the planning horizon, assuming that the available vehicle fleet is sufficiently large to meet all customer demand.

3 SOLUTION METHOD

When solving the SLRP for locker location to optimality, it is theoretically possible to approach the solution by inspection, at least in principle. Specifically, all possible $\binom{|L|}{n}$ combinations of locker activations from the set of potential sites are considered and examined individually. By defining the values of the location variables w_l for each combination, their effect on second-stage decisions can be evaluated by solving a Vehicle Routing Problem with Time Windows (VRP-TW) for each scenario. The nodes to be visited in each VRP-TW include the activated lockers and the customers who, in the corresponding scenario, are scheduled for home delivery.

After solving the VRP-TW for each scenario $s \in S$, the total solution cost for the SLRP can be calculated using equation (1).

The pseudo-code of the proposed procedure for solving the SLRP for each choice of n lockers to activate is presented below.

Algorithm 1: Procedure SLRP. ($\overline{L}$, z).

begin
  // $\overline{L}$ is the set of n lockers to be activated
  // z is the total cost of the SLRP
1 $z_3 = \sum_{l \in \overline{L}} f_l$;
2 $z = z_3$;
3 **for** $s = 1$ *to* $|S|$ **do**
4   Determine the list $C_1^{(s)}$ of customers who, according to scenario s, request home delivery;
    // $C_0^{(s)}$ is the list of customers served via lockers
5    $C_0^{(s)} = C \setminus C_1^{(s)}$
6   **for** $c = 1$ *to* $|C_0^{(s)}|$ **do**
7   Assign c to the nearest locker $l \in \overline{L}$;
8   Determine the penalty δ_{cl};
9   **end**
10    $z_2^{(s)} = \sum_{c \in C_0^{(s)}} \sum_{l \in \overline{L}} \delta_{cl}$;
    // Update z
11    $z := z + p^{(s)} z_2^{(s)}$;
    // Define the set $\overline{N}$ of nodes for which the VRP-TW will be solved
12    $\overline{N} = 0 \cup C_1^{(s)} \cup \overline{L}$;
13   Solve the VRP-TW on the complete directed graph induced by $\overline{N}$;
    // $z_1^{(s)}$ is the cost of the VRP-TW
    // Update z
14    $z := z + p^{(s)} z_1^{(s)}$;
15 **end**
16 **return** z;
end

The time complexity of the entire algorithm grows exponentially with the problem size, making it quickly impractical even for moderately sized inputs, as the execution time increases rapidly.

To keep execution times within acceptable limits, three approaches can be applied simultaneously: 1) reducing the number of configurations of n lockers to test among the $|L|$ possible choices (this reduces the number of calls of the **SLRP** procedure); 2) reducing the number of scenarios in the **for** loop $3-15$ of the **SLRP** procedure; and 3) heuristically solving the VRP-TW for each scenario (code line 13 of the **SLRP** procedure). These three proposed approaches are briefly outlined below.

3.1 Reduction of the Number of Locker Configurations

For limiting the number of locker configurations to be tested, geographical criteria are widely used and proposed in literature to optimize placement and enhance efficiency (Lagorio and Pinto (2020), Sawik (2024) and Wang et al. (2022b)). Here are the two relevant approaches:

- *optimal geographical coverage*: lockers should be strategically placed to ensure even distribution across the service area, minimizing coverage gaps and guaranteeing accessibility in all key locations;
- *proximity between lockers*: it is possible to avoid placing lockers too close to one another, preventing redundancy and enhancing the overall efficiency of the locker network.

3.2 Reduction of the Number of Scenarios

As observed in Section 2, the number of scenarios grows exponentially with the number of customers. To give an idea, considering 30 customers would result in a number of scenarios equal to $2^{30} = 1{,}073{,}741{,}824$. This implies that only a representative subset $\overline{S}$ of all possible scenarios may be considered.

To reduce the number of scenarios, the following approach is proposed. First, the most likely scenario s^* is considered. The probability $p^{(s^*)}$ of this scenario is equal to the product of the probabilities associated with each individual customer corresponding to the most probable choice:

$$p^{(s^*)} = \prod_{c \in C} \max\{\pi_c; 1 - \pi_c\}.$$

Then, $|\overline{S}|$ scenarios are randomly generated, ensuring that the overall probability (given by the sum of the scenario probabilities) is greater than or equal

Algorithm 2: Procedure Scenario_Generation.

begin
  // Σ is a binary matrix of scenarios, with size $|\overline{S}| \times |C|$
  Randomly generate Σ;
  Determine the cumulative probability $p^{(\Sigma)} = \sum_{s=1}^{|\overline{S}|} p^{(s)}$ associated with the scenarios in Σ;
  while $p^{(\Sigma)} < p^{(s^*)}$ **do**
    Randomly select an entry (s, c) of Σ;
    if $p_c^{(s)} < \max\{\pi_c; 1-\pi_c\}$ **then**
      $p_c^{(s)} = \max\{\pi_c; 1-\pi_c\}$;
      Modify scenario s in Σ accordingly;
      Update $p^{(\Sigma)}$;
    end
  end
end

to $p^{(s^*)}$. The proposed procedure is illustrated in the sequel.

Finally, to ensure consistent comparison between solutions derived from different scenario subsets, we normalize the associated probabilities so that the overall probability across all scenarios $s \in \overline{S}$ equals one. This is achieved by replacing each value of $p^{(s)}$ for all $s \in \overline{S}$ at the end of the **Scenario_Generation** procedure with the ratio $p^{(s)}/p^{(\Sigma)}$.

3.3 Heuristic Approaches for the VRP-TW

Several methods can be applied to generate feasible vehicle routes while respecting time windows constraints. As reported in Liu et al. (2023a), most of these methods fall into three categories: exact, heuristic, and metaheuristic approaches. Heuristic and metaheuristic methods, starting with Solomon's constructive heuristics (Solomon, 1987), are widely used in the literature due to their ability to efficiently solve large-scale problems.

Exact methods are also worth mentioning, though they are generally capable of optimally solving only small to medium-sized problems (Desaulniers et al., 2005) . Among them, the column generation method stands out, as it can be adapted for the VRP-TW to generate sub-optimal solutions within acceptable computational times (Kallehauge et al., 2005).

4 COMPUTATIONAL EXPERIMENTS

The solution method described in Section 3 was tested on a problem consisting of $|N| = 36$ nodes, where their spatial distribution on the plane was randomly generated (see Figure 1). Table 1 presents the Cartesian coordinates of the nodes. Node 0 represents the depot, nodes 1 to 30 correspond to the customers, and nodes 31 to 35 indicate the potential sites for locker locations.

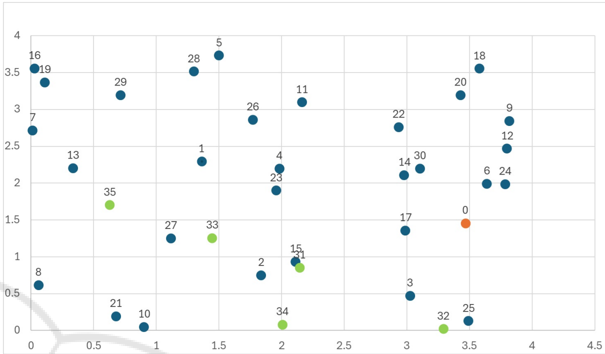

Figure 1: Spatial distribution of the nodes of the test problem.

Table 1: Cartesian coordinates (in kilometres) of the nodes of the test problem.

Node	Coordinates	Node	Coordinates
0	(3.47, 1.46)	1	(1.36, 2.29)
2	(1.84, 0.75)	3	(3.03, 0.47)
4	(1.98, 2.20)	5	(1.50, 3.73)
6	(3.64, 1.99)	7	(0.02, 2.71)
8	(0.06, 0.61)	9	(3.82, 2.84)
10	(0.90, 0.05)	11	(2.17, 3.10)
12	(3.80, 2.47)	13	(0.34, 2.20)
14	(2.98, 2.10)	15	(2.11, 0.93)
16	(0.03, 3.55)	17	(2.99, 1.35)
18	(3.58, 3.55)	19	(0.11, 3.36)
20	(3.43, 3.19)	21	(0.68, 0.19)
22	(2.94, 2.76)	23	(1.96, 1.90)
24	(3.79, 1.98)	25	(3.49, 0.13)
26	(1.77, 2.86)	27	(1.12, 1.25)
28	(1.30, 3.51)	29	(0.72, 3.19)
30	(3.11, 2.19)	31	(2.15, 0.85)
32	(3.29, 0.02)	33	(1.45, 1.25)
34	(2.01, 0.08)	35	(0.63, 1.70)

The distances between nodes (in kilometers) are Euclidean, thereby ensuring the triangular inequality property. Table 2 presents the time windows and service times for each node, expressed in hours. The time windows are set starting from 00:00.

The transportation cost γ_{ij} for each arc $(i, j) \in A$ is defined as:

$$\gamma_{ij} = 1.0 \times d_{ij},$$

where 1.0 represents the unit transportation cost (in €/km). The unit penalty cost is set to 0.60 €/km. The fixed opening cost of a locker is assumed to be identical for each potential site ($f_l = f, \forall\ l \in L$). As a result, the cost z_3 in the objective function (1) is the same for every locker configuration tested ($z_3 = fn$) and is therefore omitted. The fleet consists of $k = 4$ vehicles.

Table 2: Time windows (e_i, l_i) and service times (τ_i) for each node $i \in N$.

Node i	e_i	l_i	τ_i	**Node** i	e_i	l_i	τ_i
0	0	24	0.0	1	10	15	0.1
2	9	13	0.1	3	14	17	0.1
4	12	16	0.1	5	8	15	0.1
6	9	11	0.1	7	10	14	0.1
8	16	18	0.1	9	10	13	0.1
10	11	15	0.1	11	12	17	0.1
12	15	16	0.1	13	10	13	0.1
14	10	14	0.1	15	8	10	0.1
16	9	13	0.1	17	15	17	0.1
18	14	16	0.1	19	12	14	0.1
20	13	16	0.1	21	10	11	0.1
22	9	12	0.1	23	14	16	0.1
24	12	13	0.1	25	14	16	0.1
26	12	15	0.1	27	10	11	0.1
28	12	14	0.1	29	8	9	0.1
30	15	16	0.1	31	9	18	0.3
32	9	18	0.3	33	9	18	0.3
34	9	18	0.3	35	9	18	0.3

The probabilities of home service for the 30 customers are presented in Table 3.

Consequently, the most likely scenario s^*, corresponding to the 30 binary entries reported in Table 4, has a probability $p^{(s^*)} = 0.000354352$.

We consider the case where two lockers are to be activated out of the five potential sites. In this case, the SLRP can be solved for each possible configuration of locker activation, as the number is small, being $\binom{5}{2} = \frac{5!}{2! \times 3!} = 10$.

Regarding the scenarios, computational experiments were conducted considering two situations: 1) only scenario s^*, corresponding to the most likely service configuration, for which the probability $p^{(s^*)}$ is set to one; and 2) using a set $\overline{S}$ of 30 representative scenarios, deemed sufficient to capture the stochastic nature of the problem. These scenarios were generated using the **Scenario_Generation** procedure, which produced the binary matrix Σ of size 30×30, with a cumulative probability $p^{\Sigma} = 0.000381808$. The probabilities of each scenario in $\overline{S}$ were then normalized so that $\sum_{s \in \overline{S}} p^{(s)} = 1$.

Table 3: Probabilities of service at home for the 30 customers.

Customer (c)	**Probability** (π_c)	**Customer** (c)	**Probability** (π_c)
1	0.0449	2	0.9441
3	0.1970	4	0.6387
5	0.0678	6	0.3687
7	0.8344	8	0.8841
9	0.4342	10	0.9260
11	0.2256	12	0.0765
13	0.0425	14	0.7687
15	0.4447	16	0.7207
17	0.6687	18	0.8906
19	0.3913	20	0.2988
21	0.6518	22	0.9872
23	0.2420	24	0.9467
25	0.6288	26	0.6725
27	0.7480	28	0.0616
29	0.5331	30	0.8611

Table 4: Home delivery (1) or service via locker (0) for the 30 customers in the most likely scenario s^*.

Customer (c)	$g_c^{(s^*)}$	Customer (c)	$g_c^{(s^*)}$
1	0	2	1
3	0	4	1
5	0	6	0
7	1	8	1
9	0	10	1
11	0	12	0
13	0	14	1
15	0	16	1
17	1	18	1
19	0	20	0
21	1	22	1
23	0	24	1
25	1	26	1
27	1	28	0
29	1	30	1

It is worth emphasizing that solving the SLRP by considering only the most likely scenario effectively transforms it into a deterministic problem, where service decisions are made under the assumption that each customer will always choose their most preferred service.

From this perspective, the "single scenario" model does not account for uncertainty and is therefore referred to as the deterministic case hereafter.

The entire algorithm was implemented in Python 3 using the Visual Studio Code IDE. To solve the VRP-TW, we used the VRPy library, a package de-

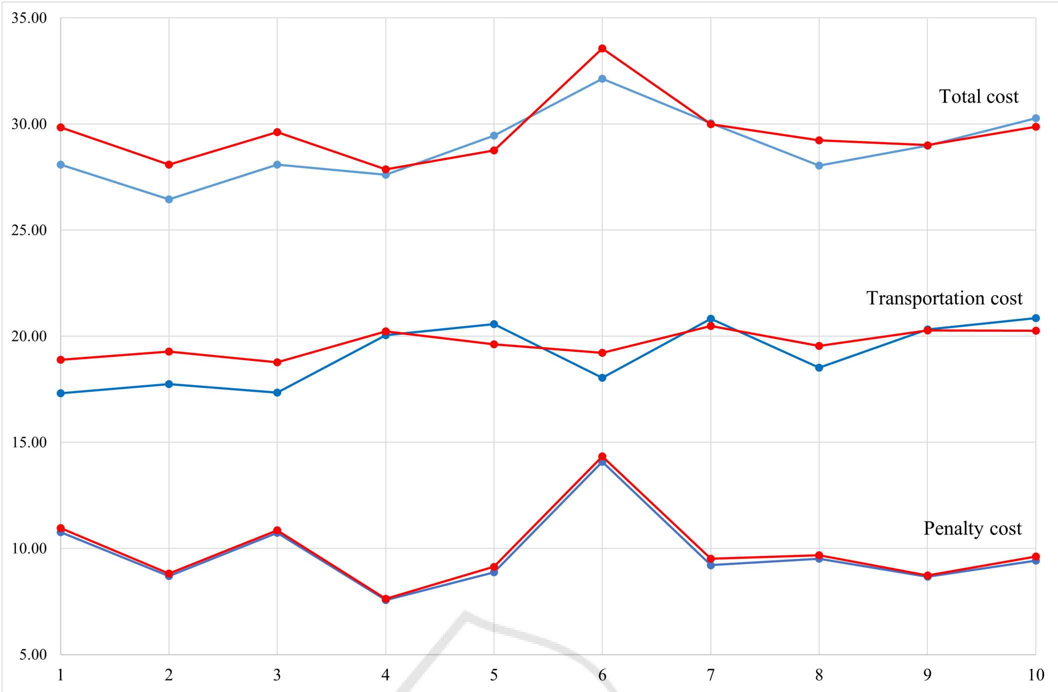

Figure 2: Costs obtained for each the ten tested locker configurations, both in the deterministic case (in blue) and the stochastic case (in red).

Table 5: Pair of activated lockers, transportation cost (z_1), penalty cost (z_2), and total cost (z) for each of the ten locker configurations in the deterministic case.

Configuration	$\overline{L}$	z_1	z_2	z
1	31, 32	17.32	10.77	28.09
2	31, 33	17.75	8.69	26.44
3	31, 34	17.34	10.74	28.08
4	31, 35	20.05	7.57	27.62
5	32, 33	20.57	8.88	29.45
6	32, 34	18.05	14.09	32.14
7	32, 35	20.83	9.21	30.04
8	33, 34	18.52	9.52	28.04
9	33, 35	20.31	8.66	28.97
10	34, 35	20.86	9.42	30.28

Table 6: Transportation costs (z_1), penalty costs (z_2), and total costs (z) for the ten activated locker configurations in the stochastic case.

Configuration	$\overline{L}$	z_1	z_2	z
1	31, 32	18.89	10.95	29.84
2	31, 33	19.27	8.81	28.08
3	31, 34	18.77	10.85	29.62
4	31, 35	20.23	7.63	27.86
5	32, 33	19.62	9.14	28.76
6	32, 34	19.22	14.34	33.56
7	32, 35	20.48	9.51	29.99
8	33, 34	19.55	9.68	29.23
9	33, 35	20.28	8.72	29.00
10	34, 35	20.26	9.61	29.87

signed for solving various vehicle routing problems through a column generation approach (Montagné et al., 2020). In this method, routes (or columns) are generated by solving a pricing problem and then passed to a master problem, which selects the best routes from a pool such that each vertex (except the depot) is served exactly once. It is important to note that VRPy does not necessarily return an optimal solution, but feasibility is guaranteed.

The results are summarized in Tables 5 and 6 for the deterministic and stochastic cases of the problem, respectively. The tables are organized as follows: column **Configuration** enumerates the ten configurations of activated lockers, column $\overline{L}$ indicates the nodes corresponding to the pairs of activated lockers, columns z_1 and z_2 report the transportation and penalty costs, respectively, while column z provides the total costs. The results are also plotted in Figure 2, where, for each activated locker configuration, the transportation, penalty, and total costs are shown for both the deterministic (in blue) and stochastic (in red) cases.

The SLRP with 30 scenarios effectively incorporates a degree of uncertainty regarding customer service choices, making the model more realistic compared to its simple deterministic counterpart. Consequently, it is expected that the optimal solutions obtained may differ. In fact, in the stochastic case, the lockers identified as the best locations are at nodes 31 and 35, whereas in the deterministic case, the optimal solution corresponds to lockers located at nodes 31 and 33.

5 CONCLUSIONS AND FUTURE DEVELOPMENTS

In this paper, we presented a stochastic location-routing problem for the optimal placement of parcel lockers, incorporating customer preferences between home delivery and locker collection under uncertain conditions. The model was formulated as a two-stage stochastic program, with the first stage determining locker locations and the second stage addressing vehicle routing based on service requests across multiple scenarios. Through computational experiments, we demonstrated the differences between deterministic and stochastic solutions, highlighting the ability of the model to account for customer behavior variability.

While the model offers a valuable framework for addressing uncertainty in last-mile delivery, several areas for future research and development remain unexplored.

One possible extension involves the inclusion of scenarios where some customers opt not to request service at all. This reflects a real-world phenomenon where, due to various factors such as pricing, delivery preferences, availability of alternatives, or personal circumstances, customers may decide not to engage with the delivery network in a given time period. Another extension would be to expand the model to multi-period scenarios while incorporating capacity constraints for both vehicles and lockers, making the model more applicable to real-world logistics settings.

Furthermore, the current model assumes a fixed number of lockers to be activated. In future research, this assumption could be relaxed, allowing the number of lockers to be determined dynamically. This would require the development of more sophisticated procedures to determine which lockers to activate, such as ADD, DROP, or ADD-DROP heuristics. Lastly, to enhance computational efficiency, the code could be adapted to run on multi-core architectures, enabling the generation of a greater number of scenarios. In this approach, the routing problem could be decomposed into scenario subsets, with each subset assigned to a different core, thereby reducing overall computational time.

These future developments aim to improve the practical applicability of the model, ensuring it remains relevant for a wide range of logistics and last-mile delivery problems under uncertainty.

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