Morphing Between Monotonic Spinner Planar Curves Through Radial-Sign Descriptors

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Abstract: This paper introduces an innovative morphing method leveraging Radial-Sign descriptors for monotonic spinner shapes, offering a robust, efficient, and computationally refined solution to shape blending challenges. The method encodes two shapes using radial distances and angular sign variations relative to the centroids, respectively, producing complete, stable, and invertible descriptions. By applying weighted interpolation directly to these two descriptors and reconstructing in-between shapes through an inverse formula, the approach ensures smooth, morphologically coherent transitions while preserving essential geometric properties. Unlike conventional curvature or registration-based techniques, which often require intensive post-processing or face limitations with significantly different shapes, the proposed method adeptly blends both similar and dissimilar shapes, including those with differing turning number, by introducing additional turns in simpler shapes to ensure continuity and coherence.

1 INTRODUCTION

Morphing, or blending, is a powerful technique widely utilized in computer vision and graphics, and it has gained substantial importance in machine learning for its ability to generate new data and, then, augment existing datasets. This process involves the smooth transformation of one shape or image into another. However, its effective implementation demands addressing several challenges. The correspondences between points on source and target shapes is critical yet challenging, especially in the presence of non-rigid deformations. Misaligned correspondences can lead to undesirable artifacts such as unnatural deformations.

In this work, we focus on planar shape Morphing. The process of planar shape morphing is commonly divided into two fundamental steps: the vertex correspondence problem and the vertex path problem (Saba et al., 2014). The vertex correspondence problem involves establishing a mapping between points, vertices, or features on the source and target shapes, ensuring that meaningful and consistent relationships are preserved throughout the transformation process. This step is crucial for handling variations in topology, geometry, or complexity between the shapes. Once correspondences are established, the vertex path problem focuses on determining the intermediate shapes that interpolate between the source and target. This phase ensures a smooth and visually coherent transition along the morphing trajectory.

Traditional shape blending techniques, such as polygon interpolation (Alexa et al., 2000; Shapira and Rappoport, 1995), often struggle with computational challenges, particularly regarding triangle compatibility in non-smooth shapes. These methods can result in discontinuities or require human-interaction.

Methods utilizing curvature descriptors (Surazhsky and Elber, 2002; Hirano et al., 2017; Saba et al., 2014; Sederberg et al., 1993a) are widely adopted and demonstrate strong performance. Thanks to arc-length parameterization, the correspondence step is simplified to aligning the starting points of the source and target shapes. However, these approaches often result in intermediate shapes that are not closed, requiring additional post-processing steps, such as B-splines, to achieve curve closure, which can be relatively computationally intensive (Hirano et al., 2017; Saba et al., 2014; Thévenaz et al., 2000).

As another common approach, Registration-based methods align shapes through non-linear optimization for correspondence purposes (Sebastian et al., 2003; Srivastava et al., 2010; Klassen et al., 2004;

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Jin et al., 2021). These methods exhibit highly logical and smoothly continuous deformations. However, it often incur significant computational overhead and may fail to guarantee smooth transitions when the source and target shapes are significantly different.

In the other hand, Fourier-based methods offer a relatively fast alternative, inherently producing intermediate closed curves due to their periodic nature while being invariant to starting point(Ghorbel et al., 2022; Ghorbel et al., 2021). However, these methods are constrained to contours limiting their applicability to open curves.

As an efficient and straightforward solution, we propose a novel blending approach leveraging the Radial-Sign descriptor (Ghorbel and Burdin, 1994; Burdin et al., 1992), a complete, stable, and invertible shape descriptor, designed for the morphing of *Monotonic Spinner* planar curves. A *Monotonic Spinner* planar curve is defined as a curve where the radial distance from the center of rotation varies monotonically with the angle. Similarly to curvature-based methods, the proposed technique requires only a starting point alignment to establish correspondence between shapes and generates deformations that are invariant under rigid transformations (translations, scales and rotations). However, unlike curvature-based approaches, the proposed method maintain a morphologically coherent transition, preserving the natural structure and flow of the shapes throughout the process. Moreover, the method effectively blend both similar and dissimilar shapes, including those with differing turning number. Indeed, it suffices to introduce additional turns in simpler shapes, ensuring continuity and coherence in the resulting transition. Therefore, it allows the interpolation between shapes with distinct topologies.

In the following, Section 2 recalls the Radial-Sign Descriptor and introduce the Blending approach. Section 3 concerns the experiments where Morphing quality, Morphing invariance, comparison with curvature methods, and application on the MPEG-7 CE Benchmark are presented. Finally, Section 4 concludes this work.

2 PROPOSED MORPHING APPROACH

In this section, we begin by recalling the Radial-Sign descriptors introduced in (Ghorbel and Burdin, 1994; Burdin et al., 1992), emphasizing their properties that are essential for shape morphing. Therefore, we propose a novel morphing method that uses Radial-Sign descriptors to achieve smooth and coherent transitions between monotonic spinner contours.

2.1 Radial-Sign Descriptor

Let $\mathcal{R} = (O, i, j)$ be a Cartesian coordinate system in the Euclidean plane, and let s denote a normalized arc length. Consider $\gamma(s)$, a monotonic spinner curve with respect to its centroid O, parameterized by its normalized arc length. In polar coordinates, this parameterization can be expressed as:

$$\gamma(s) = \big(\rho(s)\cos(\theta(s)), \rho(s)\sin(\theta(s))\big),$$

where $\rho(s)$ is the radial function and $\theta(s)$ is the polar angle. Differentiating this representation with respect to s and taking the norm of both sides of the resulting equation, we use the fact that $\|\gamma'(s)\|_2 = 1$ which leads to the following relation,

$$\theta'(s)^2 = \frac{1-\rho'(s)^2}{\rho(s)^2}.$$

The solution to this differential equation is given by,

$$\theta(s) = \theta(0) + \int_0^s \sigma(l)\frac{\sqrt{1-\rho'(l)^2}}{\rho(l)}\,dl, \qquad (1)$$

where $\sigma(l) \in \{-1, 1\}$ is the sign of $\theta'(l)$. Note that to obtain invariance to scale, it suffices to normalize the radius function by dividing $\rho(s)$ by the scale factor ρ_{scale}, defined as the maximum radius:

$$\rho_{\text{scale}} = \max_s \rho(s).$$

This normalization leads to the scale-invariant radial function,

$$\tilde{\rho}(s) = \frac{\rho(s)}{\rho_{\text{scale}}}.$$

Figures 1 and 2 show respectively a 1-turning and 4-turning monotonic spinner curves with respect to their centroids and their corresponding polar coordinates. Figure 3 illustrates the radial-sign computation framework. Starting from Cartesian coordinates, the transformation to polar coordinates is achieved by calculating the radial function $\rho(s)$ and the polar angle $\theta(s)$. Subsequently, the sign $\sigma(s)$ of the derivative of the polar angle $\theta(s)$ is computed.

Under the assumption of a normalized arc-length parameterization, the angular function $\theta(s)$ can be replaced by the sign function of its derivative, $\sigma(s)$, without any loss of information. As a result, the set of descriptors $\{(\rho(s), \sigma(s))\}$ becomes a compact and robust representation that is invariant to rigid transformations while maintaining the property of completeness (Crimmins, 1982).

A complete descriptor ensures that two shapes are identical if and only if their descriptors are equal. Beyond invariance and completeness, additional properties such as invertibility and stability are essential for tasks such as morphing and shape reconstruction. the invertibility guarantees that shapes can be reconstructed from their descriptors through explicit formulas, enabling the generation of intermediate curves by interpolating between descriptors (Ghorbel et al., 2022). The stability on the other hand, ensures that small variations in the descriptor space result in correspondingly small variations in the shape space, and vice versa, making the descriptors robust to perturbations (Ghorbel, 1992). Figures 4, 5 and 6 illustrate these properties with practical examples.

For shape morphing, these properties are particularly critical. Morphing requires descriptors that allow the reconstruction of shapes as continuous and progressive deformations, without introducing abrupt changes in shape. To achieve this, the focus is placed on monotonic spinner curves, which exhibit a constant sign for $\sigma(s)$. This constancy ensures both the invertibility and stability of the descriptors, facilitating smooth transitions between shapes during morphing.

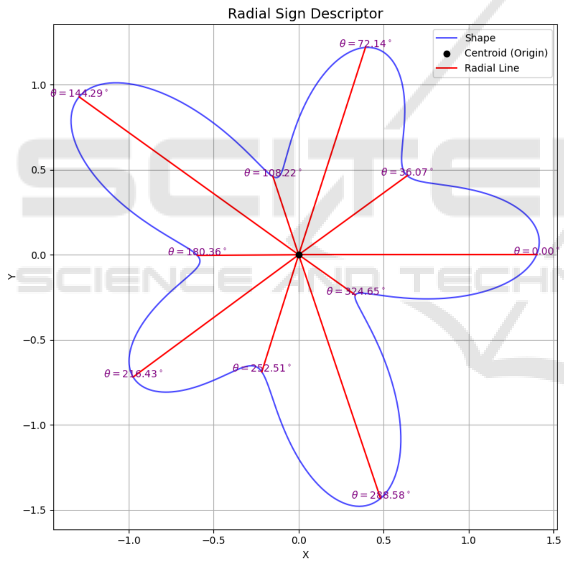


Figure 1: An example of a monotonic spinner with 1-turning number (case. star-shaped) curve with polar coordinates.

2.2 Radial-Sign Morphing

Since the Radial-Sign descriptor satisfies the four key properties of Completeness, Invariance, Invertibility, and Stability (CIIS) for monotonic spinner contours, it is well-suited for performing morphing, as introduced in (Ghorbel et al., 2022).

Let γ_0 and γ_1 be two monotonic spinner contours parameterized by their normalized arc lengths s, and described by their radial-sign functions $f(\gamma_0) =$

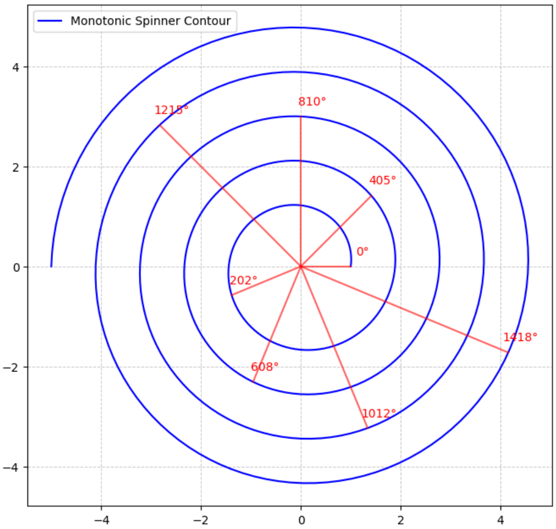


Figure 2: An example of a monotonic spinner with 4-turning number curve with polar coordinates. the red line corresponds to the Radial measure.

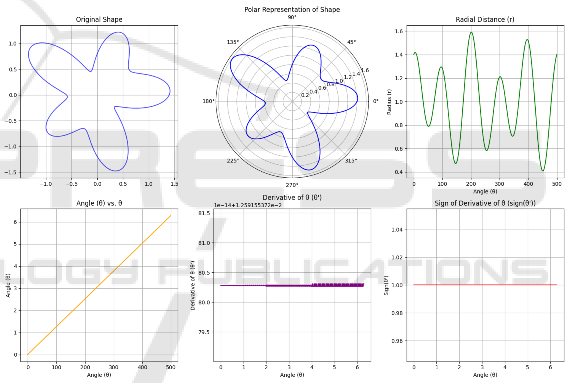


Figure 3: The radial-sign descriptor framework. Starting from Cartesian coordinates, the transformation to polar coordinates is achieved by calculating the radial function $\rho(s)$ and the polar angle $\theta(s)$. Then, the sign $\sigma(s)$ of the derivative of $\theta(s)$ is computed.

$(\rho_0(s), \sigma_0(s))$ and $f(\gamma_1) = (\rho_1(s), \sigma_1(s))$, respectively. The interpolation between the two contours is performed in the space of the Radal-sign invariant descriptors. For $t \in [0,1]$, the interpolated radial function is:

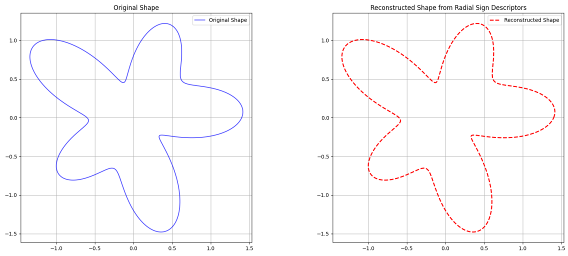
$$\rho_t = (1-t)\rho_0 + t\rho_1.$$

While the sign function should be equal,

$$\sigma_t = \sigma_1(s) = \sigma_0(s)$$

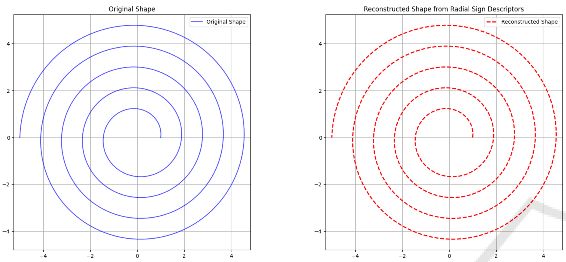
Finally, the intermediate contours γ_t are reconstructed by computing the θ_t angle from the σ_t as follows,

$$\theta_t = \int_0^s \sigma_t(l) \frac{\sqrt{1-\rho_t'(l)^2}}{\rho_t(l)} dl,$$



(a) (b)

Figure 4: Invertibility of the descriptors for star-shaped curve (1-turning number). (a) Original curve (b) Reconstructed curve.



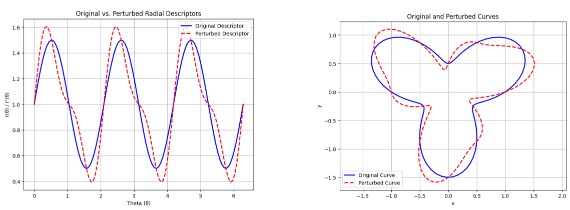
(a) (b)

Figure 5: Invertibility of the descriptors for monotonic spinner shape (4,5-turning number). (a) Original curve (b) Reconstructed curve.

This method ensures a smooth and consistent morphing process, leveraging the invariance and stability of the Radial-Sign descriptors to generate meaningful intermediate contours. Figure 7 presents two Radial-Sign morphing examples between a pair of curves.

3 EXPERIMENTS

In this section, we report and analyze the experimental results. First, we present some morphing sequences in order to evaluate the quality visually. Therefore, the robusteness of Radial-sign based deformations to rigid transformation is tested. Then, we compare the proposed approach based on the Radial-



(a) (b)

Figure 6: Stability of the descriptors. (a) Radial representation for the original and perturbed curves. (b) Reconstruction of the original and the pturbed curves.

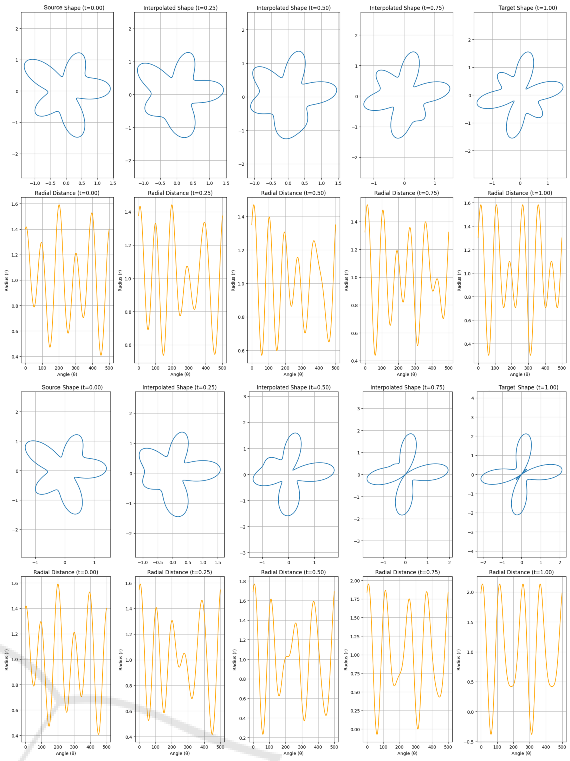


Figure 7: Two examples of Radial-sign based Morphing between a pair of curves.

sign descriptor to curvature-based methods with B-splines (Surazhsky and Elber, 2002; Sederberg et al., 1993b; Saba et al., 2014) according to turning number challenge and some qualitative properties. Finally, we provide qualitative results on the monotonic spinner shapes from the MPEG-7 CE dataset (Latecki et al., 2000).

3.1 Morphing Quality

In this part, we propose to evaluate the morphing quality visually. Let's begin by showing various examples of the Radial-sign blending under various conditions, including open, closed, and non-simple curves. Figure 8 demonstrates the deformation between two open curves, where the intermediate shapes preserve the continuity and smoothness of the curve while gradually transitioning from the source curve to to the target curve. Figure 9 showcases an example of deformation between two shapes that exhibit multiple n-turning points. This scenario highlights the robustness of the morphing process in handling shapes with intricate geometries. Finally, Figure 10 illustrates a deformation process between a simple and a non-simple shape. By introducing additional turns in the source shape to match those of the target, the method ensures both continuity and coherence throughout the morphing sequence. Despite the complexities introduced by the differences in topology, the approach successfully preserves the underlying structure of the shapes. This demonstrates the versatility and robustness of the proposed method in handling challenging geometries, enabling smooth and morphologically consistent transitions even between shapes with significant dissimilarities.

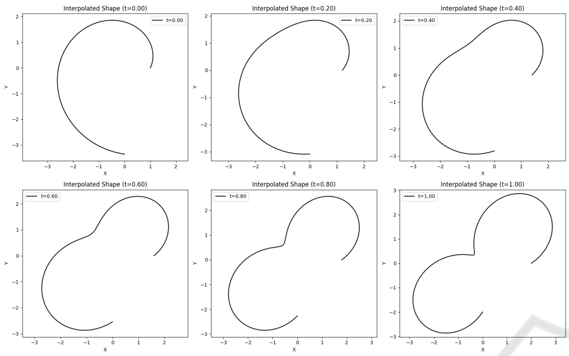


Figure 8: Example of Deformation between two open curves.

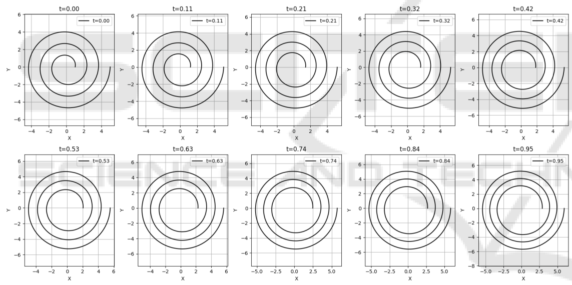


Figure 9: Example of Deformation between two n-turning number shapes.

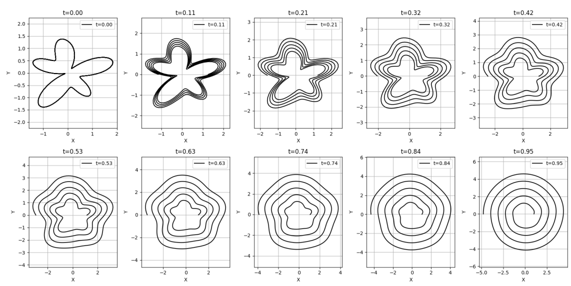


Figure 10: Example of Deformation between two a simple shape and a complex one.

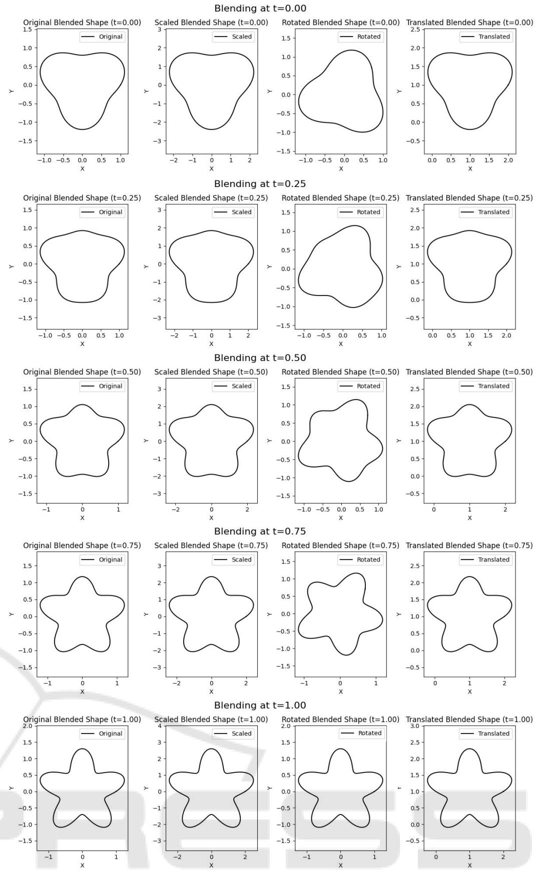


Figure 11: Invariance of the Radial-Sign deformation under rigid transformations.

3.2 Invariance to Rigid Transformations

To evaluate the robustness of the Radial-Sign morphing approach under rigid transformations, we propose to blend two monotonic spinner curves after the application of different transformations. The aim is to validate that the intermediate shapes remain invariant when the input shapes are subject to scaling, rotation, and translation. These transformations simulate practical scenarios where shapes may vary in size, orientation, or spatial location. In this specific test, the source curve was manipulated by applying a rotation of $\pi/4$, a scaling factor of $a = 2$, and a translation by $(1, 1)$ in Cartesian coordinates. Figure 11 showcases the results of this experiment. The intermediate shapes, regardless of the applied transformations, stay the same. This performance is attributed to the invariance properties of the Radial-Sign descriptors.

3.3 Comparison with Curvature-Based Methods

We compare our method to curvature-based approaches, focusing on three aspects: Turning number challenge and qualitiative properties.

3.3.1 Turning Number Challenge

Blending two curves with different turning numbers presents additional challenges. Specifically, determining the correct behavior of the interpolation method in such cases remains complex. Methods that aim to preserve the *natural* deformation of the curves often involve intricate transformations, including folding or unfolding of the curves. This behavior complicates the testing and evaluation of our approach, as it seeks to maintain as much consistency with the original curve's geometry as possible, without introducing unnatural deformations.

In our experiments, we blended a circle into an *8-shaped* curve. The results, shown in Figure 11, highlight the unpredictable behavior observed across all methods. Nevertheless, our proposed radial-sign approach mitigates issues associated with turning numbers and provides a more stable and intuitive interpolation process. The comparison demonstrates the superior performance of our method, particularly in terms of the quality of the intermediate shapes. The shapes produced by our approach are smoother and preserve their concavities throughout the transformation. In contrast, curvature-based methods often introduce distortions or lose concavities during interpolation, necessitating additional post-processing steps such as B-splines to close the curves. In another hand, our method naturally ensures closed curves without requiring such steps.

3.3.2 Qualitative Properties

To further validate the Radial-Sign descriptor's performance, we compare qualitatively the morphing approaches according to several properties.

Table 1: Comparison Between the Curvature-Based and Radial-Sign-Based Morphing Methods.

	Curvature	**Radial-Sign**
Invertibility	Numerical	Analytical
Postprocessing	Yes	No
Curve type	C^2	$Monotonic-Spinner$
Complexity	$O(n^2)$	$O(n)$
Invariance	SE(2)	SE(2)

Table 1 highlights several key differences between the curvature-based and Radial-Sign morphing methods. First, the curvature method relies on numerical approaches for invertibility, whereas the Radial-Sign method provides an analytical inversion, leading to more stable and precise results. Additionally, curvature-based methods often require postprocessing to close the curves, while the Radial-Sign approach operates without this need. In terms of smoothness, the curvature-based method is efficient on C^2 curves while monotonic spinner curves are needed for the Radial-Sign method. In another hand, for the interpolation part, the Radial-Sign method has a linear time complexity of $O(n)$, compared to the quadratic $O(n^2)$ complexity of curvature-based methods, making it significantly more efficient. Finally, Radial-Sign Morphing ensure the invariance under Euclidean transformations as well as the curvature-based approaches.

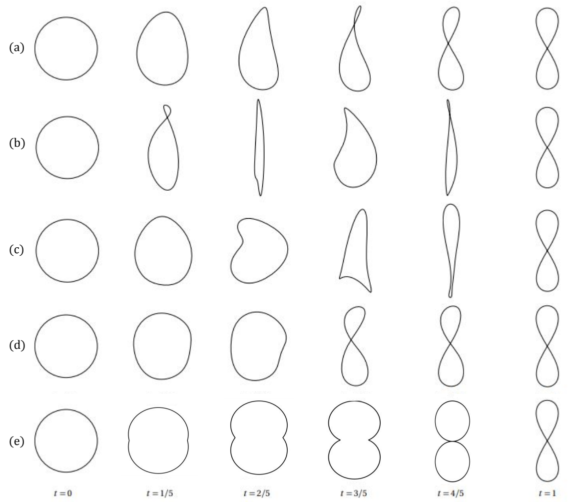


Figure 12: Comparison of the methods when interpolating two curves with different turning numbers. In this case, γ (left) has turning number 1 and γ' has turning number 0. (a) linear interpolation (b) curvature (Sederberg et al., 1993b) (c) curvature (Surazhsky and Elber, 2002) (d) curvature (Saba et al., 2014) (e) our method.

3.4 Morphing on the MPEG-7 CE Dataset

In this section, we propose extracting monotonic spinner curves, specifically star-shaped ones, from the MPEG-7 CE dataset (Latecki et al., 2000) and generating in-between curves belonging to the same category (intra-class) and in-between curves belonging to different categories (inter-class). The MPEG-7 dataset is a widely used 2D shape dataset consisting of 70 categories, with 20 images per category. Figure 13 illustrates some star-shaped samples from the dataset. After extracting contours from the black and

Figure 13: Star-shaped samples from MPEG-7 CE dataset.

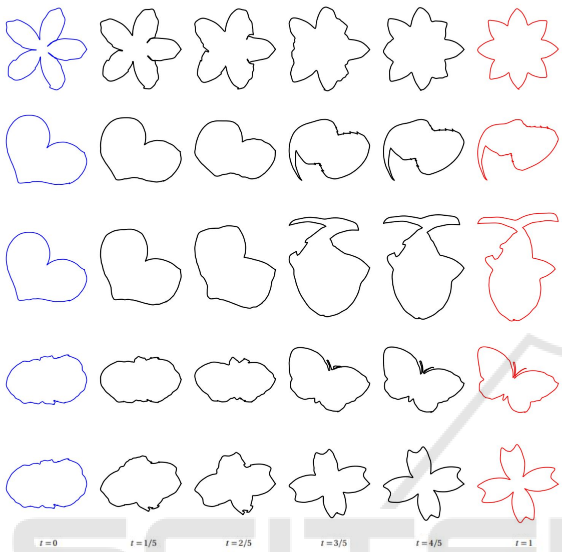


Figure 14: Examples of inter-class deformations between shapes belonging to MPEG-7 CE.

white MPEG-7 CE images, we apply the proposed morphing method. Figures 15 and 14 illustrate the results of the morphing process. Intra-class morphing produces smooth and visually coherent transitions between shapes within the same class. For inter-class morphing, the Radial-Sign blending approach generates meaningful intermediate shapes while maintaining a low computational cost, demonstrating the efficiency and practicality of the proposed method.

4 CONCLUSION

This work introduces a morphing approach based on the Radial-Sign descriptor, tailored for monotonic spinner planar curves. By leveraging the completeness, stability, and invertibility of these descriptors, the proposed method generates intermediate shapes that are invariant under rigid transformations while ensuring progressive and continuous deformations. Experimental results validate the effectiveness of this approach in achieving smooth, high-quality in-between curves, outperforming traditional curvature-based methods in terms of computational efficiency,

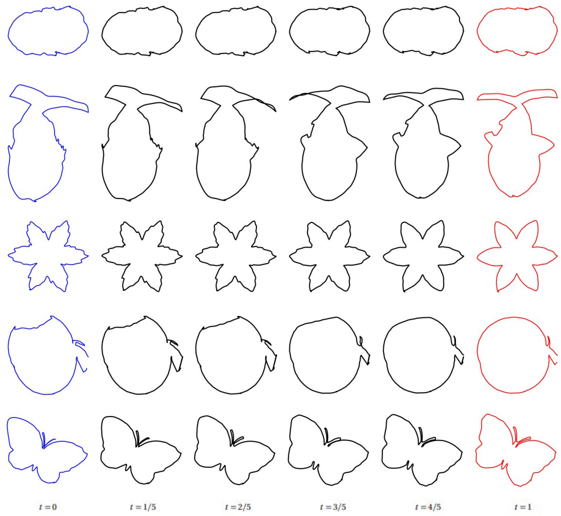


Figure 15: Examples of intra-class deformations between shapes belonging to MPEG-7 CE.

shape closure, and turning number issue. For future work, the focus will be on advancing 3D classification techniques, exploring methods for compression and reconstruction, and further enhancing morphing techniques, particularly with sliced monotonic spinner shapes.

REFERENCES

Alexa, M., Cohen-Or, D., and Levin, D. (2000). As-rigid-as-possible shape interpolation. In *Proceedings of the 27th annual conference on Computer graphics and interactive techniques*, pages 157–164.

Burdin, V., Ghorbel, F., de Bougrenet de la Tocnaye, J., and Roux, C. (1992). A three-dimensional primitive extraction of long bones obtained from bi-dimensional fourier descriptors. *Pattern Recognition Letters*, 13(3):213–217.

Crimmins, T. R. (1982). A complete set of fourier descriptors for two-dimensional shapes. *IEEE Transactions on Systems, Man, and Cybernetics*, 12(6):848–855.

Ghorbel, E., Ghorbel, F., and M'Hiri, S. (2022). A fast and efficient shape blending by stable and analytically invertible finite descriptors. *IEEE Transactions on Image Processing*, 31:5788–5800.

Ghorbel, E., Ghorbel, F., Sakly, I., and M'Hiri, S. (2021). Fast blending of planar shapes based on invariant invertible and stable descriptors. In *2020 25th International Conference on Pattern Recognition (ICPR)*, pages 10259–10265. IEEE.

Ghorbel, F. (1992). Stability of invariant fourier descriptors and its inference in the shape classification. In *INTERNATIONAL CONFERENCE ON PATTERN RECOGNITION*, pages 130–130, New York City at 3 Park Ave. IEEE COMPUTER SOCIETY PRESS.

Ghorbel, F. and Burdin, V. (1994). Invariant approximation of star-shaped form for medical applications. *Wavelets, Images, and Surface Fitting*, page 269.

Hirano, M., Watanabe, Y., and Ishikawa, M. (2017). Rapid blending of closed curves based on curvature flow. *Computer Aided Geometric Design*, 52:217–230.

Jin, L., Wen, Z., and Hu, Z. (2021). Topology-preserving nonlinear shape registration on the shape manifold. *Multimedia Tools and Applications*, 80(11):17377–17389.

Klassen, E., Srivastava, A., Mio, M., and Joshi, S. H. (2004). Analysis of planar shapes using geodesic paths on shape spaces. *IEEE transactions on pattern analysis and machine intelligence*, 26(3):372–383.

Latecki, L., Lakamper, R., and Eckhardt, T. (2000). Shape descriptors for non-rigid shapes with a single closed contour. In *Proceedings IEEE Conference on Computer Vision and Pattern Recognition. CVPR 2000 (Cat. No.PR00662)*, volume 1, pages 424–429 vol.1, New York City at 3 Park Ave. IEEE.

Saba, M., Schneider, T., Hormann, K., and Scateni, R. (2014). Curvature-based blending of closed planar curves. *Graphical models*, 76(5):263–272.

Sebastian, T. B., Klein, P. N., and Kimia, B. B. (2003). On aligning curves. *IEEE transactions on pattern analysis and machine intelligence*, 25(1):116–125.

Sederberg, T. W., Gao, P., Wang, G., and Mu, H. (1993a). 2-d shape blending: an intrinsic solution to the vertex path problem. In *Proceedings of the 20th annual conference on Computer graphics and interactive techniques*, pages 15–18, New York, NY, USA. ACM.

Sederberg, T. W., Gao, P., Wang, G., and Mu, H. (1993b). 2-d shape blending: an intrinsic solution to the vertex path problem. In *Proceedings of the 20th annual conference on Computer graphics and interactive techniques*, pages 15–18.

Shapira, M. and Rappoport, A. (1995). Shape blending using the star-skeleton representation. *IEEE Computer Graphics and Applications*, 15(2):44–50.

Srivastava, A., Klassen, E., Joshi, S. H., and Jermyn, I. H. (2010). Shape analysis of elastic curves in euclidean spaces. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 33(7):1415–1428.

Surazhsky, T. and Elber, G. (2002). Metamorphosis of planar parametric curves via curvature interpolation. *International Journal of Shape Modeling*, 8(02):201–216.

Thévenaz, P., Blu, T., and Unser, M. (2000). Interpolation revisited [medical images application]. *IEEE Transactions on medical imaging*, 19(7):739–758.