Cost Optimization Analysis of Retrial Machine Repair Problem with Warm Standby Components and Imperfect Coverage

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Abstract: In this paper, we examine the cost optimization analysis of retrial machine repair problem with warm standby components and imperfect coverage. This research suggests that failure times and repair times of the primary and warm standby components are exponentially distributed and that the coverage factor is the same for a primary component failure and a standby component failure. When the server is either busy with other tasks or is repairing a failed component, the failed component will be sent to the retrial orbit. The steady-state probabilities of the number of failed components in an orbit is developed by using the matrix-analytic method. The particle swarm optimization (PSO) algorithm is implemented to simultaneously determine the joint optimal values of the number of warm standbys, the repair rate, and the retrial rate at minimum cost. Under optimal operating conditions, numerical experiments were presented to illustrate results. Sensitivity analysis for system parameters is performed additionally.

1 INTRODUCTION

This paper studies a retrial machine repair problem (RMRP) with warm standby components and imperfect coverage. Retrial queue or RMRP with imperfect coverage is a major issue. Artalejo (1999a, 1999b), Falin (1990), and Yang and Templeton (1987) provided the foremost overall surveys and ideas for retrial queues. Queueing systems in which arriving failed components that cannot accept service immediately enter orbit and retry for service again after a random time is called retrial queue. Retrial queueing problems are increasingly important concerns and play a crucial role in many practical applications, such as message switching systems, manufacturing systems, telecommunication systems, and production management. With an imperfect coverage factor, the failed component is immediately detected, located, and recovered by standby, and the faults that exchange the failed component within the standby component are called to be not covered. Wang et al. (2014) examined an M/G/1 MRP with imperfect coverage by constructing a cost function and using direct search and the Quasi-Newton method to find the optimal number of operating components, the repair rate, and the coverage factor. Wang et al. (2013) proposed the direct search method and PSO algorithm to determine the joint optimal values at the maximum profit function. After comparing these two methods, using the PSO algorithm is a better choice when dealing with optimization problems. Yen et al. (2021) contrasted four retrial systems with imperfect coverage and warm standbys. They presented the comparative analysis of the cost-benefit ratio among four retrial systems and provided the optimal retrial system. Yen and Wang (2020) investigated the cost-benefit analysis of four retrial systems with imperfect coverage and warm standbys and made comparisons. Wang et al. (2012) compared two availability systems with imperfect coverage and warm standbys. This paper also compares five different distributions of repair time, which are exponential, normal, gamma, uniform, and deterministic. Sherbeny and Hussien (2019) studied the cost-benefit and the availability analysis for three models with imperfect coverage and mixed standby (cold and warm standby). Jain and Meena (2017) analyzed a model of a fault-tolerant system with imperfect coverage, applied the Runge-Kutta method to evaluate system performance measures, and conducted a numerical simulation of the cost and sensitivity. Wang et al. (2013) conducted a reliability and sensitivity analysis for the repair system with imperfect coverage and service pressure conditions, and studied the influence of different

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parameters on system reliability and *MTTF*.

Cost optimization is a major topic and much research has been done using particle swarm optimization (PSO) algorithm for cost optimization analysis. The PSO algorithm was first proposed by Kennedy and Eberhart (1995). Zhang et al. (2017) investigated the retrial queue under the state-dependent service policy and established a reward-cost function, using the PSO algorithm to obtain the optimal strategy. Compared with related service strategies, managers get more benefits. Wang et al. (2019) analyzed RMRP with working breakdowns under the N policy, proposing a profit function and using the PSO algorithm to determine the optimum number of warm standbys, fast service rates, and slow service rates. Yang et al. (2020) considered an M/M/2 queue with two heterogeneous servers. They constructed a cost function and determined the optimal solutions by using PSO algorithm. Zhang and Wang (2017) studied an M/G/1 retrial queue with setup times and used the PSO algorithm to find the optimal reserved idle time for maximizing profit.

The rest of the paper is organized as follows. The model descriptions and assumptions of the RMRP with warm standby components and imperfect coverage are presented in Section 2. By using the matrix-analytic method, Section 3 provides the derivations of the steady-state probabilities of the number of failed components in the retrial orbit. The effects of various system parameters on the system performance measures are investigated in Section 4. The total expected cost function to determine the optimal solutions and perform sensitivity analysis is shown in Section 5. Finally, the conclusion will be shown in Section 6 of this paper.

2 THE MODEL DESCRIPTIONS

We consider the RMRP with $N=M+S$ identical components and a single server in the repair facility. As many as M of these components can operate at the same time, while the rest of the S components are warm standbys. The assumptions of the model are described as follows:

(1) Primary components are subject to breakdowns according to the independent Poisson process with parameter λ.

(2) Warm standby components are subject to breakdowns according to independent Poisson process with parameter α ($0<\alpha<\lambda$).

(3) When one of the primary components fails, it is instantly replaced by an available warm standby component. When a warm standby moves into operating state, its failure characteristics will be that of a primary component.

(4) Whenever a component fails, it will be sent to the server immediately, where the repair is provided in the order of their breakdowns, that is, the first-come, first-served discipline.

(5) The repair times at this repair facility follow exponential distribution with parameter μ.

(6) The server can only repair one failed component at a time. Once a failed component is repaired, it becomes as good as new.

(7) The probability of successful recovery on the failure of a primary component (or warm standby component) is denoted as c. Quantity c, which is included in the probabilities of successful detection, location, and recovery from a failure, is known as the coverage factor or coverage probability

(8) The *unsafe failure* state of the system in any one of the breakdowns is *not covered*.

(9) Primary component (or warm standby component) failure in the *unsafe failure* state is cleared by a reboot. Reboot delay follows exponential distribution with parameter β.

(10) When a primary or warm standby component fails and finds that the server is busy in repairing another failed component, it will be sent to the retrial queue (orbit).

(11) Failed components in the retrial queue repeat its request for service with an exponential random period of retrial time at rate γ.

(12) When the time waiting in the retrial queue terminates, the failed component will get the repair if the server is idle; otherwise, it will again be sent to the retrial queue for another random period.

(13) The failure times, repair times, retrial times, and reboot delay times are mutually independent from each other.

3 STEADY-STATE RESULTS

In this RMRP with imperfect coverage, we describe the system states by the pairs (i,j). $i=0$ means that the server is idle, $i=1$ shows that the server is busy, $j=n$ denotes that there are n failed components in the retrial orbit and the system is in a safe failure state, and $j=uf_n$ represents that there are n failed components in the retrial orbit and the system is in an unsafe failure state.

The mean failure rate of a primary component λ_n is given by

$$\lambda_n=\begin{cases}M\lambda+(S-n)\alpha, & 0\le n\le S-1,\\ (M+S-n)\lambda, & S\le n<N=M+S.\end{cases}$$

The following steady-state probabilities are employed throughout this paper:

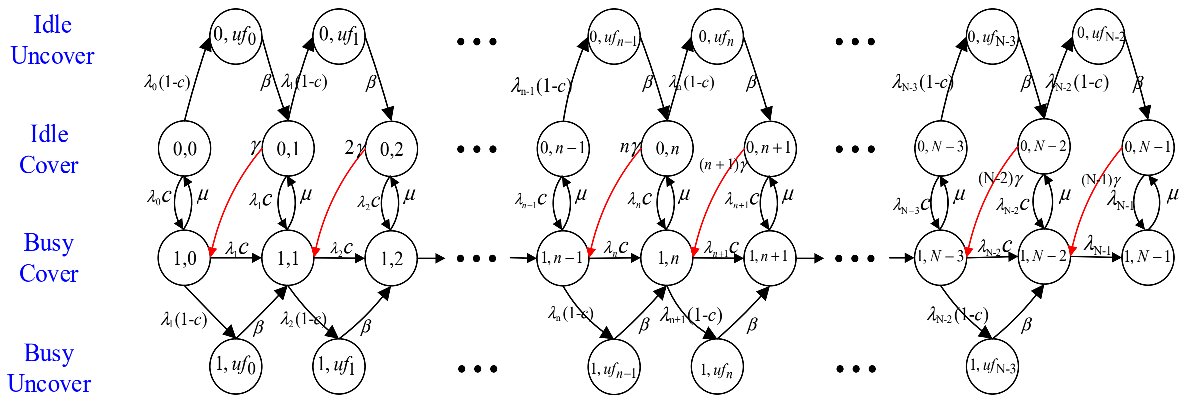

Figure 1: State-transition-rate diagram of RMRP with warm standby components and imperfect coverage.

$P_{0,n} \equiv$ probability that there are n failed units in the retrial orbit when the server is idle and the system is in a safe failure state (cover), where $n = 0, 1, 2, \ldots, N-1$;

$P_{1,n} \equiv$ probability that there are n failed units in the retrial orbit when the server is busy and the system is in a safe failure state (cover), where $n = 0, 1, 2, \ldots, N-1$;

$P_{0,uf_n} \equiv$ probability that there are n failed units in the retrial orbit when the server is idle and the system is in an unsafe failure state (not covered), where $n = 0, 1, 2, \ldots, N-2$;

$P_{1,uf_n} \equiv$ probability that there are n failed components in the retrial orbit when the server is busy and the system is in an unsafe failure state (not covered), where $n = 0, 1, 2, \ldots, N-3$.

3.1 Steady-State Equations

Referring to the diagram displayed in Figure 1, the equilibrium equations are deduced as follows:

$$\mu p_{1,0} = \lambda_0 p_{0,0} \quad (1)$$

$$\beta p_{0,\mathrm{uf}_{j-1}} + \mu p_{1,j} = (\lambda_j + j\gamma) p_{0,j}, \; 1 \le j \le N-1 \quad (2)$$

$$\lambda_0 c p_{0,0} + \gamma p_{0,1} = (\mu + \lambda_1) p_{1,0} \quad (3)$$

$$\lambda_j c (p_{0,j} + p_{1,j-1}) + (j+1)\gamma p_{0,j+1} + \beta p_{1,\mathrm{uf}_{j-1}} = (\mu + \lambda_{j+1}) p_{1,j}, \; 1 \le j \le N-2 \quad (4)$$

$$\lambda_{N-1}(p_{0,N-1} + p_{1,N-2}) = \mu p_{1,N-1} \quad (5)$$

$$\lambda_j (1-\mathrm{c}) p_{0,j} = \beta p_{0,\mathrm{uf}_j}, \; 0 \le j \le N-2 \quad (6)$$

$$\lambda_{j+1}(1-\mathrm{c}) p_{1,j} = \beta p_{1,\mathrm{uf}_j}, \; 0 \le j \le N-3 \quad (7)$$

3.2 Matrix-Analytic Method

The matrix-analytic method was first introduced by Neuts (1981) while studying the embedded Markov chains of many queueing systems. Because of the high complexity of this RMRP with imperfect coverage, the matrix-analytic method is employed to derive the steady-state probabilities $p_{i,j}$. By appropriately arranging the system states, the corresponding transition rate matrix $\mathbf{Q}$ of this Markov chain can be established as the following block tridiagonal form:

$$\mathbf{Q} = \begin{bmatrix} \boldsymbol{B}_0 & \boldsymbol{C}_1 & & & & & & \\ \boldsymbol{A}_0 & \boldsymbol{B}_1 & \boldsymbol{C}_2 & & & & & \\ & \boldsymbol{A}_1 & \boldsymbol{B}_2 & \boldsymbol{C}_3 & & & & \\ & & \boldsymbol{A}_2 & \boldsymbol{B}_3 & \ddots & & & \\ & & & \boldsymbol{A}_3 & \ddots & \ddots & & \\ & & & & \ddots & \ddots & \boldsymbol{C}_{N-4} & \\ & & & & & \ddots & \boldsymbol{B}_{N-4} & \boldsymbol{C}_{N-3} & & \\ & & & & & & \boldsymbol{A}_{N-4} & \boldsymbol{B}_{N-3} & \boldsymbol{C}_{N-2} & \\ & & & & & & & \boldsymbol{A}_{N-3} & \boldsymbol{B}_{N-2} & \boldsymbol{C}_{N-1} \\ & & & & & & & & \boldsymbol{A}_{N-2} & \boldsymbol{B}_{N-1} \end{bmatrix}. \quad (8)$$

Each element of the matrix $\mathbf{Q}$ is a submatrix, which may be listed as follows:

$$\boldsymbol{B}_n = \begin{bmatrix} -\beta & \lambda_n(1-c) & 0 & 0 \\ 0 & -\lambda_n - n\gamma & \mu & 0 \\ 0 & \lambda_n c & -\mu - \lambda_{n+1} & 0 \\ 0 & 0 & \lambda_{n+1}(1-c) & -\beta \end{bmatrix}, \; 0 \le n \le N-3,$$

$$\boldsymbol{B}_{N-2} = \begin{bmatrix} -\beta & \lambda_{N-2}(1-c) & 0 \\ 0 & -\lambda_{N-2} - (N-2)\gamma & \mu \\ 0 & \lambda_{N-2} c & -\mu - \lambda_{N-1} \end{bmatrix},$$

$$\boldsymbol{B}_{N-1} = \begin{bmatrix} -\lambda_{N-1} - (N-1)\gamma & \mu \\ \lambda_{N-1} & -\mu \end{bmatrix},$$

$$\boldsymbol{C}_n = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & n\gamma & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, 1 \le n \le N-3,$$

$$\boldsymbol{C}_{N-2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & (N-2)\gamma & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{C}_{N-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ (N-1)\gamma & 0 \end{bmatrix},$$

$$\boldsymbol{A}_n = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \beta & 0 & 0 & 0 \\ 0 & 0 & \lambda_{n+1}c & \beta \\ 0 & 0 & 0 & 0 \end{bmatrix}, 0 \le n \le N-4,$$

$$\boldsymbol{A}_{N-3} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \beta & 0 & 0 & 0 \\ 0 & 0 & \lambda_{N-2}c & \beta \end{bmatrix}, \quad \boldsymbol{A}_{N-2} = \begin{bmatrix} \beta & 0 & 0 \\ 0 & 0 & \lambda_{N-1} \end{bmatrix}.$$

It should be noted that the matrix $\mathbf{Q}$ has a non-homogeneous quasi-birth-death process. Let $\mathbf{P}$ denote the corresponding steady-state probability vector of $\mathbf{Q}$. By partitioning the vector $\mathbf{P}$ as $\mathbf{P} = [\mathbf{p}_0, \mathbf{p}_1, \ldots, \mathbf{p}_{N-3}, \mathbf{p}_{N-2}, \mathbf{p}_{N-1}]^T$, where $\mathbf{p}_n = [p_{0,uf_n}, p_{0,n}, p_{1,n}, p_{1,uf_n}]^T$, ($0 \le n \le N-3$) are column vectors with dimension 4×1, $\mathbf{p}_{N-2} = [p_{0,uf_{N-2}}, p_{0,N-2}, p_{1,N-2}]^T$ is a 3×1 column vector, and $\mathbf{p}_{N-1} = [p_{0,N-1}, p_{1,N-1}]^T$ is a 2×1 column vector, the equilibrium equation $\mathbf{QP} = \mathbf{O}_{4N-3}$ can be rewritten as follows:

$$\mathbf{B}_0\mathbf{p}_0 + \mathbf{C}_1\mathbf{p}_1 = \mathbf{O}_4, \tag{9}$$

$$\mathbf{A}_{n-1}\mathbf{p}_{n-1} + \mathbf{B}_n\mathbf{p}_n + \mathbf{C}_{n+1}\mathbf{p}_{n+1} = \mathbf{O}_4, 1 \le n \le N-3 \tag{10}$$

$$\mathbf{A}_{N-3}\mathbf{p}_{N-3} + \mathbf{B}_{N-2}\mathbf{p}_{N-2} + \mathbf{C}_{N-1}\mathbf{p}_{N-1} = \mathbf{O}_3 \tag{11}$$

$$\mathbf{A}_{N-2}\mathbf{p}_{N-2} + \mathbf{B}_{N-1}\mathbf{p}_{N-1} = \mathbf{O}_2 \tag{12}$$

where $\mathbf{O}_s$ is a zero column vector with dimensions $s\times 1$.

3.3 Steady-State Solutions

Based on Equations (9)-(12), we have the following:

$$\mathbf{p}_0 = \mathbf{X}_1\mathbf{p}_1, \text{ where } \mathbf{X}_1 = -\mathbf{B}_0^{-1}\mathbf{C}_1, \tag{13}$$

$$\begin{aligned} &\mathbf{p}_n = \mathbf{X}_{n+1}\mathbf{p}_{n+1}, \text{ where} \\ &\mathbf{X}_{n+1} = -\left(\mathbf{A}_{n-1}\mathbf{X}_n + \mathbf{B}_n\right)^{-1}\mathbf{C}_{n+1}, \ 1 \le n \le N-2 \end{aligned} \tag{14}$$

$$\begin{aligned} &\left(\mathbf{A}_{N-2}\mathbf{X}_{N-1} + \mathbf{B}_{N-1}\right)\mathbf{p}_{N-1} = \mathbf{O}_2, \text{ where} \\ &\mathbf{X}_{N-1} = -\left(\mathbf{A}_{N-3}\mathbf{X}_{N-2} + \mathbf{B}_{N-2}\right)^{-1}\mathbf{C}_{N-1} \end{aligned} \tag{15}$$

Consequently, $\mathbf{p}_n$, $0 \le n \le N-2$ can be represented in term of $\mathbf{p}_{N-1}$ as

$$\begin{aligned} \mathbf{p}_n &= \mathbf{X}_{n+1}\mathbf{X}_{n+2}\cdots\mathbf{X}_{N-1}\mathbf{p}_{N-1} = \prod_{i=N-1}^{n+1}\mathbf{X}_i\mathbf{p}_{N-1} \\ &= \mathbf{\Phi}_{n+1}\mathbf{p}_{N-1}, \end{aligned} \tag{16}$$

where $\mathbf{\Phi}_{n+1} = \prod_{i=N-1}^{n+1}\mathbf{X}_i$, $n = 0,1,\ldots,N-2.$

Finally, the probability $\mathbf{p}_{N-1}$ can be derived from Equation (15). The following normalizing equation is derived from:

$$\begin{aligned} &\sum_{n=0}^{N-3}\mathbf{e}_4^T\mathbf{p}_n + \mathbf{e}_3^T\mathbf{p}_{N-2} + \mathbf{e}_2^T\mathbf{p}_{N-1} \\ &= \left(\sum_{n=0}^{N-3}\mathbf{e}_4^T\mathbf{\Phi}_{n+1} + \mathbf{e}_3^T\mathbf{\Phi}_{N-1} + \mathbf{e}_2^T\right)\mathbf{p}_{N-1} = 1 \end{aligned} \tag{17}$$

where $\mathbf{e}_s$ denotes an identity column vector with dimensions $s\times 1$. Once the probability $\mathbf{p}_{N-1}$ has been determined, the remaining probability vectors can be derived recursively according to Equation (16). Then, the desired system performance measures can be obtained on the basis of these probability vectors. The solution algorithm for the steady-state probability vectors is described in the following subsection.

3.4 The Solution Algorithm

INPUT: Number of primary and warm standby components (M,S), $\mathbf{Q}$ matrix

OUTPUT: Steady-state probability vectors

Step 1: Set $\mathbf{X}_1 = -\mathbf{B}_0^{-1}\mathbf{C}_1$.

Step 2: For n from 1 to $N-2$, set

$$\mathbf{X}_{n+1} = -\left(\mathbf{A}_{n-1}\mathbf{X}_n + \mathbf{B}_n\right)^{-1}\mathbf{C}_{n+1}.$$

Step 3: Set $\mathbf{X}_{N-1} = -\left(\mathbf{A}_{N-3}\mathbf{X}_{N-2} + \mathbf{B}_{N-2}\right)^{-1}\mathbf{C}_{N-1}$.

Step 4: For n from 1 to $N-1$, set

$$\mathbf{\Phi}_n = \mathbf{X}_n\cdots\mathbf{X}_{N-3}\mathbf{X}_{N-2}\mathbf{X}_{N-1}.$$

Step 5: Solve $\left(\mathbf{A}_{N-2}\mathbf{X}_{N-1} + \mathbf{B}_{N-1}\right)\mathbf{p}_{N-1} = \mathbf{O}_2$ and the normalization condition

$$\left(\sum_{n=0}^{N-3}\mathbf{e}_4^T\mathbf{\Phi}_{n+1} + \mathbf{e}_3^T\mathbf{\Phi}_{N-1} + \mathbf{e}_2^T\right)\mathbf{p}_{N-1} = 1$$

simultaneously, to obtain the probability $\mathbf{p}_{N-1}$.

Step 6: For $0 \le n \le N-2$, the probability $\mathbf{p}_n$ is constructed as follows:

$$\mathbf{p}_n = \mathbf{\Phi}_{n+1}\mathbf{p}_{N-1}.$$

4 SYSTEM PERFORMANCE MEASURES

We define system performance measures of the RMRP with warm standby components and imperfect coverage as follows:

$E[N] \equiv$ the expected number of failed components in the retrial orbit;

$E[N_C] \equiv$ the expected number of failed components in the retrial orbit when the system is in a safe failure state (cover);

$E[N_{NC}] \equiv$ the expected number of failed components in the retrial orbit when and the system is in an unsafe failure state (not cover);

$E[S] \equiv$ the expected number of warm standby components in the retrial orbit;

$E[O] \equiv$ the expected number of primary components in the retrial orbit;

$P_{NC} \equiv$ the probability of failed components in the retrial orbit when and the system is in an unsafe failure state (not cover);

$AV \equiv$ the probability of the number of failed units is less than or equal to the number of warm standby units.

We can compute $E[N]$, $E[N_C]$, $E[N_{NC}]$, $E[S]$, and $E[O]$ from the following equations.

$$E[N]=\sum_{n=0}^{N-1} n(p_{0,n}+p_{1,n})+\sum_{n=0}^{N-2} np_{0,\mathrm{uf}_n}+\sum_{n=0}^{N-3} np_{1,\mathrm{uf}_n} \quad (18)$$

$$E[N_C]=\sum_{n=0}^{N-1} n(p_{0,n}+p_{1,n}) \quad (19)$$

$$E[N_{NC}]=\sum_{n=0}^{N-2} np_{0,\mathrm{uf}_n}+\sum_{n=0}^{N-3} np_{1,\mathrm{uf}_n} \quad (20)$$

$$E[S]=\sum_{n=0}^{S}(S-n)(p_{0,\mathrm{uf}_n}+p_{0,n}+p_{1,n}+p_{1,\mathrm{uf}_n}) \quad (21)$$

$$E[O]=N-E[N]-E[S], \quad (22)$$

$$P_{NC}=\sum_{n=0}^{N-2} p_{0,\mathrm{uf}_n}+\sum_{n=0}^{N-3} p_{1,\mathrm{uf}_n} \quad (23)$$

$$AV=\sum_{n=0}^{S}(p_{0,\mathrm{uf}_n}+p_{0,n}+p_{1,n}+p_{1,\mathrm{uf}_n}) \quad (24)$$

The base case for the setting of system parameters is listed below:

$M=15$, $S=10$, $\lambda=0.16$, $\mu=2.0$, $\alpha=0.08$, $\beta=6.0$, $\gamma=15.0$, $c=0.9$.

This section first studies how each parameter affects system performance measures by the change of each system parameter value. Except for M=15 (which is always fixed), each system parameter takes turn changing in a certain range while keeping other system parameters fixed at the level of the base case. We consider seven cases with various values of system parameters. The numerical results are shown in Figures 2-3.

Case 1: $\mu=2.0$, $\alpha=0.08$, $\beta=6.0$, $\gamma=15.0$, $c=0.9$, λ varies from 0.12 to 0.26;

Case 2: $\lambda=0.16$, $\alpha=0.08$, $\beta=6.0$, $\gamma=15.0$, $c=0.9$, μ varies from 1.0 to 10.0;

In Figure 2, we find that (i) $E[N]$, $E[N_C]$, $E[O]$, and AV are significantly affected by λ; (ii) $E[S]$ is slightly affected by λ; and (iii) $E[N_{NC}]$ and P_{NC} seems too insensitive to change in λ. In Figure 3, we find that (i) $E[N]$, $E[N_C]$, $E[S]$, $E[O]$, AV are significantly affected by μ; and (ii) $E[N_{NC}]$ and P_{NC} are slightly affected by μ.

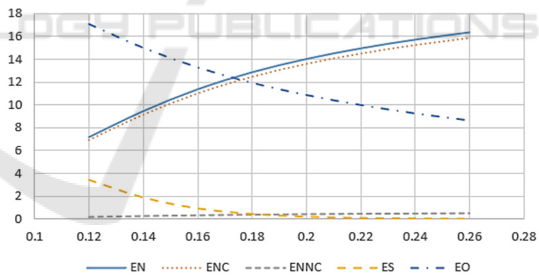

Figure 2: System performance measures versus various values of λ.

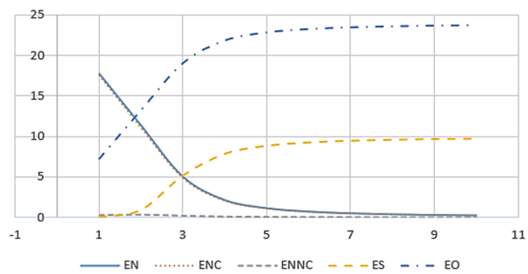

Figure 3: System performance measures versus various values of μ.

5 COST OPTIMIZATION ANALYSIS

Several researchers have investigated the study of retrial queue involving cost optimization analysis. They aimed at determining the optimal number of servers, optimal service rate, optimal repair rate, and so on. We construct the expected cost function per unit time for the RMRP with warm standby components and imperfect coverage where S, μ, and γ are decision variables. Our main goal is to determine the optimal value of (S,μ,γ), say $(\hat{S},\hat{\mu},\hat{\gamma})$, so as to minimize the cost function. The cost elements are defined as follows:

$C_1 \equiv$ cost per unit time per failed component in the retrial orbit when the system is in a safe failure state (cover);

$C_2 \equiv$ cost per unit time per failed component in the retrial orbit when the system is in an unsafe failure state (not cover);

$C_3 \equiv$ cost per unit time of probability of P_{NC};

$C_4 \equiv$ cost per unit time of unavailability $1-AV$;

$C_5 \equiv$ cost per unit time of providing the service rate μ;

$C_6 \equiv$ cost per failed component in retrial orbit by providing the retrial rate γ.

Based on all of the cost elements listed above, the expected cost per unit time is constructed as follows:

$$TC(S,\mu,\gamma) = C_1E[N_C] + C_2E[N_{NC}] + C_3P_{NC} + C_4(1-AV) + C_5\mu + C_6\gamma. \quad (25)$$

Thus, the cost minimization problem can be expressed mathematically as

$$TC(\hat{S},\hat{\mu},\hat{\gamma}) = \underset{S,\mu,\gamma}{Minimize}\; TC(S,\mu,\gamma).$$

5.1 Sensitivity Analysis

As the following numerical examples, we consider the cost elements as follows:

$C_1 = \$60, C_2 = \$120, C_3 = \$240, C_4 = \$480, C_5 = \$90, C_6 = \$30.$

To examine the effect of system parameters on the cost function, a sensitivity analysis in six cases is provided for $M=15$ with various values of $S = 3, 5, 7,$ respectively.

Case 1: $\mu = 2.0$, $\alpha = 0.08$, $\beta = 6.0$, $\gamma = 15.0$, $c = 0.9$, λ varies from 0.12 to 0.26;

Case 2: λ=0.16, $\alpha = 0.08$, $\beta = 6.0$, $\gamma = 15.0$, $c = 0.9$, μ varies from 1.0 to 10.0;

The numerical results of the above four cases are shown in Figures 4-5, which depicts the sensitivity performance of cost function TC on λ, α, β, μ, γ, and c, respectively. It is important to note that the sign of sensitivity indicates an increase or decrease in the expected cost by changing the values of system parameters. Figure 4 reveals that (i) $\partial TC/\partial\lambda$ is positive, which means that TC increases as λ increases for all S; (ii) $\partial TC/\partial\lambda$ has the highest point at around $\lambda = 0.13$ for all S; and for (iii), as λ is fixed, $\partial TC/\partial\lambda$ gets larger as S increases. Figure 5 shows that (i) $\partial TC/\partial\mu$ changes from negative to positive, which means that TC changes from a decrease to an increase on μ for all S; (ii) as μ is fixed, $\partial TC/\partial\mu$ gets smaller as S increases; and when (iii) as μ is greater than 4, $\partial TC/\partial\mu$ is similar for all S.

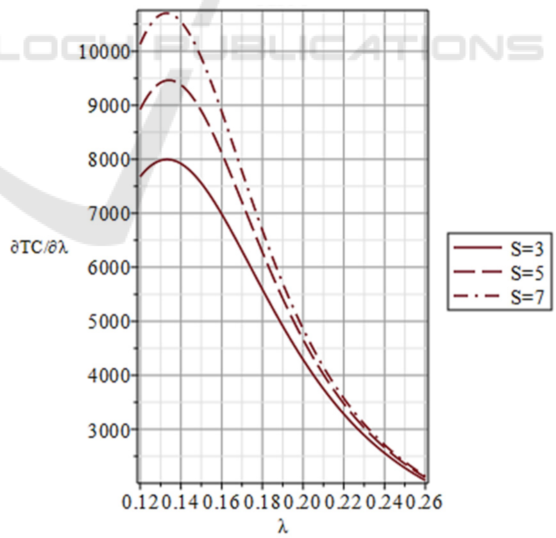

Figure 4: Sensitivity analysis of TC with respect to λ for different S.

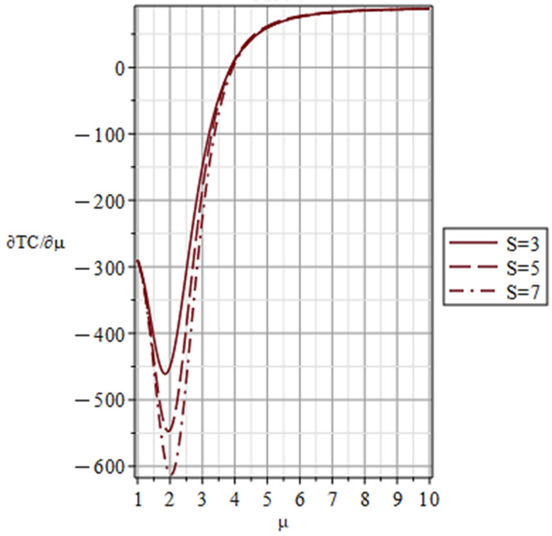

Figure 5: Sensitivity analysis of TC with respect to μ for different S.

5.2 Cost Optimization

The PSO algorithm was first proposed by Kennedy and Eberhart (1995) and works with a population of particles where one procedure includes exploitation optimization searches. As the following numerical examples, we consider the cost elements as follows:

$$C_1 = \$60, C_2 = \$120, C_3 = \$240,$$
$$C_4 = \$480, C_5 = \$90, C_6 = \$30.$$

The PSO algorithm is applied to find the approximate optimization solution $(\hat{S}, \hat{\mu}, \hat{\gamma})$ and minimum cost $TC(\hat{S}, \hat{\mu}, \hat{\gamma})$. Since the PSO algorithm does not need to compute the gradient, it is flexible for non-differentiable cost functions. Moreover, it can be implemented to handle optimization problems with a mixture of discrete and continuous decision variables. We first fix M=15 and consider various values of λ, α, β, and c. Then, after setting the different ranges of decision variables S, μ, γ by $1 \le S \le 10$, $0.1 \le \mu \le 10.0$, and $0.1 \le \gamma \le 20.0$, we use computer software Maple for numerical investigation. The detailed optimal solution $(\hat{S}, \hat{\mu}, \hat{\gamma})$, minimum cost $TC(\hat{S}, \hat{\mu}, \hat{\gamma})$ and related parameters are shown in Tables 1-4. We observe from Tables 1-4 that (i) the optimal number of warm standby components $\hat{S}$ increases as λ increases; (ii) the optimal number of warm standby components $\hat{S}$ decreases as α increases; (iii) the optimal number of warm standby components $\hat{S}$ is the same even though β varies from 4.0 to 8.0 and c varies from 0.6 to 1.0; (iv) $TC(\hat{S}, \hat{\mu}, \hat{\gamma})$ increases as λ or α increases; and (v) $TC(\hat{S}, \hat{\mu}, \hat{\gamma})$ decreases as β or c increases. Intuitively, the optimal number of warm standby components $\hat{S}$ is significantly affected by λ and α, but seems too insensitive to change in β and c.

Table 1: The optimal results for various values of λ with $\alpha = 0.08$, $\beta = 6.0$, and $c = 0.9$.

λ	$\hat{S}$	$\hat{\mu}$	$\hat{\gamma}$	$TC(\hat{S}, \hat{\mu}, \hat{\gamma})$
0.12	4	3.435	2.493	519.127
0.14	5	3.854	2.711	579.451
0.16	5	4.268	2.967	637.675
0.18	5	4.672	3.216	694.645
0.20	5	5.066	3.457	750.483

Table 2: The optimal results for various values of α with $\lambda = 0.16$, $\beta = 6.0$, and $c = 0.9$.

α	$\hat{S}$	$\hat{\mu}$	$\hat{\gamma}$	$TC(\hat{S}, \hat{\mu}, \hat{\gamma})$
0.04	6	4.215	2.899	622.923
0.06	5	4.247	2.957	631.115
0.08	5	4.268	2.967	637.675
0.10	5	4.288	2.976	644.031
0.12	4	4.300	3.052	648.612

Table 3: The optimal results for various values of β with $\lambda = 0.16$, $\alpha = 0.08$, and $c = 0.9$.

β	$\hat{S}$	$\hat{\mu}$	$\hat{\gamma}$	$TC(\hat{S}, \hat{\mu}, \hat{\gamma})$
4.0	5	4.272	2.977	644.166
5.0	5	4.270	2.971	640.304
6.0	5	4.268	2.967	637.675
7.0	5	4.267	2.964	635.770
8.0	5	4.266	2.962	634.327

Table 4: The optimal results for various values of c with $\lambda = 0.16$, $\alpha = 0.08$, and $\beta = 6.0$.

c	$\hat{S}$	$\hat{\mu}$	$\hat{\gamma}$	$TC(\hat{S}, \hat{\mu}, \hat{\gamma})$
0.6	5	4.242	3.346	689.743
0.7	5	4.250	3.223	673.331
0.8	5	4.259	3.096	656.009
0.9	5	4.268	2.967	637.675
1.0	5	4.279	2.834	618.210

6 CONCLUSIONS

This article considers the RMRP with warm standby components and imperfect coverage. Steady-state results are computed numerically with the matrix-analytic technique. We have performed the sensitivity analysis of system performance measures with respect to various system parameters. By using the PSO algorithm, we determine the joint optimal values of the number of warm standbys, the repair rate, and the retrial rate simultaneously to minimize the expected cost. The PSO algorithm can be applied to analyze the complex optimization problems that occur in various retrial queues (or RMRP). Under optimal operating conditions, we illustrate our results by discussing several cases of numerical examples. The experimented results are helpful for managers to make decisions. Moreover, the results obtained provide further insight into the RMRP with warm standby components and imperfect coverage.

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