From Coefficients to 3D Planes: A VR Mathematics Module for Teaching Systems of Linear Equations with 3D Interaction

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Abstract: Traditional approaches to teaching systems of linear equations frequently fail to connect abstract algebraic principles with students' spatial reasoning, resulting in recurring errors in solution determination. To address this, we developed a virtual reality (VR) application for Meta Quest headsets that enables immersive interaction with 3D graphical representations of systems of linear equations. The application allows users to manipulate equations as interactive planes in virtual space, adjust coefficients and visualize intersections in real time. In a study of 38 students, participants were tasked with identifying valid solutions (empty set, point, line, plane) in a paper-based exercise. Results indicated that students frequently misidentified solutions, selecting incorrect options such as two lines, three lines, several points, lines, and two triangles. Furthermore, correct solutions were not consistently identified (empty set: 60%, point: 44%, line: 51%, plane 68%). After a VR session, all students correctly recognized proper solution types (empty set, point, line, plane) in post-test scenarios, demonstrating improved conceptual mastery. Qualitative feedback highlighted heightened engagement and reduced cognitive load during VR tasks. These findings suggest that VR presentation can significantly improve algebraic problem-solving skills compared to static, paper-based methods. This study underscores the potential of immersive technologies to transform mathematics education while emphasizing the need for teacher training to integrate such tools effectively.

1 INTRODUCTION

The evolution of educational methodologies has historically mirrored technological progress. For centuries, teaching relied on oral traditions and static texts, but the 20th century introduced audiovisual tools, followed by digital platforms, each reshaping pedagogical strategies. Today, the acceleration of technologies such as artificial intelligence (AI), augmented reality (AR), and virtual reality (VR) has ushered in a paradigm shift, demanding yet another transformation in how educators teach and students learn. While these tools promise immersive, personalized learning experiences, they also create significant challenges for teachers – many of whom were trained in didactic approaches rooted in pre-digital eras (Ertmer and Ottenbreit-Leftwich, 2010), (Bieniecki et al., 2010).

Modern students, as digital natives, increasingly inhabit virtual worlds, rendering traditional methods like textbooks and lectures less engaging. For mathematics education, this disconnect is particularly acute. Abstract concepts such as systems of linear equations, often taught through paper, struggle to resonate with learners who thrive on interactivity and instant feedback (Makransky and Petersen, 2021). Simultaneously, teachers face a dual burden: mastering emerging technologies while reconciling them with pedagogical frameworks from their own training, which may lack emphasis on digital tools (Voogt et al., 2013). This tension underscores a critical responsibility for researchers – to design evidence-based techno-

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Figure 1: User wearing VR glasses and working on the given system of linear equations in the application.

logical solutions that empower educators, rather than replace them, by aligning innovation with curricular goals and classroom realities.

Virtual reality (VR) exemplifies this potential. By immersing learners in 3D environments (Dalgarno and Lee, 2010) where they can spatially manipulate equations, VR bridges the gap between abstract mathematics and tangible experience, aligning with embodied cognition theory (Johnson-Glenberg, 2017). However, its adoption hinges on addressing educators' practical concerns, such as usability, curriculum integration, and training demands (Nicholson et al., 2022). Without tools that are both pedagogically sound and teacher-friendly, even the most advanced technologies risk exacerbating, rather than alleviating, educational inequities.

This study focuses on developing and evaluating a VR application for teaching systems of linear equations – a topic foundational to algebra yet persistently challenging due to its reliance on spatial reasoning and multi-step problem-solving. By collaborating with educators during the design process, we aim to create a tool that not only enhances student engagement and understanding but also supports teachers in transitioning from traditional methods to technology-aided instruction. Our work seeks to answer following question:

- Will students, based on their knowledge of the geometric interpretation of systems of linear equations with two unknowns, be able to present and geometrically interpret solutions to systems of three linear equations with three unknowns?

The remainder of this article is organized as follows. First we synthesize existing research on VR in mathematics education, focusing on theoretical frameworks like embodied cognition and constructivist learning. The next section details the technical implementation of our VR tool and System of Linear Equations in VR introduces mathematical background in terms of VR application, emphasizing its interactive 3D graphing system and gamified problem-solving tasks. The study design then describes the methodology, including participant selection, experimental procedures, and assessment metrics. The Results section presents quantitative and qualitative findings on learning gains and engagement, followed by a Discussion interpreting these outcomes in light of pedagogical challenges and opportunities. Finally, we summarize the implications for future research and practical applications of VR in algebra education.

2 LITERATURE REVIEW

Several recent studies have explored the integration of Virtual Reality (VR) in mathematics education at universities, highlighting its potential to improve learning outcomes and student engagement. Mathematics often involves abstract concepts that can be challenging for students to grasp. VR allows learners to visualize and manipulate these concepts in a three-dimensional space, making it easier to understand complex topics such as geometry, measurement, and spatial relationships. This hands-on experience can significantly enhance comprehension and retention.

The (López and Garcia, 2023) study evaluates the impact of VR on mathematics education in a rural secondary school in Antiguo Morelos, Mexico. It highlights how VR enhances student motivation and academic performance, particularly in understanding geometric concepts such as perimeters and areas. The study employs a quasi-experimental design comparing an experimental group using VR with a control group receiving traditional instruction. The results indicate a significant improvement in the experimental group's performance, suggesting that VR can effectively bridge educational gaps, especially in resource-limited settings. The implementation of VR technology necessitates training for educators, which can lead to improved teaching practices overall. As teachers become more proficient with technology integration, they can improve their instructional methods and better support student learning outcomes.

In (Li, 2024) the author examines the potential of XR technologies, including VR, to transform mathematics assessments. They discuss how immersive environments can enhance the assessment process by making mathematical concepts more accessible and engaging. The paper outlines challenges to XR adoption in educational settings and proposes a research agenda for further exploration of its benefits in mathematics education.

The study of (Awang et al., 2016) reviews the integration of AI alongside AR and VR technologies in mathematics education, emphasizing their role in enhancing student motivation and problem-solving abilities. It presents thematic analyses from recent studies that illustrate how these technologies can reduce math anxiety and improve learning outcomes. The findings indicate a growing interest in using immersive technologies to create more effective educational experiences.

In (Quiñones et al., 2016) the authors evaluate students' experiences using a virtual reality (VR) tool and their learning of three-dimensional vectors in an introductory physics university course. The researchers used an experimental research design, with control and experimental groups, measuring student performance using a pre-post 3D vectors questionnaire. They found that students in the experimental group outperformed the control group on items in which visualization was important. The students perceived the VR tool as having a positive impact on their learning and as a valuable tool to enhance their learning experience.

In turn, in (Du and Li, 2022), the authors analyse the application of VR technology in higher education, selecting 80 empirical studies from the Web of Science database. The review finds that VR application research mainly focuses on undergraduates in science, engineering, and medical-related majors. Head-mounted devices are the primary VR equipment. Most studies indicate that VR has positive effects on higher education and teaching, primarily by influencing student behaviour and, secondarily, cognition and emotional attitudes. The authors suggest that VR is mainly used in higher operability courses and that researchers assess teaching effectiveness mainly through objective tests, subjective questionnaires, and descriptive/variance analyses.

The work of (Quiñones et al., 2016) and (Du and Li, 2022) contributes to the ongoing discourse on the integration of VR in higher mathematics education, showcasing its potential to transform teaching practices and enhance student learning outcomes.

The use of VR in mathematics education offers numerous advantages that can enhance student engagement, understanding, and overall academic performance. As this technology continues to evolve, it has the potential to transform educational practices in underserved areas, making high-quality mathematics education more accessible to all students.

3 VR APPLICATION DESIGN

The application, developed for Meta Quest 2/3 headsets (Android .apk), was built using Unity 2021.3.16 with the XR Interaction Toolkit to ensure cross-platform functionality. A modular architecture was adopted, comprising 13 distinct mathematical subjects, though this study focuses exclusively on the *systems of linear equations* module.

3.1 Technical Implementation

Custom 3D models, including dynamic coordinate grids and equation planes, were designed in Blender and integrated into Unity. Real-time interaction was enabled through a combination of Meta's native hand tracking API and controller-based input. Hand recognition allows users to manipulate variables via intuitive gestures (e.g., pinching to "grab" coefficients, swiping to rotate planes), while controller support ensures accessibility for users less familiar with gesture controls.

To mitigate motion sickness, comfort features such as snap-turning (45°increments) and teleportation-based locomotion were implemented. Users can physically walk in room-scale environments or navigate virtually in seated mode. Performance optimization was prioritized through baked light maps, reducing computational load while maintaining visual fidelity.

3.2 Module Overview: Systems of Linear Equations

The module enables users to plot and manipulate systems of two or three linear equations in 3D space. Equations are represented as interactive planes, with sliders for adjusting coefficients and constants. Real-time visual feedback, powered by custom shaders, highlights intersections between planes using colour gradients. Haptic vibrations confirm correct solutions, while incorrect attempts prompt contextual hints, such as textual cues (e.g. *"These planes are parallel – no solution exists!"*) or visual markers indicating inconsistent coefficients.

A hybrid interface accommodates diverse interaction preferences: hand tracking for direct manipulation of equation components and controllers for precision tasks like numerical input. The UI includes an undo/redo system to encourage experimentation without penalty.

4 SYSTEMS OF LINEAR EQUATIONS IN VR

4.1 Justification for the Choice of Topic

In mathematics, a system of linear equations (or linear system) refers to a set of one or more linear equations that involve the same set of variables. This concept forms the foundation of linear algebra, a discipline that underpins much of modern mathematics and finds applications in numerous fields. The development of computational algorithms to solve these systems is a cornerstone of numerical linear algebra, with critical implications for disciplines such as engineering, physics, chemistry, computer science, and economics. Furthermore, systems of non-linear equations can often be approximated by linear systems through a process known as linearization. This technique is particularly valuable when developing mathematical models or computer simulations for analysing complex systems.

Given the prevalence of linear systems across various scientific disciplines, many students encounter the challenge of solving or interpreting such systems during their education. Since linear equations possess a straightforward geometrical interpretation, VR applications offer a promising avenue for enhancing the learning experience. By leveraging VR technology, students can engage with linear systems in an immersive and interactive manner, facilitating a deeper understanding of the underlying concepts.

4.2 Learning Goals and Outcomes

When solving a system of equations, students must address three fundamental questions:

1. Does the system have a solution ?
2. How many solutions exist ?
3. What are the solutions ?

To answer these, a student needs a solid understanding of concepts like rank and reduced row echelon form, as well as the ability to interpret calculations and analyse the system's geometric properties.

The VR application provides an interactive way to explore these aspects. It enables students to visualize the system graphically, adjust coefficients, and observe how changes affect its geometry. Through this, students can explore the relationship between matrix rank, the number of solutions, and the system's geometric interpretation. This hands-on approach helps students build intuition for tackling larger systems with more variables, where direct geometric visualization is impossible.

4.2.1 Example 1

System of equations

$$\begin{cases} -25x+19y=-3 \\ x-2y=0 \end{cases} \tag{1}$$

is represented by two intersecting lines. The solution of the system, which may be obtained from the augmented matrix in its reduced row echelon form

$$\begin{bmatrix} -25 & 19 & -3 \\ 1 & -2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & \frac{6}{31} \\ 0 & 1 & \frac{3}{31} \end{bmatrix} \tag{2}$$

is the point of intersection $(x,y) = \left(\frac{6}{31}, \frac{3}{31}\right)$.

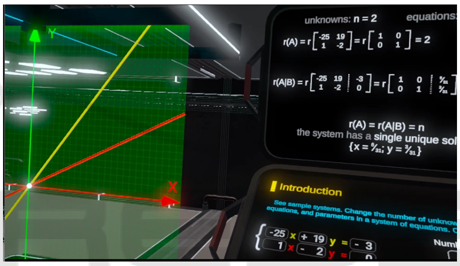

Figure 2: Example 1. Snapshot from VR application.

4.2.2 Example 2

System of equations

$$\begin{cases} 7x=-9 \\ x-3y=7 \\ -3x-5y=16 \end{cases} \tag{3}$$

is represented by three lines that do not intersect at one point. The system is inconsistent and has no solution, which can be concluded through the Kronecker-Capelli theorem, since $r(A) \neq r([A|B])$ in this case.

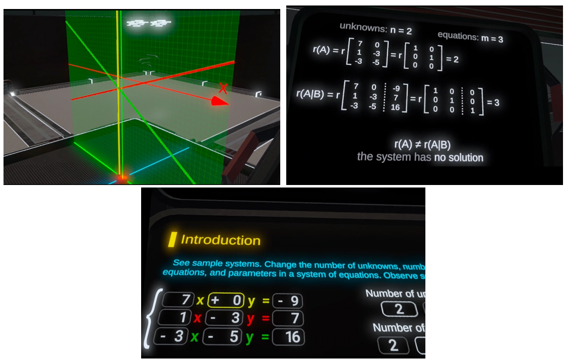

Figure 3: Example 2. Snapshots from VR application.

4.2.3 Example 3

System of equations

$$\begin{cases} -6x - 19y + 4z = -13 \\ -5x + 16z = 20 \\ 19x - 15y + 6z = 14 \end{cases} \tag{4}$$

is represented by three planes that intersect at one common point. The solution may be obtained from the reduced row echelon form of the augmented matrix.

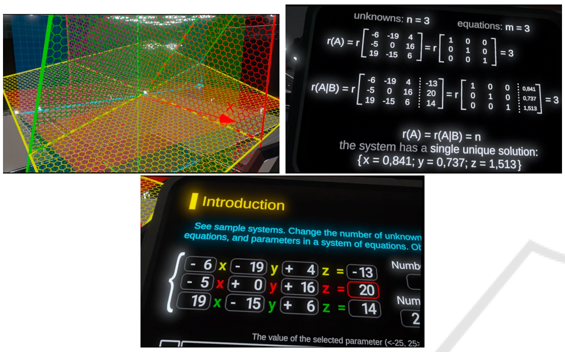

Figure 4: Example 3. Snapshots from VR application.

4.3 Some Practical Applications

We must remember that an important element of the process of acquiring mathematical knowledge during engineering studies is learning the applications of the concepts discussed. Students of various fields of science will encounter system of equations in their studies.

In economy, problems connected to cost and profit calculations appear throughout all levels of education. Even primary school students may be faced with solving problem like this one:

"The cost of a ticket to the amusement park is 25€ for children and 50€ for adults. On a certain day, attendance at the park is 2 000 and the total gate revenue is 70 000€. How many children and how many adults bought tickets?"

Quick analysis of the problem leads us to a system of two equations

$$\begin{cases} x + y = 2000 \\ 25x + 50y = 70000 \end{cases} \tag{5}$$

where x denotes the number of children and y denotes the number of adults.

Students of chemistry very often have to work out problems of balancing chemical equations, for example

$$N_2 + H_2 \rightarrow NH_3 \tag{6}$$

A student must find the quantities of molecules of nitrogen and hydrogen on the left side of the reaction that would produce a certain proportional amount of the compound on the right side of the reaction. Simple analysis of the problem leads to a system of equations that is represented by the augmented matrix

$$\begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & -3 & 0 \end{bmatrix} \tag{7}$$

The solution of the system yields a balanced equation

$$N_2 + 3H_2 \rightarrow 2NH_3 \tag{8}$$

In physics, a typical problem that leads to a system of equation is determining unknown values in a given circuit.

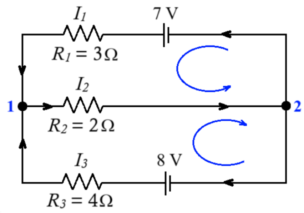

Figure 5: Example of electrical circuit that yields a system of equations.

By applying Kirchhoff's laws, one creates a system of linear equations that allow to find the unknown values, such as currents, voltages, or resistances. For example, the circuit shown in Figure 5 yields the system of equations

$$\begin{cases} I_1 - I_2 + I_3 = 0 \\ 3I_1 + 2I_2 = 7 \\ 2I_2 + 4I_3 = 8 \end{cases} \tag{9}$$

where the currents I_1, I_2, I_3 are the unknown values.

5 STUDY DESIGN

5.1 Study Group

The study participants were first-year Electrical Engineering students at the Lodz University of Technology. The research was conducted in January 2025, with a total of 38 respondents.

5.2 Course of the Study

A month before the start of the study, first-year electrical engineering students were taught the basics of analytical geometry in three-dimensional space. As part of these classes, they learned the equation of the plane, $Ax + By + Cz = D$, and its properties, as well as vectors and operations on them. The aim of these classes was to introduce students to 3D geometry and prepare them for further mathematics classes.

On the day of the study, the teacher gave an hour-long lecture on solving systems of linear equations with two unknowns. The first part of the lecture recalled the methods of solving systems of equations (substitution, Gaussian elimination), and the second part focused on the geometric interpretation of the solution. The teacher explained that the solution of a system of two linear equations with two unknowns can be the intersection point of two lines in the plane, the empty set – parallel lines that do not overlap, and lines that overlap. Students were encouraged to ask questions and actively participate in the discussion.

The following research question was posed: Will students, based on their knowledge of the geometric interpretation of systems of linear equations with two unknowns, be able to present and geometrically interpret solutions to systems of three linear equations with three unknowns?

Research hypothesis: Students, based on their knowledge of the geometric interpretation of systems of linear equations with two unknowns, will have difficulties with the full and correct presentation and geometric interpretation of solutions to systems of three linear equations with three unknowns.

In particular, they will not be able to correctly present three planes and their relative position in 3D space. They will not be able to recognize different types of solutions in terms of geometric interpretation.

5.3 Results

In order to investigate whether students are able to geometrically interpret solutions to a system of three linear equations with three unknowns based on their knowledge of the interpretation of systems of equations with two unknowns, a survey was conducted. Students were presented with two drawings:

1. A drawing of a single plane in a three-dimensional coordinate system (see Figure 6).
2. A drawing of two planes in a three-dimensional coordinate system, with a marked line of their intersection (see Figure 7).

Then the students were asked the following question: List all possible geometric solutions to a system of three planes in three-dimensional space. Describe what geometric figures could be solutions to such a system. In addition to correct answers, summarized in Table 1, students were also giving incorrect ones:

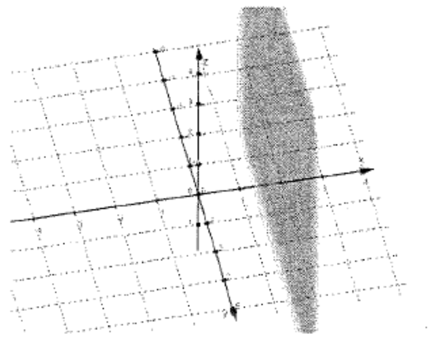

Figure 6: The first drawing shown to students during study.

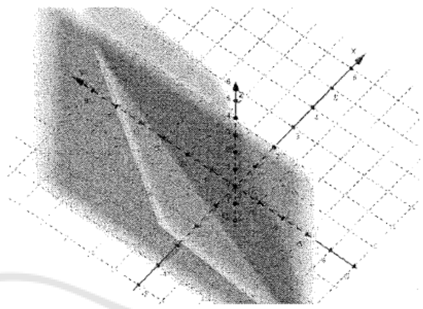

Figure 7: The second drawing shown to students during study.

Table 1: Results of the study.

Possible solution	Percentage of students who gave the answer
Empty set	60%
Point	44%
Line	51%
Plane	68%

two lines, three lines,several points, lines, two triangles.

After engaging with the VR simulation of solving systems of linear equations with three unknowns, students correctly identified all possible solutions.

5.4 Discussion

As many as 60% of students indicated that the solution to a system of three planes can be the empty set. This suggests that students have a good understanding that three planes can have no common point, which corresponds to the lack of solutions to a system of equations.

Only 44% of students correctly identified a point as a possible solution. This suggests that although students may understand the concept of no solutions, they may have difficulty visualizing a situation in which three planes intersect at a single point, which is the solution to the system.

51% of students indicated a line as a possible solution. This suggests that students have a good understanding that three planes can intersect along a line, which corresponds to an infinite number of solutions to a system of equations.

As many as 68% of students identified a plane as a possible solution. This suggests that students may understand situation with plane as a solution, which has infinite number of solutions.

These results suggest that students have difficulty understanding the geometric interpretation of systems of linear equations in three unknowns. They are good at understanding the concept of no solutions and have a reasonable understanding that a solution can be a line. However, they have considerable difficulty visualizing a point as a solution and distinguishing between a plane as a member of the system and a plane as a solution.

Thus, developing and utilizing a VR application for solving systems of linear equations in three-dimensional space is a logical and effective approach.

6 CONCLUSIONS

This study highlights the potential of VR applications in enhancing the understanding of systems of linear equations, particularly in three-dimensional space. Our findings suggest that while students can grasp certain geometric interpretations, such as the absence of solutions or solutions as plane, they struggle with visualizing points and lines as solutions.

The VR application proved effective in addressing these challenges, allowing students to interact with linear systems dynamically and gain a deeper conceptual understanding. By engaging with geometric representations in an immersive environment, students were able to correctly identify all possible solutions after using the application.

These results suggest that VR can be a valuable tool for algebra education, particularly in topics requiring spatial reasoning. Future research should explore its effectiveness across different student populations and mathematical concepts, as well as investigate how VR-based learning compares to traditional instructional methods in the long term.

6.1 Limitations

While the application currently lacks multiplayer functionality, its single-user focus aligns with the study's goal of individualized skill assessment. Future iterations could incorporate collaborative features for group problem-solving scenarios.

We recognize the limitations of our data and study design. Firstly, the sample was limited to a single field of study and one university. Conducting several experiments at different universities and taking into account students from different majors is considered to be done in the future. Secondly, we believe that the participants' motivation to fully engage in the study was moderate.

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