Heart Rate Turbulence: Wavelet Analysis of Frequency Modulated Signals

S. V. Bozhokin[1] [a], I. B. Suslova[2] [b], A. A. Riabokon[1] [c] and T. D. Shokhin[1] [d]
[1]*Institute of Physics and Mechanics, Peter the Great St. Petersburg Polytechnic University, 195251 St. Petersburg, Russia*
[2]*World-Class Research Center for Advanced Digital Technologies, Peter the Great St. Petersburg Polytechnic University, 195251 St. Petersburg, Russia*

Keywords: Heart Rate Turbulence, Continuous Wavelet Transform, Local Frequency.

Abstract: This paper presents new approaches to the analysis of non-stationary heart rate variability (HRV) taking into account strong time variation in the duration of RR intervals, which is associated with extrasystoles. A mathematical model of a frequency-modulated signal comprising of identical Gaussian peaks unevenly spaced along the time axis is applied to the phenomenon of heart rate turbulence (HRT). The maxima of the Gaussian peaks correspond to the moments of real heart contractions. A time-continuous function of local (instantaneous) heart rate frequency is calculated by analyzing the maxima of the continuous wavelet transform applied to such a model signal. The change in local frequency over time is proposed as a new characteristic of extrasystoles and compensatory pauses in the heart tachogram. The proposed method, applied in this work to study tachogram records with extrasystoles, can be used in the analysis of any other heart rhythm disturbances.

1 INTRODUCTION

Heart rate turbulence (HRT) is of considerable interest in cardiology for the diagnosis of life-threatening conditions, attracting the attention of both practicing physicians and multidisciplinary researchers. HRT is associated with significant disturbances in heart rhythm frequency, such as extrasystole (Schmidt et al., 1999; Bauer et al., 2008; Disertori et al., 2016; MA, 2004; Cygankiewicz & Zaręba, 2006; Cygankiewicz, 2013; Germanova et al., 2021; Huikuri et al., 2001). Extrasystole is a premature excitation of the heart caused by the mechanism of repeated entry of electrical excitation (re-entry). The essence of the re-entry mechanism is that the electrical impulse repeatedly enters a section of the myocardium or the conduction system of the heart, creating a circulation of the excitation wave. The relationship between cardiovascular risk factors and heart rate variability (HRV) in patients with heart failure has been shown in many scientific studies (Zeid et al., 2024; Thayer et al., 2010; Kubota et al., 2017; Huikuri & Stein, 2013; Turcu et al., 2023; Yan et al., 2023; Lombardi & Stein, 2011). Many rather complex mathematical methods have been developed and applied for quantitative analysis of HRV based on processing of electrical signals of the heart. Recently, various combinations of time-frequency, nonlinear and neural network methods have been actively used. Comparison of various methods and the results of their application in the study of HRT shows the presence of classification errors and difficulties in comparing the results obtained for different groups of patients by different methods (Blesius et al., 2020; Acharya et al., 2006; Sauerbier et al., 2024; Yin et al., 2014; Koyama et al., 2002; Tsvetnikova et al., 2008).

The problem of studying extrasystole includes both ECG analysis in terms of PQRST complex morphology and the analysis of variations in RR interval duration. It should be noted that in both atrial (APV) and ventricular (VPC) extrasystole, a

[a] https://orcid.org/0000-0001-5653-6574
[b] https://orcid.org/0000-0002-4497-1867
[c] https://orcid.org/0009-0001-6948-6200
[d] https://orcid.org/0009-0006-3348-7603

Bozhokin, S. V., Suslova, I. B., Riabokon, A. A. and Shokhin, T. D.
Heart Rate Turbulence: Wavelet Analysis of Frequency Modulated Signals.
DOI: 10.5220/0013342700003911

In *Proceedings of the 18th International Joint Conference on Biomedical Engineering Systems and Technologies (BIOSTEC 2025) - Volume 1*, pages 1038-1045
ISBN: 978-989-758-731-3; ISSN: 2184-4305

significant change in the RR intervals is observed before and after the ectopic contraction.

In this paper, we present new characteristics of non-stationary heart rate variability (NHRV) obtained from the analysis of non-stationary rhythmic features of the tachogram RR - intervals with extrasystoles. When monitoring the state of the cardiovascular system, measuring heart rate is quite convenient and simple. As a model of NHRV we consider the model of frequency modulated tachogram signal. In the proposed tachogram model, we do not exclude from the analysis the time intervals of the extrasystoles themselves, as well as the compensatory pauses following the extrasystoles.

The research works (Schmidt et al., 1999; Zeid et al., 2024; Blesius et al., 2020; Sauerbier et al., 2024; Yin et al., 2014) provide reference values for prognostic parameters: the onset of turbulence (TO)> 0%, the slope of turbulence (TS) < 2.5 ms/RR, which indicate the increase in the risk of heart failure. It should be noted that these reference values may differ for atrial and ventricular extrasystoles. The HRV Standards (*Electrophysiology*, 1996) state that when processing the ECG signal, "ectopic beats, arrhythmic events, missing data and noise effects… should be removed from the recording. Short-term recordings that are free of ectopia, missing data and noise are preferred." However, the Standards (*Electrophysiology*, 1996) allows for the influence of ectopia on the results. In this case, it is proposed to indicate the relative number and relative duration of RR intervals that were missed and interpolated. Along with this, some authors (Milaras et al., 2023) point out the necessity of taking into account the duration of the compensatory pause, and extrasystole itself, since these areas provide important information about the change in the frequency spectrum of the signal, and, consequently, about the physiological state of the patient. The authors of this article believe that the phenomenon of heart rhythm turbulence HRT occurs at the first contraction of the rhythmic interval, expressed as a premature contraction of the heart, and ends after the normalization of the sinus rhythm. The approximate transition period for heart rhythm stabilization after an extrasystole is about 20 heartbeats.

In this paper, we propose a new approach to study the properties and new quantitative characteristics of the heart rate tachogram signal to obtain diagnostically important information about HRT, not excluding the time segments of extrasystoles. Within the framework of this approach, the tachogram model is a frequency-modulated signal (Bozhokin & Suslova, 2014; Bozhokin et al., 2012, 2020, 2018), where QRS complexes are represented as Gaussian peaks unevenly distributed on the time axis. The uneven distribution of peaks reflects frequency modulation of the signal. The proposed model differs significantly from the generally accepted model of a signal with amplitude modulation (Addison, 2005; Cartas-Rosado et al., 2020; Wang et al., 2021) and leads to different spectral characteristics of the signal. This is especially noticeable in long-term cardiological tests with a strong trend in the RR-interval sequence. The signal is processed in time-frequency domain by means of the continuous wavelet transform CWT (Bozhokin & Suslova, 2014; Bozhokin et al., 2012, 2020, 2018). The proposed model allows us to derive CWT of the signal in the analytical form and to analyze the wavelet spectrum by calculating so-called local frequency $F_{max}(t)$, which corresponds to the maximum CWT for each time point throughout the entire period of the pulse rate registration without any exceptions. It should be noted that in the works (Schmidt et al., 1999; MA, 2004; Sauerbier et al., 2024; Yin et al., 2014), extrasystoles are characterized by only two parameters TO and TS, which do not take into account the extrasystoles themselves and subsequent compensatory pauses, namely RR_{ext} and $RR(0)$ intervals. In the present article, the behavior of local frequency function $F_{max}(t)$ depends significantly on both RR_{ext} and $RR(0)$, since it is calculated for any moment in time.

The change in local frequency over time can be used to solve the problem of identification and classification of extrasystoles, since it is calculated over the entire period of heart rate registration without any exceptions. This approach can be applied to study all types of arrhythmias.

2 MATERIALS AND METHODS

2.1 Tachogram Records Used in This Work

This article is based on the analysis of approximately 70 tachogram records that contain extrasystoles. The records of such tachograms are provided in the book (Shubik & Tikhonenko, 2019). This collection includes a large set of examples of various heart rhythm and conduction disturbances that are most often registered when analyzing Holter monitoring records. In this article, we take a closer look at only two examples of HRT from (Shubik & Tikhonenko, 2019): the tachogram with one extrasystole and that with three extrasystoles.

2.2 Algorithm for Calculating Local Frequency in the Case of a Tachogram with One Extrasystole

To analyze non-stationary heart rate record (HRV) with extrasystoles, we consider $Z(t)$ as a continuous frequency-modulated signal (FMS), which depends on time t, instead of the traditional HRV model as a discrete amplitude-modulated signal (AMS). The signal $Z(t)$ is a set of identical Gaussian peaks with the centers located on an uneven time grid and coinciding with the true moments of heartbeats $t_{n+1} = t_n + RR_n$, where RR_n are the time intervals between heartbeats, $n = 0,1,2, \dots N-1$, $t_0 = 0$, and N is the number of heartbeats. Note that the total number of Gaussian peaks is $N+1$, since the maximum of the first Gaussian peak happens at $t_0 = 0$.

$$Z(t) = \sum_{n=0}^{N} z_n\,(t - t_n), \tag{1}$$

$$z_n(t - t_n) = exp\left[-\frac{(t-t_n)^2}{4\tau_0^2}\right]. \tag{2}$$

For such a model, all Gaussian peaks, separated by different intervals RR_n, have the same unit amplitude and the same width $\tau_0 = 0.02\ s$, equal to the average width of QRS complex.

Let us consider a sequence of $N = 7$ heartbeats, which is a vector RR_A= {996, 981, 934, 401, 1284, 891, 895}, where all RR_n have millisecond dimension (Fig.1).

Among RR_n values, $n = 0,1,2, \dots N-1$, $t_0 = 0$, there is a premature cardiac contraction, preceded by the interval $R_{ext} = 401\ ms$. Thus, we fix the only extrasystole with the beat number $n = 4$ in the series. After a strong non-stationarity $R_{ext} = 401\ ms$, a long compensatory pause $RR(0) = 1284\ ms$ occurs, and $RR(0) \gg RR_{ext}$. Note that real cardiac contractions are located on an uneven grid of discrete times t_n (Bozhokin & Suslova, 2014; Bozhokin et al., 2012, 2020, 2018).

Fig.1 shows frequency-modulated signal $Z(t)$(FMS) related to the tachogram with a single extrasystole $R_{ext} = 401\ ms$ and the time of premature heartbeat $t_{ext} = 3.31\ s$. In this signal, the heart contractions occur at time moments $t_{n+1} = t_n + RR_n$, where RR_n are determined by RR_A.

Fig.1 clearly indicates the difference between frequency modulated signal (FMS) used in this article and amplitude modulated signal (AMS) considered in the Standards (*Electrophysiology*, 1996).

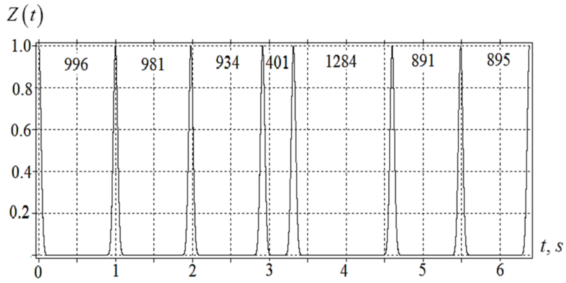

Figure 1: Continuous model of FMS for the tachogram with a single extrasystole $R_{ext} = 401$ ms.

The analytical expression $|V(\nu,t)|$ of the continuous wavelet transform with the Morlet mother wavelet function, where ν is the frequency, measured in *Hz*, and t is the time in *s*, was derived for the irregular system of Gaussian peaks in (Bozhokin & Suslova, 2014; Bozhokin et al., 2012, 2020, 2018).

Fig.2 shows the wavelet spectrum $|V(\nu,t)|$ for the continuous signal $Z(t)$ (Fig.1) in time-frequency domain. We use the Morlet mother wavelet function in the continuous wavelet transform (CWT) because such a wavelet gives the correct positions of the maxima in frequency and time for simplest non-stationary signals.

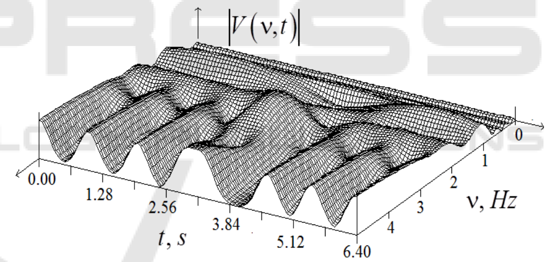

Figure 2: CWT for the system of Gaussian peaks (Fig.1), where ν is measured in *Hz*, and t in *s*.

The frequency corresponding to the maximum value of $|V(\nu,t)|$ is found. This so-called local (instantaneous) frequency $F_{max}(t)$ depends on time. Although the usual sinus rhythm contains both spectral components of low-frequency VLF= (0.015; 0.04 Hz), mid-frequency LF= (0.04;0.15 Hz), and high-frequency HF= (0.15; 0.4 Hz), the calculations of $F_{max}(t)$ in this case do not lead to great difficulties (Bozhokin & Suslova, 2014; Bozhokin et al., 2012, 2020, 2018). However, in the case of extrasystole (Fig.1), the behavior of $|V(\nu,t)|$ (Fig.2) near the extraordinary time moment $t_{ext} = 3.31\ s$ has more complex nature. This entails difficulties in calculating the local frequency $\nu = F_{max}(t)$ near the critical time moment $t_{ext} = 3.31\ s$. To calculate $F_{max}(t)$ for a signal with an ectopic Gaussian peak, we should formulate criteria that will determine upper $F_<(t)$ and lower $F_>(t)$ limits for local frequency $F_{max}(t)$: $F_<(t) \leq F_{max}(t) \leq F_>(t)$. The difficulties in determining the boundaries $F_<(t)$ and $F_>(t)$ are due to the fact that in the model of the signal $Z(t)$ as a set of Gaussians (Fig.1), along with the first frequency harmonic, which gives the maximum $|V(\nu,t)|$, there appear senior harmonics, which also give maxima to $|V(\nu,t)|$. In addition, we have the tachogram with extrasystoles (Fig.1), where time intervals RR_n between the peaks change significantly in time. This leads to a complex system of $|V(\nu,t)|$ maximal values. We are interested in the position of the first harmonic, which determines the basic frequency of the heart rate. Let us formulate an algorithm for finding $F_{max}(t)$ for a signal with ectopic heartbeats. We will consider as ectopic the interval RR_{ext} preceding the moment t_{ext} of the heartbeat if the duration between the peaks of neighboring Gaussians satisfies the condition $RR_{ext} < 0.65\ s$ (Shubik & Tikhonenko, 2019) (Shubik & Tikhonenko, 2019). First, using the known values of RR_n, we calculate the corresponding local frequencies $f_n = \frac{1}{RR_n}$, where the values of f_n have the dimension *Hz*, and the values of RR_n are measured in seconds. Our first assumption is that in the time interval close to the ectopic Gaussian peak, the value $F_{max}(t)$ should be approximately equal to f_n. This physical principle can be formulated in a simple way. If there is a short time interval RR_{ext} between Gaussian peaks, then the first harmonic of such a signal (the sum of identical Gaussian peaks) cannot be greater than $1/_{RR_{ext}}$.

However, if we have a case of several successive ectopic peaks (near the ectopic peak $RR_{ext} = RR_n$ there are other ectopic peaks with small RR_k values), then the behavior of local frequency $F_{max}(t)$ will also depend on the subsequent compensatory pause and neighboring ectopic intervals. Therefore, the algorithm to determine $F_{max}(t)$ at $t \approx t_n$ must depend on the neighboring cardiac intervals, that is, on RR_{n-1},RR_{n-2} and RR_n,RR_{n+1}, and, consequently, on the adjacent moments of time t_{n-1}, t_{n-2} and t_{n+1},t_{n+2} . It becomes especially important when a long compensatory pause followers a short ectopic interval of extrasystole. The proposed algorithm to calculate the local frequency $F_{max}(t)$ should work both for a normal sine rhythm without ectopic intervals, and for a sequence of Gaussian peaks with several ectopic intervals RR_{ext}.

Let us consider the limits f_n^{max} and f_n^{min} for finding discrete local frequencies on n - time interval. In this case, the upper f_n^{max} and lower f_n^{min} boundaries of the discrete local frequencies $f_n = \frac{1}{RR_n}$ can be determined from the relations

$$f_n^{max} = [1 + B(RR_n)]\, f_n, \tag{3}$$

$$f_n^{min} = (1 - A) f_n\ . \tag{4}$$

Note that the upper limit of the frequency search corridor f_n^{max} depends on RR_n measured in seconds. The formula for $B(RR_n)$, which is the relative excess of f_n^{max} over the value of f_n, was derived while analyzing the ectopic intervals given in (Shubik & Tikhonenko, 2019). The lower limit A of the search for $F_{max}(t)$, which characterizes the difference between f_n^{min} and f_n in (4), is a constant value $A = 0.25$.

$$B(RR_n) = B_{min} + \frac{B_{max} - B_{min}}{1 + \exp\left[-\frac{RR_n - RR_{cr}}{\tau_{RR}}\right]}, \tag{5}$$

where RR_n is measured in s, B_{min}=0.001; $B_{\max}$=0.21; RR_{cr}=0.840 s, τ_{RR}=0.12 s.

The given numerical data were obtained from the analysis of approximately 70 tachogram records with various cardiac dysfunctions (Shubik & Tikhonenko, 2019).

For short (extrasystolic) time intervals $RR_n \approx 0.4\ s$, the excess $B(RR_n)$ of the discrete f_n^{max} over f_n has a small value $B(RR_n)$<<1. For long time intervals $RR_n \gg 1.4\ s$, the value of $B(RR_n)$ reaches its asymptotic value $B_{\max} = 0.21$. Thus, at $RR_n \leq RR_{ext}$, the upper search limit f_n^{max} for the continuous function $f_{max}(t)$ slightly exceeds the discrete frequency $f_n = \frac{1}{RR_n}$. For such intervals $f_n^{max} = 1.001 f_n$. For intervals with $RR_n > 1.4\ s$, the discrete upper search boundary tends to $f_n^{max} = 1.21 f_n$. Note that after the ectopic interval $RR_{ext} = RR_n = 0.401\ s$ with number n, we observe long compensatory pause $RR_{n+1} = 1.284\ s$ with number $n+1$. As a result, the ratio of adjacent local frequencies will also be large $\frac{f_n}{f_{n+1}} = \frac{RR_{n+1}}{RR_n} \approx 3.2$. Consequently, the constant A, which determines the lower search limit for the local frequency, should be increased to the value $A = 0.7$.

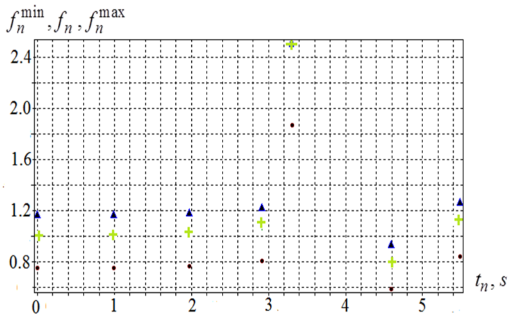

Figure 3: Time dependence of discrete local frequencies for RR_A tachogram: f_n^{min} are indicated by dots, $f_n = \frac{1}{RR_n}$ – by crosses, f_n^{max} – by triangles. For $t = t_{ext} = 3.31\ s$ (the moment of ectopic heartbeat) $f_n^{max} = f_n = \frac{1}{RR_{ext}}$.

Fig.3 shows the discrete local frequencies for the sequence of cardio intervals in the RR_A record.

The next task is to find new smooth boundaries $F_<(t)$ and $F_>(t)$ depending on time continuously in the interval $0 \le t \le T$ (T is the observation period for the signal), based on f_n^{max} and f_n^{min} frequencies specified for discrete n. The required formulas for upper $F_<(t)$ and lower $F_>(t)$ continuous boundaries of the local frequency have the form of sigmoid functions

$$F_>(t) = f_0^{max} + \sum_{n=1}^{N-1} \frac{f_n^{max} - f_{n-1}^{max}}{1+\exp\left(-\frac{t-t_n^C}{\tau_n}\right)}, \quad (6)$$

$$F_<(t) = f_0^{min} + \sum_{n=1}^{N-1} \frac{f_n^{min} - f_{n-1}^{min}}{1+\exp\left(-\frac{t-t_n^C}{\tau_n}\right)}, \quad (7)$$

where $t_n^C = \frac{(t_n + t_{n-1})}{2}$ is the center of the interval; $\tau_n = \frac{(t_n - t_{n-1})}{10} = RR_{n-1}/10$ is the characteristic time depending on the duration of the interval between heartbeats. If the time interval ends with extrasystole $RR_{ext} = 0.401\ s$, then the characteristic time τ_n will be small $\tau_n \approx 0.04\ s$. If the time interval ends with compensatory pause $RR_n = 1.284\ s$, then the transition region will be large $\tau_n \approx 0.13\ s$.

Approximation by smooth sigmoid functions has an advantage over approximations using splines. In the case of strong heterogeneity in the position of discrete points, spline approximation leads to strong oscillation of a smooth curve for intermediate values of the argument.

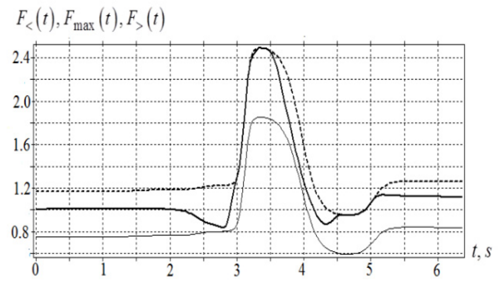

Figure 4: Dependence on time of the lower limit of frequency search $F_<(t)$ (thin line), the sought local frequency $F_{max}(t)$ (thick line), and the upper limit of frequency search $F_>(t)$ (dashed line) for tachogram RR_A

The final step is to determine $F_{max}(t)$ by finding the maximum of $|V(\nu, t)|$ (CWT of frequency modulated signal $Z(t)$). Fig.4 shows the frequency corridor $F_<(t) \le F_{max}(t) \le F_>(t)$ for local (instantaneous) frequency $F_{max}(t)$ in the case of a single extrasystole $RR_{ext} = 0.401\ s$ with compensatory pause $RR(0) = 1284\ ms$.

2.3 Algorithm for Calculating Local Frequency in the Case of a Tachogram with Three Extrasystoles

Let us apply the algorithm to determine $F_{max}(t)$ in the case of tachograms with multiple ectopic intervals. As an example, we consider the tachogram record $RR_B = \{1072,\ 544,\ 1056,\ 552,\ 1124,\ 548,\ 1129\}$ with three extrasystoles: $RR_{ext}(1) = 544\ ms$, $RR_{ext}(2) = 552\ ms$, $RR_{ext}(3) = 548\ ms$ given in (Shubik & Tikhonenko, 2019). Each of the extrasystoles is followed by the compensatory pause.

The processing sequence of such a tachogram RR_B is similar to the example of the tachogram RR_A discussed above. The analysis of $|V(\nu, t)|$ (CWT for the tachogram RR_B) shows the existence of three vertices $i = 3$ located at points with specific fixed frequencies and times $\{\nu_{ext}(i), t_{ext}(i)\}$, measured in Hz and s, respectively. These vertices relate to the extrasystoles with the characteristic frequency $\nu_{ext}(i) = \frac{1}{RR_{ext}(i)}$.

Here we have $\{\nu_{ext}(1) = 1.838\ Hz,\ t_{ext}(1) = 1.616\ s\}$; $\{\nu_{ext}(2) = 1.811\ Hz,\ t_{ext}(2) = 3.224\ s\}$; $\{\nu_{ext}(3) = 1.825\ Hz,\ t_{ext}(3) = 4.896\ s\}$.

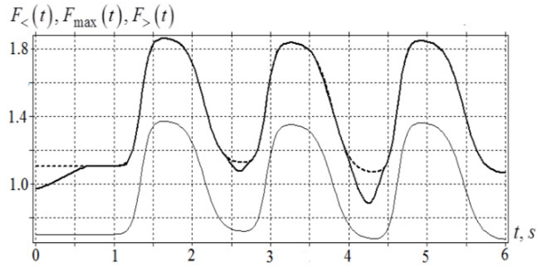

Figure 5: Dependence on time of the lower limit of frequency search $F_<(t)$ (thin line), the sought local frequency $F_{max}(t)$ (thick line), and the upper limit of frequency search $F_>(t)$ (dashed line) for tachogram RR_B.

The analysis of Fig.5 reveals a sharp increase in the sought local frequency $F_{max}(t)$ at the moments of extrasystoles for the RR_B tachogram with three extrasystoles, and then a decrease during the subsequent compensatory pauses. Three maxima of the continuous function $F_{max}(t)$ exactly correspond to the discrete values $\{\nu_{ext}(i), t_{ext}(i)\}$. Based on the processing of the data taken from (Shubik & Tikhonenko, 2019), we can conclude that each individual extrasystole, characterized by its own interval and compensatory pause, differs from another extrasystole in the duration and the range of oscillations of the corresponding local frequency function. Thus, the time behavior of local frequency near extrasystoles is closely related to the properties of extrasystoles. This fact can serve as a basis for classifying ectopic beats.

3 DISCUSSION

For many functional tests (bicycle ergometry, treadmill, orthostatic, respiratory, glucose-tolerant, pharmacological, and psychoemotional tests) the quantitative description of non-stationary heart rate variability (HRV) is of great importance. The non-stationary nature of HRV is reflected in the significant dependence of spectral and statistical properties of the processed signals on time. It is especially difficult to process HRV signals with extrasystoles - premature contractions of the heart, in which the local (instantaneous) frequency of the signal changes by 3-4 times over a time interval of $RR_{ext} \approx 0.5\ s$. As a basis for the analysis of HRV with extrasystoles, this paper uses frequency-modulated signal model (FMS) instead of the traditional amplitude-modulated signal model (AMS). AMS tachogram model (*Electrophysiology*, 1996) assumes that peaks of different heights RR_n are uniformly located at a time grid, and separated by equal time intervals $\Delta t = RRNN$, where the $RRNN$ value is the average duration of RR_n intervals over the entire observation period. Frequency modulated signal (FMS) is a set of identical Gaussian peaks whose centers are located on an uneven time grid and coincide in time with the true moments of heartbeats $t_{n+1} = t_n + RR_n,\ n = 0,1,2, \dots N-1$, N is the number of heartbeats. In contrast to AMS model, FMS model used in the article makes it possible to find the true frequencies of heart rate oscillations. The differences between traditional AMS model and FMS model are especially noticeable when analyzing functional tests, in which the tachogram trend is clearly visible over the entire testing period

In this paper, we propose to analyze the time behavior of local (instantaneous) frequency $F_{max}(t)$ as a new characteristic of the heart rhythm with strong non-stationarity (arrhythmia, extrasystole). The behavior in time of the continuous signal $F_{max}(t)$ shows both the presence of extrasystoles and their difference from each other.

An essential advantage of this approach is in the fact that FMS model allows us to obtain an analytical expression for the continuous wavelet transform (CWT) with the Morlet mother wavelet function. The analytical expression for the wavelet spectrum allows for the efficient calculation and analysis of the local frequency function.

The advantage of considering local frequency as a new characteristic of the tachogram becomes relevant when studying various cardiac arrhythmias. In this case the normal sinus rhythm is disrupted and the true moments of cardiac contractions become important. An important application of the method proposed in this article is the study of single and repeated extrasystoles associated with the appearance of an ectopic focus of trigger activity, as well as with the existence of a repeated reverse excitation entry (re-entry mechanism). The behavior of local frequency function over time is different for different types of ectopic beats. Therefore, this characteristic can serve as a basis for classifying arrhythmias. The main prognostic parameters for HRT are: TO - the onset of extrasystole and TS - the extrasystole slope, which do not take into account the extrasystoles themselves and subsequent compensatory pauses, namely RR_{ext} and $RR(0)$ intervals. We believe that the study of HRT should include the intervals of the extrasystole and the compensatory intervals, as they contain important information about the characteristic features of the rhythm disturbance.

4 CONCLUSIONS

The article proposes a mathematical model in which the tachogram signal is considered as FMS (frequency-modulated signal), which is a superposition of identical Gaussian peaks. The maxima of the Gaussian peaks are located at the moments of real heart contractions. To study HRT (heart rate turbulence), both the durations of the ectopic intervals between the peaks RR_{ext} and the duration of the subsequent compensatory pauses $RR(0)$ are taken into account.

For quantitative analysis of HRT, we propose to calculate the time behavior of local frequency $F_{max}(t)$ at any time both before and after ectopic beats. This allows us to classify the ectopic intervals and subsequent compensatory pauses. In the change of $F_{max}(t)$, one can identify a trend, as well as fluctuations relative to this trend.

The proposed method based on the analysis of local heart rate can be applied to study non-stationary cardiac tachograms of various patients with normal sinus rhythm both at rest and during functional tests. We can also propose to use the developed method for classification of cardiac rhythm with ectopic intervals for patients suffering from congestive heart failure (CHF), atrial fibrillation (AF), atrial premature contraction (APC), ventricular premature contraction (VPC), left bundle branch block (LBBB), ischemic/dilated cardiomyopathy (ISCH) and sick sinus syndrome (SSS).

ACKNOWLEDGEMENTS

The research is funded by the Ministry of Science and Higher Education of the Russian Federation as part of the World-class Research Center program: Advanced Digital Technologies (contract No. 075-15-2022-311 dated 20.04.2022)

REFERENCES

Schmidt, G., Malik, M., Barthel, P., Schneider, R., Ulm, K., Rolnitzky, L., ... & Schömig, A. (1999). Heart-rate turbulence after ventricular premature beats as a predictor of mortality after acute myocardial infarction. *The Lancet*, *353*(9162), 1390-1396.

Bauer, A., Malik, M., Schmidt, G., Barthel, P., Bonnemeier, H., Cygankiewicz, I., ... & Zareba, W. (2008). Heart rate turbulence: standards of measurement, physiological interpretation, and clinical use: International Society for Holter and Noninvasive Electrophysiology Consensus. *Journal of the American College of Cardiology*, *52*(17), 1353-1365.

Disertori, M., Masè, M., Rigoni, M., Nollo, G., & Ravelli, F. (2016). Heart rate turbulence is a powerful predictor of cardiac death and ventricular arrhythmias in postmyocardial infarction and heart failure patients: a systematic review and meta-analysis. *Circulation: Arrhythmia and Electrophysiology*, *9*(12), e004610.

MA, W. (2004). Heart rate turbulence: a 5-year review. *Heart Rhythm*, *1*, 732-738.

Cygankiewicz, I., & Zarȩba, W. (2006). Heart rate turbulence-an overview of methods and applications. *Cardiology Journal*, *13*(5), 359-368.

Cygankiewicz, I. (2013). Heart rate turbulence. *Progress in cardiovascular diseases*, *56*(2), 160-171.

Germanova, O., Shchukin, Y., Germanov, V., Galati, G., & Germanov, A. (2021). Extrasystolic arrhythmia: is it an additional risk factor of atherosclerosis? *Minerva Cardiology and Angiology*, *70*(1), 32-39.

Huikuri, H. V., Castellanos, A., & Myerburg, R. J. (2001). Sudden death due to cardiac arrhythmias. *New England Journal of Medicine*, *345*(20), 1473-1482.

Zeid, S., Buch, G., Velmeden, D., Söhne, J., Schulz, A., Schuch, A., ... & Wild, P. S. (2024). Heart rate variability: Reference values and role for clinical profile and mortality in individuals with heart failure. *Clinical Research in Cardiology*, *113*(9), 1317-1330.

Thayer, J. F., Yamamoto, S. S., & Brosschot, J. F. (2010). The relationship of autonomic imbalance, heart rate variability and cardiovascular disease risk factors. *International journal of cardiology*, *141*(2), 122-131.

Kubota, Y., Chen, L. Y., Whitsel, E. A., & Folsom, A. R. (2017). Heart rate variability and lifetime risk of cardiovascular disease: the Atherosclerosis Risk in Communities Study. *Annals of epidemiology*, *27*(10), 619-625.

Huikuri, H. V., & Stein, P. K. (2013). Heart rate variability in risk stratification of cardiac patients. *Progress in cardiovascular diseases*, *56*(2), 153-159.

Turcu, A. M., Ilie, A. C., Ștefăniu, R., Ţăranu, S. M., Sandu, I. A., Alexa-Stratulat, T., ... & Alexa, I. D. (2023). The impact of heart rate variability monitoring on preventing severe cardiovascular events. *Diagnostics*, *13*(14), 2382.

Yan, S. P., Song, X., Wei, L., Gong, Y. S., Hu, H. Y., & Li, Y. Q. (2023). Performance of heart rate adjusted heart rate variability for risk stratification of sudden cardiac death. *BMC Cardiovascular Disorders*, *23*(1), 144.

Lombardi, F., & Stein, P. K. (2011). Origin of heart rate variability and turbulence: an appraisal of autonomic modulation of cardiovascular function. *Frontiers in Physiology*, *2*, 95.

Blesius, V., Schölzel, C., Ernst, G., & Dominik, A. (2020). HRT assessment reviewed: a systematic review of heart rate turbulence methodology. *Physiological Measurement*, *41*(8), 08TR01.

Rajendra Acharya, U., Paul Joseph, K., Kannathal, N., Lim, C. M., & Suri, J. S. (2006). Heart rate variability: a review. *Medical and biological engineering and computing*, *44*, 1031-1051.

Sauerbier, F., Haerting, J., Sedding, D., Mikolajczyk, R., Werdan, K., Nuding, S., ... & Kluttig, A. (2024). Impact of QRS misclassifications on heart-rate-variability parameters (results from the CARLA cohort study). *PloS one*, *19*(6), e0304893.

Yin, D. C., Wang, Z. J., Guo, S., Xie, H. Y., Sun, L., Feng, W., ... & Qu, X. F. (2014). Prognostic significance of heart rate turbulence parameters in patients with chronic heart failure. *BMC cardiovascular disorders*, *14*, 1-8.

Koyama, J., Watanabe, J., Yamada, A., Koseki, Y., Konno, Y., Toda, S., ... & Shirato, K. (2002). Evaluation of heart-rate turbulence as a new prognostic marker in patients with chronic heart failure. *Circulation journal*, *66*(10), 902-907.

Tsvetnikova, A.A., Bernhard,t E.R., Parmon, E.V., Aseev, A.V., Treshkur, T.V.. (2008). Turbulentnost' serdechnogo ritma: metodicheskie aspekty *(Inkart, SPb.) (in Russian*).

Electrophysiology, T. F. O. T. E. S. O. C. T. N. A. S. O. P. (1996). Heart rate variability: standards of measurement, physiological interpretation, and clinical use. *Circulation*, *93*(5), 1043-1065.

Milaras, N., Dourvas, P., Doundoulakis, I., Sotiriou, Z., Nevras, V., Xintarakou, A., ... & Gatzoulis, K. (2023). Noninvasive electrocardiographic risk factors for sudden cardiac death in dilated cardiomyopathy: is ambulatory electrocardiography still relevant? *Heart Failure Reviews*, *28*(4), 865-878.

Bozhokin, S. V., Suslova, I. B. (2014). Analysis of non-stationary HRV as a frequency modulated signal by double continuous wavelet transformation method. *Biomedical Signal Processing and Control*, *10*, 34-40.

Bozhokin, S. V., Lesova, E. M., Samoilov, V. O., Tolkachev, P. I., (2012) Wavelet Analysis of Nonstationary Heart Rate Variability in a Head-Up Tilt-Table Test, Biophysics, 57(4), 530-543.

Bozhokin, S. V., Lesova, E. M., Samoilov, V. O., & Barantsev, K. A. (2020). Nonstationary heart rate variability during the head-down tilt test. *Biophysics*, *65*, 151-158.

Bozhokin S. V., Lesova E. M., Samoilov V. O., Tarakanov D. E. (2018) Nonstationary heart rate variability in respiratory tests, Human Physiology, 44(1), 32-40.

Addison, P. S. (2005). Wavelet transforms and the ECG: a review. *Physiological measurement*, *26*(5), R155.

Cartas-Rosado, R., Becerra-Luna, B., Martínez-Memije, R., Infante-Vazquez, O., Lerma, C., Perez-Grovas, H., & Rodríguez-Chagolla, J. M. (2020). Continuous wavelet transform based processing for estimating the power spectrum content of heart rate variability during hemodiafiltration. *Biomedical Signal Processing and Control*, *62*, 102031.

Wang, T., Lu, C., Sun, Y., Yang, M., Liu, C., & Ou, C. (2021). Automatic ECG classification using continuous wavelet transform and convolutional neural network. *Entropy*, *23*(1), 119.

Shubik, Yu.V., Tikhonenko, V.M. (2019). Holter monitoring for arrhythmia. Kholterovskoe monitorirovanie pri aritmiyakh. OOO"Nevskij rakurs", Saint-Petersburg,(in Russian).